Identifying and Addressing

Conceptual Difficulties

In Electronics

Operational Amplifier and Diode circuits

C. P. Papanikolaou

to my family

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Chapter 1:
Introduction

Until very recently, physics education researchers have focused relatively little attention on the upper-division and advanced laboratory courses that are ubiquitous in undergraduate physics programs across the nation and abroad.[1] In particular, the analog (or hybrid analog and digital) electronics courses typically offered by physics departments have remained largely unstudied. Thus, while there is an extensive body of research on student understanding of introductory electric circuits, both at the precollege and undergraduate levels, very little research exists that can be used to guide instruction in upper-division analog electronics.

There are many important learning goals associated with upper-division laboratory instruction, including the development of: experimental design abilities; proficiency in the areas of measurement and uncertainty; troubleshooting expertise; data analysis skills; familiarity with new tools, devices, and experimental techniques; and a deeper understanding of physics content. While it is often the case that student conceptual understanding of the relevant physics isn't the primary focus of upper-division and advanced laboratory courses, one can argue that analog electronics courses differ substantively from these other courses for a number of reasons. Perhaps most importantly, the development of a functional understanding of the behavior of circuits is typically an expected outcome; namely, students should be able to construct useful circuits for a variety of practical applications after electronics instruction. This is particularly relevant because the junior-level electronics course may serve as a gateway course in that it is often required for subsequent laboratory courses (in which students must apply their electronics knowledge) and may be critical for success in undergraduate research experiences in which custom electronics play a key role in data collection and analysis. Lastly, it is well documented that students leave most introductory physics courses with a rather poor understanding of basic dc circuits;[2,3,4] some of these difficulties have been shown to persist both during and after upper-division electronics courses.[5]

For the reasons described above, we have been conducting a multi-institutional investigation of student conceptual understanding of analog electronics. In our investigation, we have focused both on foundational circuits concepts (*e.g.*, Kirchhoff's junction and loop rules) and canonical topics in analog electronics (*e.g.*, diode, transistor, and op-amp circuits). Examining student ability to apply foundational circuits concepts in the more advanced context of common electronic circuits has been of particular interest.

In this text, we describe an in-depth, multi-year empirical study of student understanding of basic operational-amplifier (or op-amp) circuits and diode circuits. The multi-year investigation was guided by the following research questions:

1. To what extent do students develop a functional understanding of basic op-amp and diode circuits after relevant instruction in an electronics course? In particular:
   a. To what extent are students able to reason productively and / or correctly about op-amp and diode circuits that correspond to "perturbations" of canonical op-amp and diode circuits covered in the course?
   b. To what extent are students able to correctly describe (qualitatively and/or quantitatively) the currents and voltages in a canonical op-amp (such as the inverting amplifier) or diode circuit?
2. What ideas and approaches, both correct and incorrect, do students employ when analyzing op-amp and diode circuits?

The investigation was designed and conducted through the lens of the specific difficulties empirical framework used by the University of Washington Physics Education Group;[6,7,8] the goal was to characterize student thinking in sufficient detail to help guide instructional interventions. The primary participants in the study were undergraduates enrolled in upper-division physics courses on analog electronics at three different institutions. Most of the data were obtained from several written tasks administered as both graded and ungraded questions, although we discuss additional data from think-aloud student interviews as appropriate. Given the pragmatic motivation underlying our investigation, this text highlights the most prevalent conceptual and reasoning difficulties identified over the course of the study.

1.1 Overview of relevant previous research.

Student understanding of introductory electric circuits has received considerable attention by the physics education research community as well as the broader science education research community. Indeed, there is an extensive body of research on student understanding of electric circuits, both at the precollege and undergraduate levels. This brief overview is intended to set the stage for our upper-division electronics work on operational amplifiers and diodes, and is therefore not exhaustive.

In the 1980s, several studies were reported that focused on student conceptions of electric circuits at the pre-college and university levels, both in the US and internationally.[9] In 1992, McDermott and Shaffer published a pair of articles that described: (1) an in-depth investigation of the ability of undergraduates in introductory physics courses and of K-12 teachers to predict and explain the behavior of simple electric circuits, as well as (2) the development and testing of research-based instructional materials aimed at improving student understanding.[2,3] It was found that, after all lecture and laboratory instruction, many students lacked a coherent framework for thinking about simple dc electric circuits. A number of specific conceptual and reasoning difficulties were identified. In 2004, Engelhardt and Beichner reported on the development and testing of the Determining and Interpreting Resistive Electric Circuits Concepts Test (DIRECT), and their analysis similarly revealed that both high school and university students struggled with many circuits concepts after relevant instruction.[4]

Several articles describe the difficulties that introductory students encounter when working with multiple-battery circuits.[5,10] In a multi-year investigation conducted at the University of Washington, it was found that undergraduates enrolled in introductory physics had trouble applying the concept of a complete circuit and often did not draw upon Kirchhoff's junction rule when analyzing single-loop, multiple-battery circuits.[5]

Although there have been some investigations of slightly more advanced circuits topics in introductory physics, such as transients in RC circuits,[11] which are also covered in upper-division courses, the PER community as a whole has conducted relatively little research in the context of upper-division physics courses on electronics. Moreover, much of the work that has been done in upper-division electronics courses has focused on foundational circuits concepts. For example, Getty used the DIRECT instrument to assess the impact of inquiry-oriented course modifications in the first semester of a full-year electronics laboratory sequence.[12] Stetzer *et al.* found that over half of upper-division students enrolled in either a junior-level electronics course or a computer measurement laboratory course (for which electronics is a prerequisite) gave responses on multiple-battery questions that were inconsistent with Kirchhoff's junction rule, highlighting the persistence of basic difficulties.[5] Some additional work has used electronics content as a context for examining other phenomena in science education.[13]

In the engineering education literature, much of the focus has been on introductory electrical engineering courses on circuits and circuit analysis. Indeed, papers have been published on the development of the Electric Circuits Concept Inventory (ECCI), which focuses on dc circuit analysis at the introductory engineering level.[14] As in the PER literature, difficulties with foundational circuits concepts have also been documented in first-year engineering courses.[15] Some advanced circuits topics typically covered in upper-division electronics courses in physics have also been a focus of research. Kautz reported on an investigation of student understanding of phase relationships in ac circuits as part of a larger effort to develop research-based instructional materials for introductory electrical engineering courses.[16] Several student difficulties with the topic of RC filters were identified in a related study involving students enrolled in a second-year electronics laboratory course.[17]

Very little engineering education literature has focused on student learning of canonical electronics topics (*e.g.*, diode circuits, transistor circuits, and operational amplifier circuits). Rather, the emphasis has often been on pedagogical approaches or instructor resources for these topics.[18] Some work, however, has been aimed at examining student learning. For example, in 2004, Simoni *et al.* reported on the development of an Electronics Concept Inventory (ECI).[19] More recently, Hudson *et al.* examined the impact of exposure to a conceptual analysis of transistor circuits on student confidence and comfort levels when approaching new circuits.[20]

Of greatest relevance to the present investigation, Mazzolini *et al.* developed and administered a conceptual test on operational amplifier (op-amp) circuits in order to assess the impact of a sequence of interactive lecture demonstrations on student learning in a first-year unit on electronics.[21] While their study was not designed to be a detailed examination of student thinking about, and student difficulties with, these circuits, the authors state in the introduction that students tend to employ "'shallow learning' approaches" in which they memorize standard op-amp "circuit configurations and the gain formulas that apply to these particular configurations." They note specific observations of students using incorrect gain formulas when the standard resistor labels (*e.g.*, R_1 and R_2) were swapped in the diagram and of students encountering difficulties when canonical op-amp circuits were drawn in a non-traditional manner, and argue that a "true understanding of the concepts" is needed. Indeed, our investigation was designed to probe, in detail, student conceptual understanding of op-amp and diode circuits, thereby complementing the work of Mazzolini *et al.*

1.2 Context for investigation

The research reported in this text was primarily conducted in three upper-division physics courses on analog electronics, one offered at the University of Washington (UW), one at the University of Athens (UA) in Greece, and one at the University of Maine (UM). Although many aspects of the courses differed (*e.g.*, the ordering of material and the texts used), all three courses included a substantial laboratory component. Details of the courses are provided below.

1.2.1 Upper-division laboratory course at the University of Washington

The analog electronics course at UW is one quarter in length (~10 weeks), and consists of two 50-minute lectures each week along with one weekly laboratory session that is approximately three hours in length. While analog electronics is officially a junior-level course, the students enrolled are typically a mixture of sophomores, juniors, and seniors. The course is required for all physics majors and is also a prerequisite for other upper-division physics laboratory courses offered at UW. The text for the course is *The Art of Electronics* by Horowitz and Hill.[22] The course begins with voltage division, equivalent circuits, and ac circuits, and then moves into electronics content by covering diode circuits, transistor circuits, op-amp circuits, comparators, timer circuits, *etc.* The laboratories are drawn and/or adapted from the *Student Manual for The Art of Electronics* by Hayes and Horowitz.[23] Lab reports are submitted at the end of each laboratory session and do not require extensive write-ups. There are weekly homework assignments and two 50-minute exams. Approximately half of a student's course grade comes from the laboratory component of the course.

1.2.2 Upper-division laboratory course at the University of Athens

At UA, the analog electronics course is one semester in length (about 10 weeks in practice). There are two lectures each week, resulting in a weekly total of approximately 3.75 hours of lecture instruction. The two-hour laboratory sessions are biweekly. The course is obligatory for all students in the UA Physics Department (*i.e.*, physics majors), and is typically taken by juniors. The text (in Greek) for the course is *Introduction to Electronics* by George S. Tombras.[24] In the course, op-amp circuits are the first electronic circuits introduced once the students have studied voltage division, equivalent circuits, ac circuits, and general circuit theory and analysis approaches. The course then focuses on diode and transistor circuits. The laboratories were developed by the author of the text and are used exclusively at UA. Formal lab reports are required. (At the beginning of this study, however, students were not asked to submit laboratory reports.) There are final exams for both the lecture and laboratory components of the course. Homework assignments are not given.

1.2.3 Upper-division course at the University of Maine

At UM, the physical electronics course is one semester in length (approximately 15 weeks), and is primarily devoted to analog electronics. While this investigation was being conducted, the course format was modified. Originally, the course consisted of one 50-minute lecture per week and one weekly laboratory session that was about three hours in length. In the modified course, there are two 50-minute lectures per week and one weekly two-hour laboratory. The course is required for all physics majors and it is the first half of a full-year junior laboratory experience. The text for the course was originally *Principles of Electronic Instrumentation* by Diefenderfer and Holton,[25] but *Electronics with Discrete Components* by Galvez is now used,[26] after being adopted midway through our investigation. The sequence of topics covered is essentially identical to that at UW, although op-amp circuits were introduced prior to diode and transistor circuits (as at UA) for the first time during the final year of the investigation. Formal lab reports are required for approximately half of the laboratories. Although there are no regular homework assignments, some of the laboratories require students to complete pre-lab assignments. Students are typically given one midterm exam and one cumulative final exam.

1.3 Research methods

Given that very little research has been conducted on student learning of analog electronics, we were interested in examining the level of understanding of key course concepts after relevant instruction in lecture and laboratory. To accomplish this, we used a combination of (a) written free-response questions and (b) task-centered clinical interviews. As will be discussed later, the interview findings often helped illuminate particular categories of written responses and were thus instrumental in facilitating the identification of conceptual and reasoning difficulties that students encounter when studying op-amp and diode circuits.

The free-response written probes developed as part of this investigation were administered after lecture and laboratory instruction on op-amp circuits. These probes were included on course exams and given as short ungraded conceptual questions administered at the beginning of laboratory or lecture sessions. While the conceptual questions were not graded, students received participation credit provided that their responses included explanations of reasoning.

The task-centered clinical interviews were conducted in the quarter/semester *after* students had completed the analog electronics course. The interview tasks were drawn from the written probes. In the interviews, students were asked to think aloud while responding to these questions. No incentives were provided for the interview participants. The average course grades of the interview participants (at both UA and UW) were considerably higher than those of their respective classes. In most cases, interview participants were among the top students. All interviews with students at UA were transcribed and translated into English.

1.4 Overview of dissertation

In chapter 2, we will study conceptual difficulties associated with basic operational amplifier circuits taught to the students of three different Universities (UA, University of Washington, Seattle and University of Maine, Orono). In particular, we will study how students are able to handle perturbations of canonical circuits, as well as how they think of voltages and currents in these circuits.

In chapter 4, we will study conceptual difficulties associated with diode circuits taught to the students of these three Universities (UA, UW and UM).

In chapter 4, we will look at conceptual difficulties faced by junior year students of the Physics Department of the National and Kapodestrian University of Athens (UA), enrolled in the Electronics course. This preliminary work focusing on foundational circuits concepts was conducted in the beginning of the investigation, yet it is included at the end due to the focus on upper-division topics.

In chapter 5, we will talk about the implications for instruction suggested by our findings in the previous chapters.

At last, in chapter 6, we will summarize our results and make suggestions for relevant future work, in an effort to achieve better student understanding through electronics instruction.

Chapter 2:

Operational Amplifier circuits

2.1 Introduction

In this chapter we describe a systematic investigation of student understanding of the behavior of simple operational-amplifier (op-amp) circuits. The participants in this study were undergraduates enrolled in upper-division physics courses on analog electronics at three different institutions, as well as undergraduates in introductory and upper-division electrical engineering courses at one of the institutions. We focus on the use of slightly modified or “perturbed” canonical op-amp circuits in research tasks designed to provide insight into student thinking about basic op-amp circuits. The findings indicate that, after instruction, many students in both physics and engineering courses are unable to productively analyze circuits that differ only slightly from those explicitly covered in lecture, laboratory, and the textbook. The most prevalent conceptual and reasoning difficulties identified in this part of the investigation are described and related implications for electronics instruction are discussed.

We first focus on the use of slightly modified or “perturbed” canonical op-amp circuits in research tasks designed to provide insight into student thinking about basic op-amp circuits, thereby addressing research questions 1a and 2. Then we examine, in depth, student understanding of canonical op-amp circuits.

In section 3.2, we begin with a brief overview of previous research on introductory electric circuits and electronics conducted in both physics and engineering.

We then discuss our research methodology and context (Section III).

In Section IV, we present two free-response questions used to probe student understanding along with the data collected and difficulties identified.

We then describe the extension of this investigation to electrical engineering courses as well as the associated findings (Section VI).

Finally, in Sections VII and VIII, we discuss implications for instruction and summarize our findings in the conclusion.

2.2 Brief overview of op-amp coverage

In all courses, students learn that an operational amplifier is a high-gain differential amplifier, with a non-inverting (+) input and an inverting (-) input, and is typically powered by connections to positive and negative rails (*e.g.*, ±15 V). See Fig. 2.1.

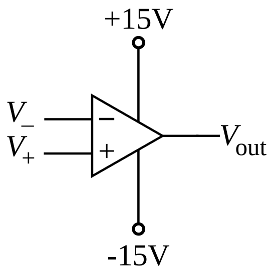


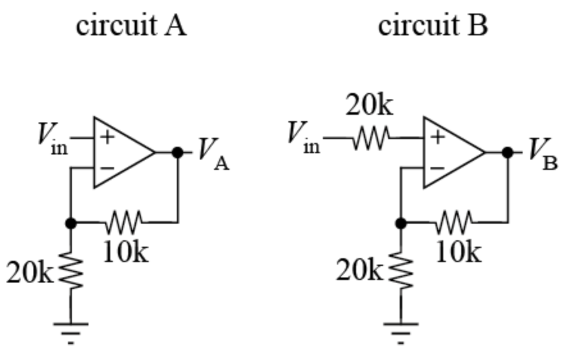
Fig. 2.1. Standard schematic of an operational amplifier or op-amp. The op-amp has two input terminals (the non-inverting input indicated by a "+" and the inverting input indicated by a "–") and one output terminal.

The non-inverting and inverting inputs are characterized by extremely large input impedances (modeled as ∞ in an ideal op-amp) and therefore negligible currents. When the op-amp is placed in a circuit in which there is negative feedback (see, for example, circuit B in Fig. 2.2, in which there is a connection between the op-amp output and the inverting input), the output voltage V_{out} of the op-amp will quickly adjust until $V_- \approx V_+$, if possible.[27] Horowitz and Hill summarize op-amp behavior under these conditions via the Golden Rules:[22] "I. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero…. II. The inputs draw no current." At UW and UM, these Golden Rules are incorporated into instruction explicitly; at UA, although the same ideas are motivated and covered in instruction, they are not explicitly referred to as rules for op-amp behavior.

2.3 Perturbations of canonical circuits (the non-inverting amplifier)

The work of Mazzolini *et al.* indicated that students encountered difficulties when they were asked to analyze standard op-amp circuits drawn in non-traditional ways.[21] This suggested that memorization of specific circuits, gain formulas, and key results may play a substantive role in student ability to solve *standard* op-amp circuits successfully. For this reason, we were interested in exploring the extent to which students could predict the behavior of circuits that were slight perturbations of standard op-amp circuits. It was hoped that student responses to such tasks would provide deeper insight into the extent to which students were simply applying memorized results rather than reasoning through the behavior of the "perturbed" circuit from foundational principles. The circuits discussed in this section are all perturbations of the non-inverting amplifier.

Is the absolute value of V_A *greater than*, *less than*, or *equal to* V_B?



op-amps ideal and identical
source voltage same for both and constant

Fig. 2.2
Two amplifiers question, in which students are asked to compare the absolute values of the output voltages from two non-inverting amplifier circuits with identical input voltages V_{in}.

2.3.1 Two amplifiers question

In the two amplifiers question, students were shown a standard non-inverting amplifier (circuit A) and a similar circuit that also includes a 20-kΩ "input resistor" between V_{in} and the non-inverting input of the op-amp (circuit B); both circuits are shown in Fig. 2.2. Students were told that the op-amps in both circuits are identical and ideal and that the input voltages V_{in} for both circuits are constant and identical. Students were then asked whether the absolute value of V_A (the output of circuit A) is greater than, less than, or equal to that of V_B (the output of circuit B).

2.3.1.1 Correct response

There are many approaches students can use to determine the correct value of V_A. For example, students can simply apply the gain formula for the standard non-inverting amplifier ($1+R_1/R_2$, where R_1 in this case corresponds to the 10-kΩ resistor in the feedback loop and R_2 corresponds to the 20-kΩ resistor between V_- and ground), and correctly determine that the output of circuit A is $3/2V_{in}$. It is important to note, however, that the use of such an approach does not necessarily reflect a robust understanding of op-amp circuits, as students may simply be applying a formula that they do not understand and could not easily derive. Alternatively, students may apply Golden Rule I to conclude that the voltage at the inverting input must be V_{in}. Thus, there is a voltage drop of V_{in} across the 20-kΩ resistor. Since the current flowing through the 20-kΩ resistor is equal to that flowing through the 10-kΩ resistor (due to Golden Rule II and Kirchhoff's junction rule), there must be a drop of $½V_{in}$ across the 10-kΩ resistor and V_A is equal to $3/2V_{in}$.

In order to determine the value of V_B, a student needs to recognize that there will be no current through the 20-kΩ input resistor due to Golden Rule II. Thus, V_+ is equal to V_{in} since there is no voltage drop across the resistor; it follows that V_B is equal to $3/2V_{in}$ and is therefore also equal to V_A. Given the similarities between the two circuits, a complete explanation only requires the recognition that there will be no current through and thus no voltage drop across the 20-kΩ input resistor due to Golden Rule II, so the output voltages of the two circuits must be the same.

2.3.1.2 Overview of student performance and reasoning

This question was administered on the second (and final) course exam at UW to 54 students. All instruction on analog electronics had been completed; the instruction included approximately four 50-minute lectures on op-amp circuits, two labs, and relevant reading and homework questions.

Only 70% of the students correctly recognized that V_A is equal to V_B. Just under three-quarters of these students (or 52% of all students) provided a correct and complete or partially complete explanation. Explanations that argued that the 20-kΩ input resistor in B simply doesn't matter or do anything (~7% of total responses) and those that used a gain expression without justifying its applicability in circuit B (~6% of total responses) were not considered to be correct and complete (or partially complete). While these types of responses could be consistent with a robust understanding, it was not possible to rule out the possibility that they stemmed from a superficial/incorrect line of reasoning (*e.g.*, the input resistor isn't relevant because it is not a variable in the standard non-inverting amplifier gain expression).

Table 2.1. Overview of student performance on the two amplifiers question in a physics course on analog electronics. The question is shown in Fig. 3.2.

	Percentage of total responses UW (N=54)
V_A=V_B (Correct)	**70%**
Correct reasoning	52%
Gain expression(s) only	6%
Input resistor doesn't matter	7%
V_A>V_B	**30%**
Voltage drop due to input	20%

Of more interest, however, is the fact that the remaining 30% of students indicated that V_A is greater than V_B. These students tended to provide similar arguments. For example, one student wrote:

> *"These circuits are non-inverting amplifiers that multiply the voltage at the + terminal by 3/2 so V_A > V_B because the voltage at the + terminal in B has already lost voltage because of the resistor."*

Another student noted that:

> *"[t]he 20k resistor between V_{in} and V_+ in circuit B reduces the input to the op amp, reducing the output as well."*

In total, approximately 20% of all students argued that V_A is greater than V_B because there is a voltage drop across the input resistor.

2.3.1.3 Specific difficulties identified

In this section, we classify the specific conceptual and reasoning difficulties that emerged from our in-depth analysis of student responses to the two amplifier question.

Lack of a functional understanding of Golden Rule II.

At the most basic level, these findings call into question the extent to which students have developed a truly functional understanding of Golden Rule II (*i.e.*, there is no current into the inverting and non-inverting inputs due to their high input impedances) over the course of instruction. Students should recognize that a voltage drop across the 20-kΩ input resistor must correspond to a current through the resistor (due to Ohm's law) and into the non-inverting input; the existence of this current, however, is in contradiction with Golden Rule II. The fact that 20% of the students did not appear to recognize the conflict between Golden Rule II and the ascribed voltage drop across the 20-kΩ input resistor suggests that students may not be comfortable applying Golden Rule II when reasoning through unfamiliar op-amp circuits.

Tendency to ascribe a voltage drop to a resistor through which there is no current.

As discussed above, from Golden Rule II, there cannot be a voltage drop across the input resistor since there is no current into the non-inverting input. (This line of reasoning was offered by approximately three-quarters of those students giving a correct answer.) However, on this question, *none* of the students who claimed that V_A is greater than V_B because of a voltage drop across the input resistor explicitly mentioned a current through that resistor. See Fig. 2.3 for an illustrative response that explicitly highlights currents elsewhere in both circuits but does not indicate current through the input resistor in circuit B. Thus, after all instruction, 20% of the students argued that there would be a voltage drop across the input resistor but did not explicitly refer to current through that resistor.

The absence of such a justification in most of these incorrect responses is of particular interest. This responses pattern suggests that students may in fact be automatically (and possibly subconsciously) ascribing a voltage drop to the resistor without analyzing the situation through the more formal lens of Ohm's law. Such behavior is consistent with a "knowledge in pieces" or resources model of student thinking (in which, for example, a student might draw upon a more informal notion that "increased resistance leads to less result") and dual process theories of reasoning.[28,29,30] Indeed, a significant percentage of students simply attributed a voltage drop to the resistor without explicitly considering the presence or absence of current through the resistor. This tendency to expect a voltage drop to be measured across all resistors (regardless of circuit and arrangement) may also be reinforced by many of the circuits the students construct and explore in introductory physics and in analog electronics courses. The discussion of the three amplifiers post-test in section IV.B as well as the interviews reported in section IV.C provide further insight into student thinking about the relationship between resistors and voltage drops.

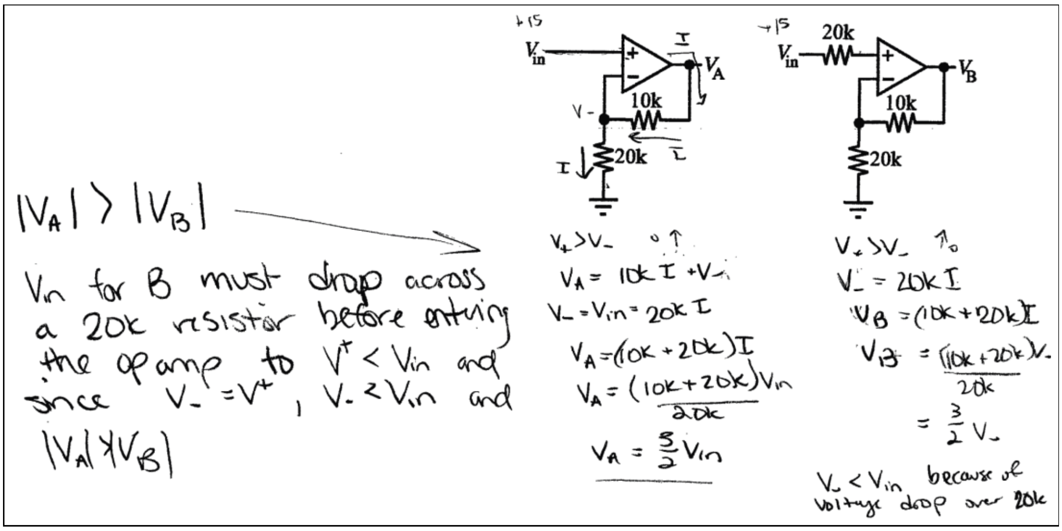


Fig. 2.3. Student response to the two amplifiers question that explicitly indicates currents elsewhere in the circuit but not through the input 20-kΩ resistor, despite the claim that there is a voltage drop across this resistor.

2.3.1.4 Discussion

Even with an op-amp circuit that has only been modified slightly from one of the canonical circuits studied in analog electronics (the non-inverting amplifier), analysis of this “exploratory” question, administered to a single class at a single institution (UW), revealed that nearly one third of students encountered difficulties when trying to compare the output of the modified circuit to that of the unmodified circuit. A significant percentage provided reasoning that is in apparent contradiction with Golden Rule II. Moreover, almost a quarter of all students claimed that a potential difference would be measured across the input resistor without mentioning anything about current through that resistor. In order to explore and document these difficulties more thoroughly, we developed a closely related question (the three amplifiers question) that involved two different perturbations of the standard inverting op-amp circuit. The question was administered in multiple classes at three different institutions so that we could probe the prevalence of identified difficulties in a variety of different courses that employed different instructional approaches and/or sequences.

2.3.2 Three amplifiers question

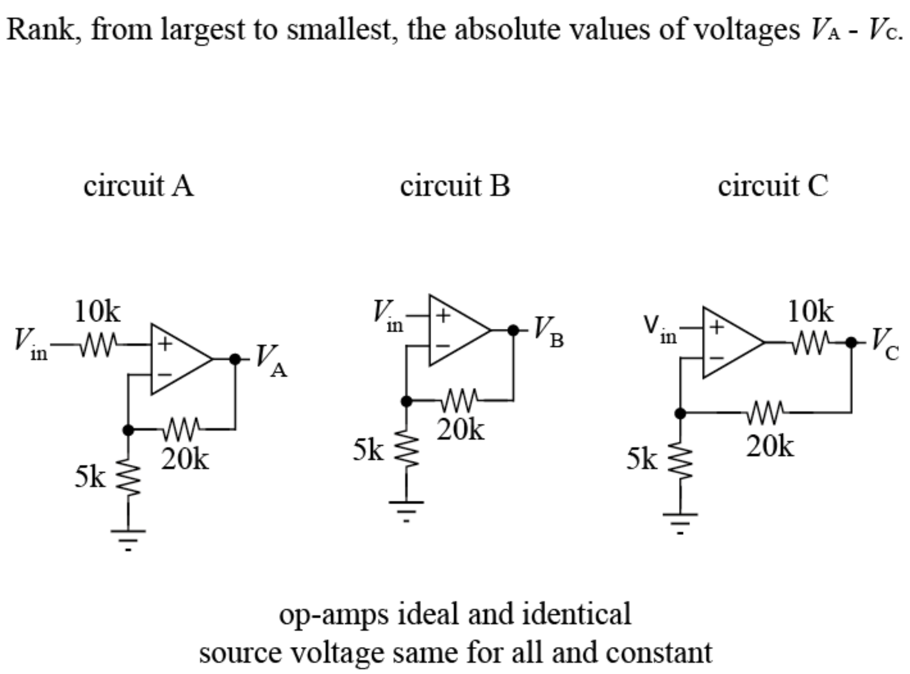


Fig. 2.4

Three amplifiers question, in which students are asked to compare the absolute values of the output voltages from three non-inverting amplifier circuits with identical positive input voltages V_{in}.

In the three amplifiers question (Fig. 2.4), students are shown three circuits that are all non-inverting amplifiers. In circuit A, a single 10-kΩ resistor is inserted between V_{in} and the non-inverting input of the op-amp. In circuit C, the same resistor is inserted between the output of the op-amp and the output of the circuit (the point at which V_C is measured). Students are told that all op-amps are identical and ideal, and that all three circuits have identical and unchanging positive input voltages V_{in} (from ideal voltage sources). They are told to assume that no loads are connected to the outputs of the circuits. In the most recent version of the question, students are asked (a) to compare the absolute value of the output voltage V_B to that of V_A, and (b) to compare the absolute value of the output voltage V_C to that of V_B. In the original version of the question, students are asked to rank, from largest to smallest, all three output voltages (V_A, V_B, and V_C) according to absolute value. Since student performance was similar on both versions, all results are presented together for simplicity.

2.3.2.1 Correct response

Like the two amplifiers question, a correct response to the three amplifiers question does not necessarily require the explicit determination of the output voltages of the three amplifiers, but instead relies on a careful analysis of whether or not the modifications to the canonical inverting amplifier (circuit B) will impact the output voltages. Through the application of the Golden Rules and foundational circuits concepts (or the appropriate gain formula) to circuit B, it can be determined that $V_B = 5V_{in}$. In circuit A, since there is no current through and therefore no voltage drop across the 10-kΩ input resistor (due to Golden Rule II), $V_A = V_B = 5V_{in}$. In circuit C, the voltage at the inverting input is once again equal to V_{in} (from Golden Rule I), and the subsequent analysis is identical to that for circuit B and leads to the conclusion that $V_C = V_B = 5V_{in}$. Note that the output voltage of the op-amp in circuit C ($7V_{in}$ in this case) must be larger than in circuit B since there is a single current through all three resistors and there will be a voltage drop across the 10-kΩ output resistor. Therefore, the correct ranking is that the output voltages (V_A, V_B, and V_C) of all three circuits are equal in absolute value and are non-zero.

2.3.2.2 Overview of student performance

The three amplifiers question has been administered at UW (N = 160), UA (N = 181), and UM (N = 49). At UW, both written versions of this task have been administered as ungraded conceptual questions after all relevant instruction on op-amps; 28 students in a single class were given the single-ranking version whereas all other students at UW were given the version with two comparisons. Since performance on both versions was similar, the results are reported jointly. At UA and UM, the two-comparison version was administered to all students. Results from all three institutions are presented below.

Table 2.2 Overview of student performance on the three amplifiers question in physics courses on analog electronics physics at three different institutions. The question is shown in Fig. 2.4.

	Percentage of total responses		
	UW (N=160)	UA (N=181)	UM (N=49)
V_A=V_B=V_C (Correct ranking)	**23%**	**9%**	**33%**
Correct reasoning	8%	2%	29%
V_B=V_A (Correct)	**40%**	**40%**	**49%**
Correct reasoning	*29%*	*11%*	*45%*
V_B>V_A	54%	48%	41%
Voltage drop due to input	*44%*	*39%*	*35%*
V_C=V_B (Correct)	**39%**	**18%**	**61%**
Correct reasoning	*10%*	*2%*	*51%*
V_C<V_B	46%	42%	31%
Voltage drop due to output	*31%*	*16%*	*27%*
V_C>V_B	15%	23%	8%
Circuit vs. op-amp output	*5%*	*8%*	*6%*

Approximately one-quarter to one-third of UW and UM students correctly ranked the absolute values of all three circuits ($V_A = V_B = V_C$), as shown in Table II. At UA, only approximately 10% of the students gave the correct ranking. The percentages of students who supported a correct ranking with correct reasoning ranged from 2% to about 30% at the different institutions. As an illustrative response classified as correct with correct reasoning, one student noted that $V_B = V_A$ because of the following justification:

> *"The op-amps are ideal, so there is no input current and hence no voltage drop across the 10k resistor. Therefore, since the circuits are identical the output voltages should be identical."*

Similarly, the student argued that $V_C = V_B$ as follows:

> *"The output current is determined by the 5 kΩ resistor* $\left(I_{out} = \frac{V_{in}}{5k}\right)$. *The opamp will supply whatever voltage is necessary to produce this current, which means the voltage drop across the 20k will be* $I_{out} \cdot 20$ *kΩ.* $V_{out} = V_{in} + 4V_{in} = 5V_{in}$*."*

This student also added that the voltage at the output of the op-amp "will be higher."

At all three institutions, only about 40 - 50% of the students correctly recognized that that $V_B = V_A$.[31] A similar percentage (~40 - 55%) of the students indicated that $V_B > V_A$. Note that this comparison is the same as that in the two amplifiers question; circuit A is a simple modification of circuit B (the standard non-inverting amplifier) in which a 10-kΩ input resistor has been added. Approximately 35 - 45% of all students justified this incorrect ranking by explicitly focusing on a voltage drop across the input resistor, providing the same types of written explanations that were observed on the more exploratory two amplifiers question. For example, one student wrote:

> *"Because* V_{in} *is the same for all circuits, since A has a 10k resistor before the noninverting input,* $V_{+A} < V_{+B}$*, thus* $V_{-A} < V_{-B}$*. Since* $V_{-A} < V_{-B}$*, more current in case B flows through the 5k. By V = IR, if R is the same, but I* ↑*, V* ↑*, so* $V_B > V_A$*."*

Again, there is no mention of current through the input 10-kΩ resistor. Upon examining all responses to both this question and the original two amplifiers question at UW and UM, there were 121 responses in support of this incorrect comparison; only *two* out of those responses explicitly attributed the voltage drop to an input current, suggesting an almost automatic mapping of a voltage drop to the input resistor as discussed earlier. At UA, however, of the 86 written responses in support of this incorrect comparison, 26 explicitly argued that there was a voltage drop due to the input current, whereas 43 solely spoke of a voltage drop. The source of this discrepancy between the UA responses and those given by the UW and UM students is not clear. The UA course emphasizes the very large input impedances but does not explicitly give students Golden Rule II, which states that the "inputs draw no current."[22] It is possible that some of the UA students were less concerned about invoking input currents for this reason. At the same time, it is also possible that the UA students were more attentive to the relationship between the voltage across and the current through an ohmic resistor.

Performance on the comparison of circuits C and B was somewhat more varied. The percentage of correct comparisons ranged from about 20% at UA to roughly 60% at UM. At all three institutions, however, approximately 30 - 45% of the students stated that $V_C < V_B$. Responses in support of $V_C < V_B$ tended to draw on productive elements of reasoning related to voltage dividers. One student explicitly mentioned the divider chain in the circuit, writing:

> *"In C, the voltage divider now has a voltage drop across 10k as well as 20k + 5k so less voltage is dropped across 20k + 5k and V_c is less."*

This student argued that the addition of a third resistor to the divider chain meant that less voltage was dropped across the original two resistors, which is consistent with an incorrect assumption that the voltage across the entire chain remains constant. Other students were more explicit about this assumption. For example, one student wrote:

> *"Circuit C is similar to B, but the input resistor from A has taken up residence between the op-amp output and V_{outC}, thus creating a voltage divider…."*

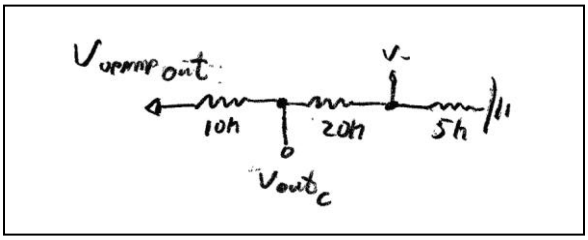


Fig. 2.5. Divider chain drawn by student in support of the incorrect comparison $V_C < V_B$ on the three amplifiers question. The student indicated that "$|V_{\text{opamp out}}| = |V_B|$."

This student correctly drew the divider chain for the circuit, shown in Fig. 2.5, but noted that *"$|V_{opamp\ out}| = |V_B|$."* Indeed, between roughly 15% and 30% of all students focused on the voltage drop due to the output resistor and appeared to be implicitly assuming that the outputs of the op-amps in circuits B and C were identical in the reasoning they provided. Relatively few of the written responses offered insight into the thinking behind this assumption. In one response, however, a student wrote:

> *"… there is a potential drop across the 10k resistor in circuit C between the output of the op-amp and the output V_C, but circuit B is identical otherwise, so $V_B > V_C$."*

This student appeared to be arguing that since most of the circuit (*i.e.,* that to the left of and below the op-amp) is the same in both cases, both op-amps should have the same output. Such responses seem to be drawing on a combination of localized and sequential reasoning, arguing that any change in the circuit *after* the op-amp shouldn't impact its output. Of course, this line of reasoning is inconsistent with the notion of negative feedback, which plays a critical role in the behavior of most op-amp circuits (including the non-inverting amplifier).

Between approximately 10% and 25% of all students incorrectly claimed that $V_C > V_B$. The most prevalent line of incorrect reasoning supporting this comparison (given by about 5-10% of all three populations) involved the erroneous claim that the additional output resistor increased the gain of the circuit. For example, one student wrote:

> *"In circuit C the 10 kΩ resistor is being added in series to the others.*
> *So $1/A_{Vo} = R_1/(R_1 + R_2 + R_3)$…. So $V_C = A_{Vo}V_{in} = 7V_{in} > 5V_{in}$…."*

If the output of the op-amp were also the output of the circuit, such reasoning would be correct. For the given circuit, however, such responses suggest a failure of students to differentiate between the output of the circuit and the output of the op-amp.

2.3.2.3 Related interview task

As part of this investigation, think-aloud interviews were conducted with 31 students: 29 at UA and 2 at UW. All but three were undergraduates who had just recently completed the electronics course. Three of the interview participants at UA were first-year physics graduate students who were either working as TAs in the electronics course for the first time or beginning experimental research in electronics. During the interview, students were presented with circuits identical to those in the three amplifiers task (Fig. 2.4). Students were first shown the standard non-inverting amplifier configuration, circuit B in Fig. 2.4, and asked to determine the output of the circuit. The students were then asked to compare the absolute values of the outputs of circuit A (with the input resistor) and circuit B. Lastly, students were asked to compare the absolute values of the outputs of circuit C (with the output resistor) and circuit B. In all transcript excerpts included in this manuscript, we use S to indicate the student being interviewed and I to indicate the interviewer.

Only about half of the interview participants were able to determine the correct output of the canonical non-inverting amplifier without explicit assistance and/or explanation from the interviewer. Of those participants, the majority used gain expressions, whereas only 3 used foundational principles and op-amp rules to derive the gain. Thirteen (of 31) students could not answer the question, typically because they didn't remember the formula and couldn't derive it.

Roughly half of the interview participants arrived at a correct output comparison for circuits B and A; the remaining students all argued that the output of circuit A would be less due to a voltage drop across the input resistor. Approximately half of the students who discussed a voltage drop explicitly mentioned that there would be a current through the input resistor; these students were all from UA.

The following interview transcript illustrates the most prevalent incorrect line of reasoning in which a voltage drop across the input resistor is invoked without any consideration of whether or not there is current through the resistor.

S: *I do have a voltage drop here, of course.*

I: *How do you know that?*

S: *Because of the resistor! This voltage will be* $V_{in} - V_R$.

I: *And how do we get that voltage drop? That is my question.*

S: *Because of the resistor.*

I: *I see.*

Even the interviewer's subtle prompting did not elicit a statement about current through the resistor. In accordance with the protocol adopted, the interviewer let the student proceed through the rest of the three amplifiers task, and only revisited this issue more explicitly near the end.

I: *Let us go back to something you said before. You said that there is* *a voltage drop here before the + input. I asked you why and you replied that it happens because of the resistor. Right?*

S: *Yes.*

I: *Remind me: What is the current there, before the + input?*

S: *It must be zero… Wait a minute… Now that I think of it, there can be no voltage drop! There is no current, no voltage drop. I guess I assumed there was a current before.*

Only after explicit questioning about current into the non-inverting input did the student consider the current (or lack thereof) through the input resistor, recognize the inconsistency in reasoning, and revise his response. Even students who arrived at a correct response without explicit prompting often struggled to reconcile their initial claims about the voltage at the non-inverting input with an analysis of the current through the input resistor, as illustrated in the transcript that follows:

S: *This is a voltage amplifier, so we care about the voltage in the input… not the current… ehhh. This is a voltage divider before the V+, so it will not take all the V_{in} inside… the voltage here [draws a source]…. I am confused…. It is sure that I am not having the same current going into the op-amp at +. It will be less now… since the resistor is in series with our source. This means…. Is the V+ the same as before? There must be a voltage drop… but I remember that the op-amp has a high input resistance… so it must be zero current there! But then you have a potential difference across the 10k without a current, but that cannot be done! If I think about Ohm's law… I think there is no difference, so [V_A] will be equal to [V_B], which is $5V_{in}$.*

For this student, the presence of the input resistor immediately led to an incorrect voltage divider analysis prior to any examination of the current. Thus, even in the interviews, we observe a tendency to (at least initially) associate a voltage drop with the input resistor, which is consistent with the informal notion that "increased resistance leads to less result."[29] In some cases (like the one illustrated above), students' subsequent analysis of the constraints on the current through the input resistor led to a refinement of their responses.

The interviews were also extremely helpful in clarifying student thinking about how the behavior of the canonical op-amp circuit (circuit B) is impacted via the addition of an output resistor (circuit C). While roughly half of the students correctly argued that the output of circuit C is equal to that of circuit B, four students confused the circuit output and the op-amp output, thereby incorrectly concluding that the absolute value of V_C is greater than that of V_B (which is true for the *op-amp* outputs). Roughly one-third of the students claimed that V_C is less than V_B. All but two of the students who concluded that V_C is less than that of V_B focused on the voltage drop associated with the 10-kΩ output resistor. One such student's reasoning about the output of circuit C is illustrated in the transcript below.

> S: *Basically it will be the $5V_{in}$ minus the voltage drop across this resistor.*
>
> I: *Why do you decide to say $5V_{in}$?*
>
> S: *We have the same circuit up to here. So it must be it…*

This student focused on the fact that the rest of the circuit (to the left and below the op-amp) is the same, and incorrectly concluded that the op-amp output must be the same ($5V_{in}$), thereby demonstrating the same kind of the local and sequential reasoning observed in the written responses. Another student initially argued that *"we should have a smaller current, and therefore V_C should be smaller,"* but then continued thinking about the question and noted that he *"will need to do it brute force"* by writing equations relating the current and resistances in the lower part of the divider chain to V_C.

> S: *It seems like it's going to be the same. Yeah. From these equations. So it's [the output voltage of the op-amp] that's going to be regulated.*

Although the student arrived at a correct answer with correct reasoning and told the interviewer that *"[i]t's such an interesting problem,"* it was clear that there was still some uncertainty and tension between his original thinking and his final conclusion. Indeed, the student added, *"But yeah, now it makes sense, but only through equations."* It seems as though the student, even at the end of this portion of the interview, wasn't fully comfortable with the result; this may speak to the tenacity of the intuitively appealing argument that a resistor added after the op-amp will diminish the circuit's output because the op-amp's output remains unchanged.

2.3.2.4 Specific difficulties identified

Interestingly, student performance on the three amplifiers task suggests that students at all three institutions may, in fact, struggle more with the application of some of these basic circuits concepts and op-amp rules to slightly perturbed circuits than was originally suggested by the performance of a much smaller group of UW students on the two amplifiers question. To begin, we find additional evidence in support of the same difficulties identified in the two amplifiers question.

Lack of a functional understanding of Golden Rule II.

Roughly half of the students at all three institutions provided reasoning when comparing circuits B and A that would only be appropriate if there were a current into the non-inverting input of the op-amp. The reasoning given by all of these students is inconsistent with Golden Rule II, and suggests, at the very least, that students are not drawing on the Golden Rules to check the feasibility of their responses. It is worth noting that some students did, in fact, change their responses in the interviews after considering the high input impedance of the non-inverting input.

Tendency to ascribe a voltage drop to a resistor through which there is no current.

As discussed above, when examining *all* explanations given in support of the $V_B > V_A$ ranking on both this question and the analogous comparison on the two amplifiers question (N = 207), only 28 students explicitly mentioned a current through the 10-kΩ or 20-kΩ input resistor in their written responses. While this doesn't preclude the possibility that many of the other students thought there was a current through the resistor (and into the non-inverting terminal), it indicates that at least some students were willing to ascribe a voltage drop to a resistor through which the current was either zero or not explicitly considered. Moreover, the interviews suggest that even students giving correct responses may have struggled with these same issues.

Lack of a functional understanding of Golden Rule I.

Between approximately 30% and 45% of all students incorrectly claimed that $V_C < V_B$. Roughly 15-30% of the students at both institutions indicated either implicitly or explicitly through their reasoning that the output voltages of the op-amps in both circuits are the same. If this were the case, however, the potential at the inverting input (V_-)

would be less than V_{in} since the same total voltage is necessarily split over three resistors rather than two. Thus, the reasoning given by up to nearly one-third of the students and the answers given by up to 45% of the students are inconsistent with Golden Rule I, which states that (with negative feedback) the op-amp output will "do whatever is necessary" to ensure that the potential difference between the two inputs is zero.[22] Again, students did not appear to draw on the Golden Rules in order to test the viability of their responses.

Tendency to reason locally and sequentially about the behavior of op-amp circuits.

In addition to failing to apply Golden Rule I while analyzing circuit C, up to one-third of the students made the assumption that the addition of a resistor after the op-amp in circuit C would not change the output of the op-amp. Reasoning that a change "downstream" cannot affect the "upstream" behavior of the circuit is typically referred to as local or sequential reasoning, and it is well documented in the literature on student understanding of introductory circuits.[32] Although this particular instantiation is a relatively clear-cut example of local reasoning, it is somewhat more surprising given the emphasis on negative feedback and feedback loops in basic op-amp circuits. If anything, one might have expected that the emphasis on feedback in electronics courses to highlight the impact of small changes to the feedback loop (*e.g.,* the addition of a resistor) on the behavior of the op-amp. In these less familiar situations, however, students appear to be relying on reasoning that they have largely abandoned for simpler circuits.[33]

2.3.2.4 Discussion

Collectively, the findings from the two and three amplifiers questions suggest that students lack a robust conceptual understanding of the standard non-inverting amplifier circuit after all relevant instruction. Small perturbations to this basic circuit (*e.g.,* the addition of a resistor immediately before or after the op-amp) typically result in up to one-half of the students making incorrect predictions about the behavior of the modified circuits. Perhaps most importantly, the types of predictions made by students were broadly inconsistent with the two Golden Rules that may be used (in conjunction with Kirchhoff's rules and other basic circuits concepts) to analyze the behavior of op-amp circuits with negative feedback. Indeed, the essential characteristics of ideal op-amps represented in these Rules (*i.e.,* high input impedance and the notion that the output will vary in order to ensure that the inverting and non-inverting outputs are at the same potential) were not reflected in approximately half of the student responses. Our analysis of the written responses and interview transcripts also revealed that a significant number of students were willing to ascribe a voltage drop to a resistor without clearly indicating or thinking about whether or not there was a current through it. In addition, up to one-third of the students incorrectly drew upon local (or sequential) reasoning when claiming that a resistor added after an op-amp would not change the output of the op-amp. On both written and interview tasks, a surprisingly large percentage of students experienced considerable difficulties when attempting to analyze circuits that represented small perturbations of the canonical non-inverting amplifier circuit.

2.3.3 Extension to electrical engineering courses

We recently began work on an NSF-supported project to investigate the learning and teaching of thermodynamics and electronics in undergraduate programs in both physics and engineering. As part of this effort, we have had the opportunity to examine student understanding of basic op-amp circuits in electrical engineering courses at UM.

2.3.3.1. Context for investigation

Data were collected after relevant instruction in three courses: an introductory circuits course required for all electrical and computer engineering (ECE) majors (typically taken in the sophomore year), an introductory circuits courses taken by *other* engineering majors (typically in the junior or senior year), and a junior-level analog electronics course required for all ECE majors. Both introductory level courses introduce op-amp circuits after covering basic dc circuit analysis but before covering ac circuits. Neither introductory course has a formal lab component. However, students in the introductory course for majors purchase a portable circuits kit and are asked to assemble basic op-amp circuits. The junior-level electronics course, which has a significant laboratory component, focuses on semiconductor devices and begins with a treatment of non-idealized operational amplifiers before moving into diode and transistor circuits.

2.3.3.2. Three-amplifiers question

The three amplifiers question (Fig. 2.4) was administered in all three engineering courses. The results, along with those from the UM electronics courses in physics, are presented in Table III. (It is worth noting that, although explanations of reasoning were explicitly requested, approximately 30% of the responses from students enrolled in the engineering courses did not contain any explanations.) The percentage of fully correct rankings ranged from 15% to 20%. In all populations, a significant percentage of students argued that $|V_B| > |V_A|$ due to the voltage drop across the input resistor. As was the case in the physics courses at UM and UW, essentially none of these students mentioned currents through the input resistor. The most prevalent incorrect ranking of circuits C and B given by students in all three courses was that $|V_C| < |V_B|$. The reasoning offered was largely similar to that given in the physics courses. While there was considerable variation in the exact percentages of students giving particular incorrect responses, our results indicate that the difficulties identified in our studies in physics courses are shared by both introductory and upper-division electrical engineering students. Moreover, we find that additional instruction in the more advanced context of non-ideal op-amps in the junior-level electronics course does not appear to be effective in addressing difficulties with basic op-amp circuits; this is consistent with findings from other studies in PER.[5,34]

Table 2.3. Overview of student performance on the three amplifiers question in electrical engineering and physics courses at the University of Maine. The question is shown in Fig. 2.4.

	Percentage of total responses			
	Engineering			Physics
	Circuits Non-majors (N=63)	Circuits Majors (N=97)	Electronics (N=59)	Electronics (N=49)
V_A=V_B=V_C (Correct ranking)	**24%**	**19%**	**14%**	**33%**
Correct reasoning	*5%*	*1%*	*0%*	*29%*
V_B=V_A (Correct)	**44%**	**46%**	**53%**	**49%**
Correct reasoning	*25%*	*25%*	*32%*	*45%*
V_B>V_A	43%	48%	42%	41%
Voltage drop due to input	*29%*	*34%*	*24%*	*35%*
V_C=V_B (Correct)	**40%**	**29%**	**22%**	**61%**
Correct reasoning	*10%*	*2%*	*2%*	*51%*
V_C<V_B	38%	57%	61%	31%
Voltage drop due to output	*33%*	*44%*	*29%*	*27%*
V_C>V_B	16%	11%	8%	8%
Circuit vs. op-amp output	*0%*	*3%*	*3%*	*6%*

2.4 Behavior of canonical circuits (the inverting amplifier)

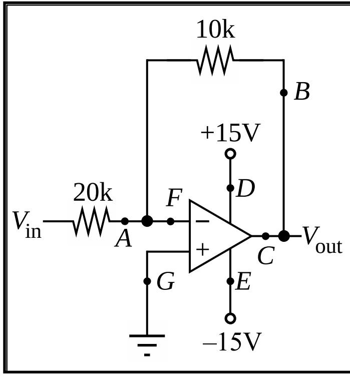
As part of our broader investigation of student understanding of op-amp circuits, we found that students really struggled with tasks involving "perturbed" or modified non-inverting amplifier circuit (which is a canonical op-amp circuit). Our results suggested that many students likely did not possess a robust understanding of the behavior of the non-inverting amplifier circuit itself, even after all instruction on basic op-amp circuits. For this reason, we were interested in developing a question in which students would be forced to think deeply about the currents and voltages in another canonical op-amp circuit – the inverting amplifier. In essence, we wished to document the extent to which students possessed the level of understanding required to derive the inverting amplifier's gain formula. The task we developed and the associated results are the focus of this section.

2.4.1 Inverting amplifier question

In this section, we describe the standard inverting amplifier question developed for this investigation and then discuss some modified versions of the question that have enhanced our insights into student thinking.

2.4.1.1 Overview

In all versions of the inverting amplifier question, students are shown the inverting amplifier circuit in Fig. 2.6. They are told that the op-amp is ideal and that there is no load connected to the output of the circuit. The input voltage V_{in} is constant and is equal to –5 V. In part 1, students are asked to find the value of the circuit's output voltage V_{out}. There are seven points *(A - G)* labeled on the diagram (Fig. 2.6); it is worth noting that points *D* and *E*, corresponding to the power rails, are not referenced in all versions of the question. In part 2, students are asked to indicate the direction of the current through point *A* or to state explicitly if there is no current through that point. In part 3, students are asked to compare the absolute values of the currents through points *F* and *G* (corresponding to the inverting and non-inverting op-amp inputs) and to indicate explicitly if any current is equal to zero. Finally, in part 4, students are asked to rank, from largest to smallest, the absolute values of the currents through points *A*, *B*, and *C*. If any of the currents are equal in absolute value or are equal to zero, students are prompted to indicate that explicitly. For all parts of the question, students are required to either explain or briefly explain their reasoning.



V_{in} = -5 V.

1. What is V_{out}?
2. Is I_A *to the right, to the left,* or *equal to zero?*
3. Is $|I_F|$ *greater than, less than,* or *equal to* $|I_G|$?
4. Rank the currents through points *A*–*C* according to absolute value.

Fig. 2.6. One version of the inverting amplifier question. For V_{in} = -5 V, students are asked to determine V_{out}, indicate the direction of the current, if any, through point *A*, compare the absolute values of the currents through points *F* and *G*, and rank the currents through points *A-D* according to absolute value.

2.4.1.2 Correct response

For part 1, there are many different approaches that may be used to determine V_{out}. Students could, for example, simply employ a memorized gain expression for the inverting amplifier. Indeed, this first part was designed so that it could be answered without a deep understanding of op-amp circuits.

In order to most clearly outline the reasoning required for all parts of this question (including additional prompts included in modified versions), however, we present a detailed analysis of the entire circuit. From Golden Rule II, the currents through points *F* and *G* are both equal to zero due to the high input impedance of the inverting and non-inverting inputs (part 3). From Golden Rule I, the electric potential at point *F* is 0 V, so current through point *A* is to the left because the potential at point *F* is higher than V_{in} = –5 V (part 2). From Kirchhoff's junction rule, the current through the 20-kΩ resistor is equal to that through the 10-kΩ resistor, so the current through point *B* is up. Since there is a single current through both resistors, a voltage drop of 5 V across the 20-kΩ resistor implies that there is a voltage drop of 2.5 V across the 10-kΩ resistor (from Ohm's law). Thus, V_{out} = +2.5 V (part 1). Because no load is attached to the output of the circuit, the current through *B* must equal that through *C* via Kirchhoff's junction rule. Thus, $|I_A| = |I_B| = |I_C|$ (part 4). Since the direction of current is from high to low potential, the currents through points *D* and *E* are both oriented down the page. By recognizing that the total current into the op-amp must equal the total current out of the op-amp (from Kirchhoff's junction rule) and that the currents through points *F* and *G* are both zero (from Golden Rule II), the current into the op-amp through point *D* must split into the current down through point *E* to the negative rail and the current to the right through point *C*. Thus, $|I_D| > |I_A| = |I_B| = |I_C| > 0$ and $|I_D| > |I_E| > 0$.

2.4.1.3 Modified versions of the inverting amplifier question

Over the course of this investigation, specific sub-parts of the question have been modified and/or added in order to probe student thinking about particular aspects of the inverting amplifier circuit. Below, we provide a brief overview of those modifications designed to examine student understanding of the rail currents.

Rail currents: Directions and magnitudes. In some versions of the question, students were explicitly asked to indicate the directions of the currents through points *D* and *E* (*i.e.*, the rail currents).[35] In other versions, students were asked to compare the absolute values of the currents through points *D* and *E*, and to indicate explicitly if any current is equal to zero.

Current ranking for points A, B, C, and D. In questions administered at UW and UM, students were asked to rank, according to absolute value, the currents through points *A*, *B*, *C*, and *D* from largest to smallest. This question replaced the somewhat simpler question in which students were asked to rank the absolute values of the currents through points *A* – *C* according to absolute value. It should be noted, however, that *A* – *C* rankings were also extracted from students' rankings for all four points on this more challenging version.

2.4.2 Overview of student performance on basic inverting amplifier task

Versions of the inverting amplifier question have been administered at UW (N = 183), UA (N = 242), and UM (N = 45) after all relevant instruction. In this section, we describe student performance on the standard version shown in Fig. 2.6. Versions targeting the rail currents, including the ranking of currents through points A - D, are discussed in part C.

Table 2.4 Overview of student performance on the inverting amplifier question in physics courses on analog electronics physics at three different institutions. The question is shown in Fig. 2

	Percentage of total responses		
	UW (N=183)	UA (N=242)	UM (N=45)
1. V_{out} = +2.5 V (Correct)	**55%**	**57%**	**69%**
Sign error	17%	5%	13%
2. Left (Correct)	**63%**	**21%**	**69%**
Correct reasoning	*50%*	*14%*	*60%*
Right	27%	59%	29%
Current from V_{in} or V_{in} to V_{out}	*5%*	*18%*	*9%*
Zero	10%	12%	2%
Golden Rule II	*4%*	*4%*	*2%*
3. $\|I_F\| = \|I_G\| = 0$ (Correct)	**79%**	**50%**	**84%**
Correct reasoning	*55%*	*27%*	*73%*
$V_F = V_G = 0$	*5%*	*5%*	*2%*
4. $\|I_A\| = \|I_B\| = \|I_C\| > 0$ (Correct)	**37%**	**7%**	**53%**
Correct reasoning	*27%*	*4%*	*36%*
$\|I_A\| = \|I_B\| > \|I_C\| = 0$	16%	16%	11%
$\|I_A\| = \|I_B\| = \|I_C\| = 0$	1%	8%	0%
Reasoning for $\|I_C\| = 0$:			
Overgeneralization of Golden	*6%*	*5%*	*9%*
Junction rule with rail	*5%*	*2%*	*0%*
$\|I_C\| > \|I_A\| = \|I_B\| > 0$	11%	8%	16%
$\|I_A\| = \|I_B\| > \|I_C\| > 0$	8%	2%	0%
$\|I_D\| > \|I_A\| = \|I_B\| = \|I_C\| > 0$ (Correct)	**13% (of N=56)**	**N/A**	**16%**

On part 1, between approximately 55% and 70% of students at all three institutions gave correct values or expressions for V_{out}. (See Table I for results by institution.) An additional 5 - 15% of students made a sign error, indicating that the output voltage would be negative. On what is arguably the most basic (and standard) question one can pose about an op-amp circuit, approximately 20% or more of all populations gave fundamentally incorrect responses for the amplifier's output voltage.

On part 2, approximately two-thirds of the students at UW and UM correctly recognized that current is to the left through point *A*, with roughly half of all students in these populations giving correct reasoning. Only about 20% of UA students indicated the correct direction, with nearly 15% supporting their correct answers with correct reasoning Approximately 25 - 30% of the students at UW and UM and 60% of the students at UA incorrectly indicated that the current through point *A* is to the right. About 20 - 30% of all such incorrect responses were supported by statements indicating that there will necessarily be current either from V_{in} into the circuit or from V_{in} to V_{out}. For example, one student wrote:

> *"Current flows from the power supply through the 20k R then through the 10k R."*

Another simply stated that current *"flows from in toward out."* This idea that current comes from the voltage source seemed to be the most prevalent explanation offered for a current to the right through point *A*. In addition, approximately 20 - 30% of these incorrect responses at UW and UM were supported by *correct* reasoning (*e.g.*, the direction of the current is from high to low potential), suggesting that some students may have been treating V_{in} as a positive voltage, which may or may not have been consistent with their responses to part 1. It is also conceivable, however, that some of these students were trying unsuccessfully to reconcile correct formal reasoning with a perhaps more intuitive sense that current should come from the voltage source, V_{in}. Between 2% and 12% of the students at all three institutions claimed that there was no current through point *A*. A large portion (from about 40 - 100%) of these responses were incorrectly justified on the basis of Golden Rule II (*i.e.*, that the op-amp inputs draw no current), suggesting that many students either failed to recognize that point *A* is located to the left of the junction or did not seem to realize that it is possible to have current through the feedback loop containing the 10-kΩ resistor. For example, one student wrote:

> *"Zero. The op-amp draws no current."*

A few students claimed that there was no current through point *A* because $V_A = 0$ V:

> *"Zero. Since 'A" is @ 'ground,' the entire V_{in} is dropped across the 20k resistor, so there is no voltage at 'A', and thus no current."*

This kind of reasoning also appeared in student responses to other parts of the inverting amplifier question.

On part 3, the majority of students (80 - 85% at UW and UM, 50% at UA) correctly indicated that the currents through points *F* and *G* were both zero. Roughly 55 - 60% of all students at UW and UM gave correct answers with correct reasoning; just over 25% of students gave correct reasoning at UA. It is also worth noting that roughly 5% of all students justified the absence of currents through points F and G exclusively on the basis of the fact that both points are grounded. For example, one student wrote:

> *"The current through F & G are both equal to zero because F & G's potentials are equal (Golden rule of op amps) & the value of V_G is zero, because it's grounded."*

We have also found evidence on this question and on others of students ranking the currents through specific points based on the relative potentials at those points. These kinds of reasoning are likely related.

On part 4, only about 35 - 55% of all students at UW and UM and 7% of students at UA correctly determined that, for the currents through points *A*-*C*, $|I_A| = |I_B| = |I_C| > 0$. The most prevalent incorrect current ranking, given by about 10 - 15% of all students, was that $|I_A| = |I_B| > |I_C| = 0$. An additional 10 - 15% of all students indicated that $|I_C| > |I_A| = |I_B| > 0$, while considerably fewer students (0 - 10%) stated that $|I_A| = |I_B| > |I_C| > 0$ (without indicating that the current through *C* was zero). Nearly 10% of the students at UA stated that $|I_A| = |I_B| = |I_C| = 0$, whereas almost no one at UW and UM gave this response.

When examining student responses in support of $|I_A| = |I_B| > |I_C| = 0$, the explanations tended to focus on why the current through point *C* must be zero. After a careful analysis of *all* student justifications for $I_C = 0$, regardless of ranking or performance on other parts of the question, two distinct (though related) categories emerged.

Tendency to generalize Golden Rule II inappropriately (i.e., assumption that there is no current into or out of any terminal of the op-amp).

Many students explicitly cited the rules or properties of the op-amp as justification for $I_C = 0$. For example, one student wrote:

> *"We know because of the axioms of the op-amp, that there is no*

> *current flowing in or out of the op-amp at the inputs/outputs."*

This student appears to have incorrectly generalized Golden Rule II to the op-amp *output* as well as to its inverting and non-inverting inputs. Some student explanations, such as the following, were considerably more specific.

> *"Op amp output gives no current because it has infinite output impedance."*

This student explicitly claimed that there is no current through point *C* because the output of the op-amp has infinite output impedance. Although students are typically taught that the op-amp's extremely low output impedance is an important characteristic of the device that makes it particularly useful, this student appears to have applied the idea of infinite input impedance, or Golden Rule II, to the output of the op-amp. Approximately 5-10% of all student explanations fell into this category.

Failure to account for the correct behavior of the rails when applying Kirchhoff's junction rule to the op-amp (e.g., incorrectly stating $I_F + I_G = I_C$ or treating $I_D = I_E = 0$).

Several students explanations for $I_C = 0$, however, differed substantively from those described above. In particular, some students gave responses similar to the following:

> *"No current at V_{in}± because op amp doesn't intake current. The op-amp has 0 current flow through it so all outputs and inputs of op-amp [are] 0 current."*

This student emphasized that, since no current enters the inverting (–) or non-inverting (+) inputs, there is no current through the op-amp, and therefore there is no current through the output of the op-amp. It is important to stress that this student recognized the importance of applying Kirchhoff's junction rule to the terminals of the op-amp and tried to ensure that the junction rule is satisfied; however, the student did not correctly account for those currents entering and exiting the op-amp via the power rails. (Given that these rails are often omitted from many diagrams of op-amp circuits, it is perhaps not surprising that some students struggle to apply Kirchhoff's junction rule to the op-amp. Indeed, if a student solely focuses on the inverting and non-inverting inputs and the output, the op-amp's behavior appears to violate Kirchhoff's junction rule.) Up to 5% of all students failed to account for the power rails correctly when applying Kirchhoff's junction rule to the op-amp. A more detailed discussion of student understanding of the rail currents is presented with the results from modified versions of the inverting amplifier question.

While the $|I_A| = |I_B| > |I_C| = 0$ response was the most common incorrect ranking, approximately 10 - 15% of all students indicated that $|I_C| > |I_A| = |I_B| > 0$. Despite the apparent prevalence of this ranking, our analysis suggests that many different lines of reasoning (all at very low percentages) are used to justify this response. Some students appeared to be treating the circuit as if there were a load, whereas others focused on the low output impedance of the op-amp or the absence of an output resistor. A few students emphasized the notion of feedback and the idea that only a part of the output is fed back to the op-amp, but they incorrectly applied these feedback arguments to currents rather than voltages.

Although not common at UW or UM, 8% of students at UA stated that $|I_A| = |I_B| = |I_C| = 0$. The most common justification given by 6% of Greek students was that there is no current in the circuit due to the absence of a load. For example, one student indicated that all three currents were zero *"because we have not attached a load to the output."* At first, we suspected that such responses stemmed from a possible overgeneralization of the fact that there will be no current through the *circuit's* output terminal in the absence of a load. However, interview data revealed that it is likely that many students argued there is no current through point C due to their incorrect understanding of the op-amp (as in the $|I_A| = |I_B| > |I_C| = 0$ responses) *and* their recognition that, in the absence of a load, all three currents must be the same as a result of Kirchhoff's junction rule. A more detailed discussion is presented in Section V.D.

2.4.3 Overview of student performance on questions involving rail currents

In the previous section, we focused on student performance on the standard version of the inverting amplifier task. In this section, we discuss student performance on questions that were included on modified versions of the task in order to explore student understanding of currents to and from the power rails.

When asked about the directions of the rail currents *(i.e.*, the directions of the currents through points D and E*)*, only 21% of UA students (N = 242) and 54% of UW (N = 56) students correctly indicated that both currents were directed down the page. Approximately 20% of both UA and UW students indicated that the currents through both points were zero. Typical reasoning for this kind of response is described in the discussion of student performance on the rail current comparison task. Sixteen percent of UW students and 28% of UA students indicated that the currents through both points were directed toward the op-amp *(i.e.,* down through point *D* and up through point *E).* While explanations in support of this answer were often unclear, up to 5% of all students focused on the idea that the rails supply power to the op-amp *(e.g., "E and D are going into the op amp to power it.").*

On the question about the relative magnitudes of the rail currents, only 1% of UA students (N = 242) and 13% of UW students (N = 76) correctly recognized that the absolute value of the current through *D* is larger than that through *E.* Approximately 45% of all students indicated that the absolute values of the currents through *D* and *E* are equal and non-zero. Twelve percent of the UW and UA students supported this comparison by noting that the rail voltages are identical in absolute value *(e.g., "Both of the absolute values of voltages are the same, and they have the same resistance so they must have the same current.").* In addition, 12% of the UW students simply focused on the role of the rails in providing power when arguing that the two current are equal *(e.g., "The abs value of current through D & E are the same & the function of this power source is to power the op-amp, so the currents will be the same.").* Roughly 15% of all students said that both currents are equal to zero. Approximately two-thirds of these responses at UW and 5% of these responses at UA were supported via an apparent overgeneralization of Golden Rule II. For example, one student wrote:

> *"Under ideal operation, op amps sink & source no current. Since power dissipation = I^2R, the op amp receives no power at inputs and sources no power. This leads me to conclude that $|I_D| = |I_E| = 0$ assuming no heat losses etc."*

An additional 15% of the $|I_D| = |I_E| = 0$ responses given by UA students were supported by the argument that there is no current anywhere in the circuit in the absence of a load.

In general, however, the explanations offered by students responding to the two questions on rail currents were rather unclear and hard to categorize; many students did not justify their answers at all. The results (including both answers and explanations) from these questions suggest that most of the students had not had pervious opportunities to think carefully and deeply about the role of the power rails in op-amp circuits.

It is therefore perhaps not surprising that only approximately 15% of the students at UW

(N = 56) and UM (N = 45) gave correct rankings of the currents through points *A* – *D* according to absolute value. Moreover, only 3 UW students and one UM student gave correct explanations for why the current through *D* must be larger than that through points *A*, *B*, and *C*. Most of the incorrect explanations for the correct current ranking focused on the higher potential of point D, the point's location on a power rail, or the fact that there would be less resistance in that part of the circuit.

2.4.4 Related interview task

The circuit shown in Fig. 2 was also used for interviews at UA (N = 29) and UW (N = 2). At UA, V_{in} = -4 V, whereas V_{in} = -5 V (as in the written task) at UW. In the interviews, students were asked to: (1) determine the output voltage, (2) find the voltages at and the currents through points *F* and *G*, (3) rank the absolute values of the currents through points *A*, *B*, and *C*, and (4) compare the absolute values of the currents through points *D* and *E*. Given that performance on parts 1 – 3 was largely similar to that reported in section III.B, we limit our discussion to student reasoning about the op-amp as a junction (from part 3) and student understanding of the power rails (in part 4).

The interviews provided considerably more insight into student thinking, particularly with respect to how students are apply Kirchhoff's junction rule to the op-amp. For example, the interview excerpt below describes the subsequent discussion that resulted after one interview participant indicated that there was no current through point *C*.

> I: *Why did you think that current is not coming out of the op-amp?*
>
> S: *Since no current is coming in! We said that the potential at F and G is zero, so there is no current coming in the inputs.*
>
> I: *Ok. Are you thinking of some op-amp rule or maybe a Kirchhoff's knot?*
>
> S: *No, just Kirchhoff.*

This student claimed that since no current is coming into the op-amp through points F and G (*i.e.*, through the non-inverting and inverting inputs), there is no output current from the op-amp. (In reasoning about the input currents, this student argued incorrectly that there is no current through a point of zero potential; this line of reasoning was also identified in student responses to the written task.) In this and other interviews, students frequently applied Kirchhoff's junction rule to the op-amp without accounting for the correct behavior of the power rails.

Students who concluded that there was no current through points *A* – *C* due to the absence of a load (a line of reasoning only identified at UA) also drew upon the junction rule in the same manner.

> S: *Sure, A = B. But there is no load. Kirchhoff must hold, the op-amp is a knot, so no current anywhere. Because nothing comes into F and G, nothing comes out at C… so A = B = 0. There is no way we can have a current.*

This student applied Kirchhoff's junction rule at multiple points in the circuit, including the op-amp itself; recognized that the currents through points A, B, and C must be equal; and then incorrectly concluded that all three currents must be equal to zero since there can be no current through points *F* and *G*. By failing to recognize that the op-amp output can serve as a source and sink of current due to the power rails, this student argued that a load was required for non-zero current. Indeed, in another interview, a student who erroneously claimed that the current in the circuit comes from the source *(i.e.,* to the right at point *A)* also concluded that there was no current anywhere in the circuit because "there is nowhere to go" without a load; in this case, the student incorrectly argued that the resistance of the op-amp output was infinite (consistent with an overgeneralization of Golden Rule II) so "current cannot go into" the output. The interviews therefore suggest that the responses from UA students claiming that $|I_A| = |I_B| = |I_C| = 0$ in the absence of a load likely stem from previously documented difficulties with the analysis of the op-amp itself (including both overgeneralization of Golden Rule II and failure to account for proper rail behavior when applying the junction rule to the op-amp) combined with a more thorough application of Kirchhoff's junction rule to the circuit (*i.e.*, recognizing that all three currents must be the same). In many ways, the $|I_A| = |I_B| = |I_C| = 0$ response is somewhat more sophisticated than the $|I_A| = |I_B| > |I_C| = 0$ response that is prevalent in all three populations.

In part 4 of the interview task, students struggled with the comparison of the rail currents through points *D* and *E*. Only 5 of 31 students correctly concluded that the absolute value of the current through *D* must be greater than that through *E* because it must also supply the op-amp's output current. Twelve students argued that the currents through points *D* and *E* are equal and non-zero for a variety of reasons, including the claim that they are the "only" currents, the fact that the absolute values of both rail voltages are equal, and a variety of other, often unclear, reasons. Two students said that both currents were equal to zero, focusing on the related ideas of high input impedance and no input current. In general, there was a sense that the students hadn't really though about the rail currents before. One student, who ultimately concluded that the current through *"D is downwards and E upwards,"* gave the following response when first asked about the current through *D:*

> S: *I have no idea. I have not found that anywhere, nobody explained them to us….*

This student's statement is consistent with the typical treatment of the power rails in most analog electronics courses; they are introduced and briefly discussed (primarily due to the constraints they impose on V_{out}), but are never analyzed in detail when considering the behaviors of canonical op-amp circuits.

2.4.5 Difficulties identified

In addition to the difficulties highlighted in Section III.B, several additional difficulties were identified through the use of the inverting amplifier task.

Tendency to apply Kirchhoff's junction rule inconsistently in op-amp circuits.

On the questions asking students to rank the absolute values of the currents through points *A*, *B*, and *C*, a significant percentage of the responses (even from students who correctly determined V_{out} and indicated that $I_+ = I_- = 0$) included rankings for which all three currents were not equal. Thus, Kirchhoff's junction rule was applied to certain junctions but not others. In some cases, a focus on students' rules about the op-amp (*e.g.,* an overgeneralized Golden Rule II) seemed to preclude the application of Kirchhoff's junction rule to the junction between points *C* and *B.* While it is clear that students in electronics courses have a solid understanding of the junction rule, the salience of specific features of these more advanced circuits seems to trigger alternative lines of reasoning that make it more difficult for students to recognize the need to apply Kirchhoff's junction rule in such cases. Kautz reports similar phenomena in the context of ac circuits.

Tendency to assume current always comes from V_{in} or that it always goes from V_{in} to V_{out}.

On the inverting amplifier question, a large percentage of students from all populations expressed the idea that current always comes from the power supply, apparently ignoring the sign of V_{in} and treating the supply as though it is only able to output current. Moreover, for some students, the voltage input and output of an op-amp circuit seemed to correspond to the input and output of current, respectively. As a result, all of these students struggled to analyze the currents in the circuit in a productive manner and typically failed to draw on foundational circuits concepts such as the relationship between electric potential difference and the direction of current.

Tendency to argue that $I = 0$ if $V = 0$ when considering voltages at a point.

On these op-amp questions, some students claimed that there is no current through points that are at electrical ground (*i.e.*, $V = 0$ V). While students rarely articulate this line of reasoning in detail, we suspect that it may possibly stem from confusion between voltage at a point (an electric potential) and voltage across an element (an electric potential difference) or from an incorrect application/generalization of Ohm's law to current through and voltage at a point. Approximately 5% of all responses to the inverting amplifier question included such arguments. (On this question and on others, some students ranked the currents through specific points based on the relative potentials at those points, which is a very similar line of reasoning.)

2.5 Comparison of Inverting and non-inverting Amplifiers

Given that, as noted by Mazzolini *et al.*, many students rely on memorized gain formulas and are easily confused by minor aesthetic modifications of canonical circuits, we were interested in ascertaining the extent to which students could differentiate the behavior of two different canonical op-amp circuits containing the same two resistors.

2.5.1 Overview of question and correct response

To accomplish this, we included the inverting and non-inverting amplifier circuits shown below (Fig. 2.7) in ranking tasks in which students were asked to compare the absolute values of V_{out} from several different circuits with identical positive values of V_{in}. Here, we limit the discussion to comparisons of the absolute values of the output voltages V_A and V_B from the inverting and non-inverting amplifiers, respectively.

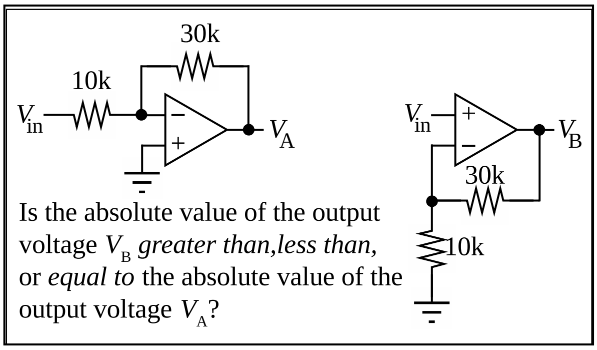


Fig. 2.7. One version of the inverting and non-inverting amplifiers comparison question. For identical and unchanging positive input voltages, students are asked to compare the absolute values of V_B and V_A.

In order to compare the absolute values of the circuits' output voltages, students may either draw on the appropriate gain formulas or reason based on the Golden Rules and foundational circuits concepts. Since $V_A = -3\ V_{in}$ and $V_B = 4\ V_{in}$, the absolute value of V_B (from the non-inverting amplifier) is greater than that of V_A (from the inverting amplifier).

2.5.2 Overview of student performance and reasoning

This comparison was included on tasks administered after relevant instruction to a total of 225 students at UW over a period of many years. See Table II. Only 56% of the students made the correct comparison ($|V_B| > |V_A|$), with roughly 30% of all students supporting the correct comparison with correct reasoning. (Due to the fact that students were typically asked to rank the outputs of four different circuits, the justifications for specific comparisons were not always clear.) Very few students (~1%) appeared to arrive at a correct comparison by erroneously comparing the output voltages rather than the absolute values of the output voltages.

Of greatest interest is the fact that approximately one-quarter of all students indicated that $|V_B| = |V_A|$. Of those students who did include explanations supporting this comparison, many (7% of all students) simply used the same gain formula for both circuits or noted that the resistors were the same in both circuits. For example, one student wrote:

> *"I believe that V_B is equal to V_A because the 10k & 30k resistors both are connected to the negative side of the op-amp. V_{out} will be the same."*

Table 2.5 Overview of student performance on the inverting and non-inverting amplifiers comparison question in a physics course on analog electronics. The question is shown in Fig.3.?.

	Percentage of total responses UW (N=225)
$\|V_B\| > \|V_A\|$ (Correct)	**56%**
Correct reasoning	*30%*
$\|V_B\| = \|V_A\|$	23%
Same gain formulas or same	*7%*
$\|V_B\| < \|V_A\|$	17%

Consistent with the observations of Mazzolini and colleagues, such students appeared to be drawing upon memorized gain formulas and failing to recognize the features of the op-amp circuits that lead to these expressions.

Roughly 15% of the students concluded incorrectly that $|V_B| < |V_A|$. While this comparison corresponds to a significant percentage of the responses, it appeared to stem from a wide variety of incorrect gain expressions and/or analyses for one or more of the circuits. As such, it was not possible to attribute this incorrect response to one or more well-defined lines of reasoning, *etc*.

2.6 Extension to electrical engineering courses.

At UM, we are currently investigating the learning and teaching of thermodynamics and electronics in both physics and engineering courses. As part of this cross-disciplinary project, we have been examining student understanding of basic op-amp circuits in electrical engineering courses.

2.6.1 Context for investigation.

Data were collected in three courses: an introductory circuits course required for all electrical and computer engineering (ECE) majors (typically taken in the sophomore year), an introductory circuits courses taken by *other* engineering majors (typically taken in the junior or senior year), and a junior-level analog electronics course required for all ECE majors. Neither introductory circuits course has a formal lab component, but students in the introductory course for majors purchase a portable circuits kit and are asked to assemble basic op-amp circuits.

2.6.2 Inverting amplifier question.

The inverting amplifier question (Fig. 2) was administered in all three engineering courses after relevant instruction. Results are shown in Table III. While student performance on part 1 (V_{out}) was quite strong, a large percentage of students in all three populations incorrectly concluded that the current through point *A* is to the right; roughly 5 - 25% of students in all three courses explicitly noted that current comes into the circuit from the power supply (or V_{in}). When students were asked to compare the absolute values of the currents through points *F* and *G*, typically less than 5% of students appeared to be reasoning that the currents were zero because of the fact that the voltages at those points were also zero. Out of those students who gave correct responses to parts 1 and 3 (typically 50 - 70%), only approximately 10 - 50% correctly recognized that the absolute values of the currents through points *A*, *B*, and *C* are equal and non-zero. Indeed, a large percentage (65 - 90%) of students who were correct on parts 1 and 3 gave rankings inconsistent with Kirchhoff's junction rule.

Table 2.6 Overview of student performance on the inverting amplifier question in electrical engineering and physics courses at the University of Maine. The question is shown in Fig. 3.?2.

	Percentage of total responses			
	Engineering			Physics
	Circuits Non-majors (N = 76)	Circuits Majors (N = 101)	Electronics (N = 68)	Electronics (N = 45)
1. V_{out} = +2.5 V (Correct)	**54%**	**68%**	**82%**	**69%**
Sign error	9%	12%	13%	13%
2. Left (Correct)	**29%**	**46%**	**62%**	**69%**
Correct reasoning	*14%*	*22%*	*50%*	*60%*
Right	53%	44%	28%	29%
Current from V_{in} or V_{in} to V_{out}	*22%*	*17%*	*6%*	*9%*
Zero	13%	10%	7%	2%
Golden Rule II	*3%*	*6%*	*1%*	*2%*
3. $\lvert I_F\rvert = \lvert I_G\rvert = 0$ (Correct)	**67%**	**70%**	**79%**	**84%**
Correct reasoning	*46%*	*48%*	*54%*	*73%*
$V_F = V_G = 0$	*1%*	*5%*	*4%*	*2%*
4. $\lvert I_A\rvert = \lvert I_B\rvert = \lvert I_C\rvert > 0$ (Correct)	**11%**	**9%**	**33%**	**53%**
Correct reasoning	*7%*	*8%*	*9%*	*36%*
$\lvert I_A\rvert = \lvert I_B\rvert > \lvert I_C\rvert = 0$	21%	7%	12%	11%
$\lvert I_A\rvert = \lvert I_B\rvert = \lvert I_C\rvert = 0$	0%	1%	0%	0%
Reasoning for $\lvert I_C\rvert = 0$:				
Overgeneralization of Golden	*8%*	*6%*	*6%*	*9%*
Junction rule with rail	*1%*	*1%*	*0%*	*0%*
$\lvert I_C\rvert > \lvert I_A\rvert = \lvert I_B\rvert > 0$	4%	13%	3%	16%
$\lvert I_A\rvert = \lvert I_B\rvert > \lvert I_C\rvert > 0$	22%	10%	33%	0%
$\lvert I_D\rvert > \lvert I_A\rvert = \lvert I_B\rvert = \lvert I_C\rvert > 0$	**0%**	**2%**	**3%**	**16%**

Roughly 15 - 25% of the students in all three populations claimed there was no current through point *C*. About 5% of students in each population gave reasoning consistent with overgeneralizing Golden Rule II. It is also worth noting that about 5 - 20% indicated that the currents through points *C, D, F,* and *G* are all zero, possibly suggesting that the op-amp is electrically isolated (which again suggests an overgeneralization of Golden Rule II). About 1 - 5% of the students in all three courses concluded that the current through point *C* was zero because they applied the junction rule but failed to account for the correct behavior of the rails. Indeed, between 20 - 40% of the responses from each course indicated that the positive rail current through point *D* was zero; a careful analysis of student explanations suggests that most students in all three engineering courses did not understand the role of the power rails.

While the exact percentages varied, results from the inverting amplifier question suggest that engineering students encounter essentially the same conceptual and reasoning difficulties as those identified in physics courses. Taken together with the electrical engineering results reported in Ref. **Σφάλμα! Δεν έχει οριστεί σελιδοδείκτης.**, it is clear that the difficulties identified in the physics populations at UA, UM, and UW are not simply artifacts of pedagogical approaches to op-amp circuits solely employed in physics courses.

2.7 Implications for Instruction.

In all populations studied, we have found evidence of substantive difficulties associated with the analysis of slightly modified, yet basic, op-amp circuits after relevant (and, in many cases, all) instruction on op-amps. Our findings are largely consistent with informal observations reported by Mazzolini *et al.*, who noted that introductory engineering students seemed to memorize gain formulas for canonical circuits and struggled when those circuits were drawn or labeled in a different manner.[21] While specific difficulties are described throughout this text, we strongly believe that our findings may inform instruction in both physics and engineering courses in a productive manner. Although we are currently in the process of developing and refining laboratory-specific activities and research-based tutorials on op-amp circuits that employ strategies similar to those in *Tutorials in Introductory Physics,*[36] here we offer some general recommendations based on our multi-year, multi-institutional investigation.

Provide additional support for students as they attempt to apply foundational circuits concepts in these more advanced contexts.

As an electronics instructor, it is sometimes tempting to gloss over the details of the application of Kirchhoff's rules, Ohm's law, *etc.* to a given electronic circuit because students indicate that they have a solid understanding of these foundational concepts. However, as described in both this article and a companion article focusing on canonical circuits, new device rules, conventions, and representations (*e.g.*, absence of rails in op-amp circuit diagrams) can make it more challenging for students to apply these concepts successfully. For example, in the two amplifier and three amplifier questions (Figs. 2 and 4), we found that many students did not appear to consider Ohm's law when deciding whether or not there is a voltage drop across the input resistor. Providing students with opportunities to apply foundational concepts in these more challenging contexts and to receive feedback on their efforts to do so can help students recognize the applicability of foundational circuits concepts throughout the course and the subtleties associated with the application of these concepts in many electronic circuits.

Use "perturbed" circuits in instruction, even if they aren't necessarily touchstone circuits.

Given time constraints, there is often a tendency in electronics courses to focus on a small number of touchstone or standard circuits that may serve as building blocks in complex designs. Our research, however, has indicated that slight modifications of standard circuits, even if those modifications seem to be pointless or non-ideal from a circuit design perspective, can be productive in that they force students to set aside memorized gain formulas and reason from foundational concepts and rules. They also may serve as excellent contexts in which students can practice both applying the Golden Rules and checking to ensure that their solutions are in fact consistent with these rules. Moreover, analyzing these "perturbed" circuits may be viewed as a first step in thinking about chaining one or more standard circuit chunks together as part of a larger design.

Provide explicit prompts for students to check that their circuit predictions are consistent with the Golden Rules and foundational circuits concepts.

Over the course of this investigation, we have collected considerable evidence indicating that students make predictions about basic op-amp circuit behavior that are inconsistent with either the Golden Rules or foundational circuits concepts. While we have found that "perturbed" circuits are productive contexts in which to elicit such inconsistencies, it may also be beneficial to prompt students explicitly to verify that their predictions are consistent with the Golden Rules, *etc.* Indeed, these kinds of basic consistency checking strategies may be used productively both in and out of the laboratory.

Emphasize the role of the op-amp's power rails.

Though they are often not included on diagrams of simple op-amp circuits, our investigation revealed that the majority of students struggled with the role of the power rails in the behavior of the op-amp circuit. Even those students who attempted to apply Kirchhoff's junction rule to the device were frequently unable to do so productively because they did not realize that the power rails enable the op-amp output to source and sink current. Without a robust understanding of the power rails, the op-amp may incorrectly be viewed as a device that violates Kirchhoff's junction rule. This, in turn, may make students more willing to abandon the very foundational circuits concepts (such as Kirchhoff's rules) upon which we hope they will build in our electronics courses. Simple measurements of rail currents in the laboratory for either inverting or non-inverting amplifiers with values of V_{in} that are positive, negative, and zero can be particularly valuable for helping students recognize that the relationship between the two rail currents changes, in accordance with Kirchhoff's junction rule, depending on whether the op-amp is sinking or sourcing current. Questions similar to the inverting amplifier question (Fig. 2) can be used to provide students with additional practice with these rail currents and op-amp as a junction.

Examine currents and voltages in one or more canonical circuits.

While students will eventually need to think about a given op-amp circuit as a single "chunk" in a larger circuit or circuit design, our findings suggest that students often are unable to think about what is actually going on in some of these very basic op-amp circuits. Providing students with an opportunity to think about the currents and voltages at various locations in a circuit for one or more values of V_{in} can help students reason through the behavior of the circuit on their own and effectively derive (or re-derive) its gain formula through the application of the Golden Rules and Kirchhoff's rules.

Provide additional support for students as they attempt to apply foundational circuits concepts in these more advanced contexts.

Even though our students may be very comfortable applying Kirchhoff's rules, Ohm's law, *etc.* to introductory-level dc circuits, it is important to remember that the kinds of circuits covered in our analog electronics courses provide additional challenges to the successful application of these concepts. Indeed, students are encountering new devices/elements, each with its own set of rules; new conventions (*e.g.*, voltage at a point); and new and increasingly abstract representations (*e.g.*, the absence of rails in many op-amp circuit diagrams, or the obfuscation of closed loops in circuit diagrams due to the use of input and output voltages and ground symbols). The results from the inverting amplifier task (Fig. 2) highlighted many of these challenges. For example, we have found that students apply Kirchhoff's junction rule to certain junctions but not to others; in some cases, overgeneralized op-amp rules appeared to be more salient for students and somehow hindered their ability to apply the junction rule to the *B-C* junction. Kautz has reported similar phenomena in the context of phase relations in ac circuits. By providing students with opportunities to apply these foundational concepts in particularly challenging contexts (*e.g.*, the inverting amplifier task) throughout the course, students can receive productive feedback and be guided to recognize the importance of and subtleties associated with applying foundational circuits concepts in all electronic circuits.

Provide explicit prompts for students to check that their circuit predictions are consistent with the Golden Rules and foundational circuits concepts.

We have found that students often make predictions about the behavior of basic op-amp circuits that are inconsistent with either the Golden Rules or foundational circuits concepts. We believe that, after students have analyzed (either individually or collaboratively) a circuit that elicits such inconsistencies, it may be beneficial to prompt students explicitly to verify that their predictions are consistent with the Golden Rules, Kirchhoff's rules, and Ohm's law. This strategy may help students explore lines of reasoning that they did not originally consider so that they can work to resolve any inconsistencies. (These kinds of strategies are frequently used in *Tutorials in Introductory Physics.*)

Chapter 3:

Diode circuits

3.1 Introduction

In this chapter we describe a systematic investigation of student understanding of the behavior of simple diode circuits. The participants in this study were undergraduates enrolled in upper-division physics courses on analog electronics at three different institutions. We focus on the use of basic diode circuits in research tasks designed to provide insight into student thinking about these circuits. The findings indicate that, after instruction, many students in physics courses are unable to productively analyze circuits that differ only slightly from those explicitly covered in lecture, laboratory, and the textbook. The most prevalent conceptual and reasoning difficulties identified in this part of the investigation are described and related implications for electronics instruction are discussed.

In section 3.2, we begin with a brief overview of previous research on introductory electric circuits and electronics conducted in both physics and engineering.

We then discuss our research methodology and context (Section III).

In Section IV, we present two free-response questions used to probe student understanding along with the data collected and difficulties identified.

Finally, in Sections VII and VIII, we discuss implications for instruction and summarize our findings in the conclusion.

3.2 Brief overview of diode coverage

In all courses, students learn that a diode is an electronic device, built by semi-conducting materials. A diode is represented by the symbol shown in Fig. 3.1, in which the *IV* characteristic of an ideal diode is also shown.

Ideal diode symbol and *IV* characteristic

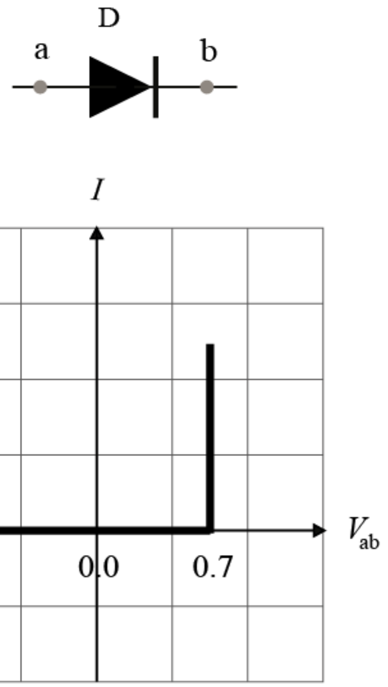


Fig. 3.1

Diode circuits are ubiquitous in electronics, particularly in power circuits. In most electronics courses, students are introduced to this passive semiconductor device by studying a number of basic circuits it is used in.

Students learn that, as derived from its *IV* characteristic, an ideal diode only allows current through it in one direction (e.g. from a to b in Fig. 3.1). That happens if the potential at point a is greater greater than the potential at point b by 0.7 V. In this case, the diode is said to be forward-biased, the voltage across it will be equal to 0.7 V and its resistance will be zero. In case the potential at point a is less than the potential at point b, then the diode's resistance is infinite and no current is allowed through the diode.

In the course, students investigate the behavior of many standard diode circuits, including the rectifier circuits.

In order to probe student understanding of diode circuits, we developed a number of research tasks, two of which are described in the sections that follow.

3.2 Questions on a simple Diode-circuit

3.2.1 Questions

In this question, students were shown a simple circuit with a diode and two resistors (Fig. 4.1). Students were told that the diode is ideal, the input voltage V_{in} is constant and equal to + 5 V, there is no load in the circuit's output and the two resistors are identical.

Students were then asked a number of questions, always explaining their reasoning. In part 1 they were asked to predict the direction of current through point a, if any. In part 2, students were asked to rank the currents through points a, b, c, and d from highest to lowest, and to explicitly state if any of the currents is zero. In part 3, they were asked to rank the absolute values of the voltages across the two resistors and the diode from highest to lowest, and to explicitly state if any of these voltages is zero.

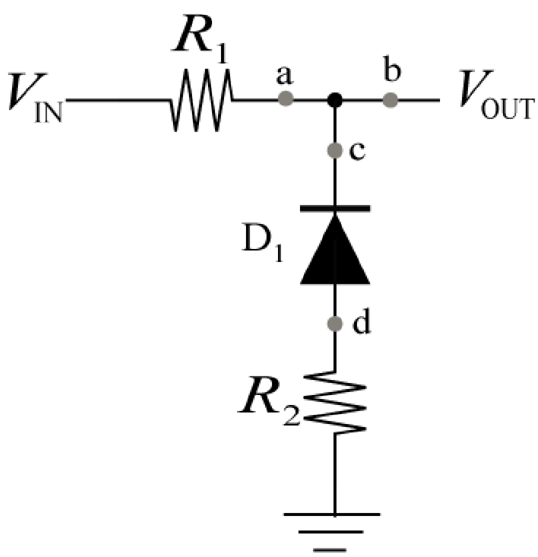


Fig. 4.1

3.2.2 Correct responses

To answer the first question, students have to understand that there is no closed loop for the current to follow. Current cannot come out of the output or come from the output because there is no load attached there. Also there cannot be current flowing from the source to point a to the ground because the diode is back-biased. So there is no current through point a.

It follows from the above reasoning that current cannot flow anywhere in this circuit, so all currents are equal to zero. So the correct answer to the second question is $I_a = I_b = I_c = I_d = 0$.

To answer question 3, students could follow one of the following lines of reasoning. Since there is no current anywhere in the circuit, there can be no voltage drop across any of the two resistors. Using Kirchhoff's loop rule, students could find that the voltage drop across the diode is equal to the source voltage, thus equal to + 5 V. Alternatively, students could say that the voltage at point a is equal to the source voltage, because there is no voltage drop across the resistor R_1. Since there is also no voltage drop across resistor R_2, then voltage at point d is equal to zero. Since voltage at point a is equal to voltage at points b and c, and the diode is placed between points d and c, the absolute value of its voltage is + 5 V. Remember that the diode is back-biased and the voltage across it should be considered as negative (V_{dc} = - 5 V). So the correct ranking is $V_D > V_{R1} = V_{R2} = 0$.

3.2.3 Overview of student responses on questions involving currents (1 - 2)

The two questions involving current (1 and 2) were administered to 343 students of three different institutions (UA, UW, and UW), in various examination periods between the years 2011 and 2012.

On question 1, only 10% - 55% of the students indicated that there is no current through point a. The majority of students claimed that there is current through point a to the right. Most of these students indicated that the diode is back-biased and there is no closed loop in this circuit. See table 3.1.

Close to how they performed on question 1, 10% - 55% of the students could answer question 2 correctly, being able to identify that there is no current anywhere in the circuit. See table 3.2. Most of the rest, about 30% of all students, would correctly say that there can be no current through points c and d, but would identify a current through points a and b, giving a ranking of a = b > c = d = 0. Students justified this by noting that by saying that the diode does not allow current in the lower part of the circuit, but treat the no-load terminal as a viable path for current. So, reasoning focuses on no current through the bottom branch due to the reverse biased diode. For example, one student wrote:

> *"We know $I_c = I_d$. Since the diode restricts the current, both are zero. Thus, all the current that flows through a, continues to b."*

Table 3.1. Responses to Question 1 (direction of current through point a).

	UA	**UW**	**UM**
	N=177	N=146	N=20
Correct	**12%**	**35%**	**55%**
no closed loop, D back-biased	6%		
D back-biased	4%		
Right	**82%**	**60%**	**45%**
from source	45%		
from high to low	6%		

Table. 3.2. Responses to Question 2 (rank currents through points a, b, c, and d, if any).

	UA	**UW**	**UM**
	N=177	N=146	N=20
Correct	9%	30%	55%
no closed loop, D back-biased	6%		
D back-biased	3%		
a=b>c=d=0	37%	35%	35%
D back-biased	29%		

3.2.4 Overview of student responses on question involving voltage (3)

Question 3 described above was involving understanding of voltages in a diode circuit. This question (3) was administered to students of three different institutions, in various examination periods between the years 2011 and 2012.

Only 5% - 35% of the students could indicate that the input voltage has to be dropped across the inversely biased diode.

One could argue that the problem is set from the poor understanding of currents. Looking at how many people are saying that there is current through point a going to the circuit's output and no current through the diode and R_2, one could expect that the $R_1 > D > R_2 = 0$ would be more prevalent. Yet, it is not, revealing a poor understanding of voltages and Kirchhoff's rules.

Table 3.3. Responses to Question 3 (rank the voltages across the two resistors and the diode).

	UA	UW	UM
	N=177	N=146	N=20
Correct	**5%**	**15%**	**35%**
$\boldsymbol{R_1 > R_2 = D = 0}$	**20%**	**35%**	**25%**
no current through D or D is back-biased	12%		
no current through R_2	8%		
ground	2%		
$\boldsymbol{R_1 = R_2 = D = 0}$	**2%**	**10%**	**10%**
no currents, no voltages	1%		
$\boldsymbol{R_1 > D > R_2 = 0}$	**11%**	**<5%**	**0%**
no current through R_2	5%		
$V_D = 0.7$ V	1%		

3.3 Related interview tasks

We interviewed UW students in Seattle (Summer 2010), as well as UA students (Spring 2011 and 2012).

In general, interviews provided insights on student thinking and explanations on how they answered the written questions.

The circuit shown in Fig. 3.1 was also used for interviews at UA (N = 17) and UW (N = 5). In both institutions, V_{in} = +6 V. In the interviews, students were asked to: (1) decide on the direction of current through point a, if any, (2) rank the currents through points a, b, c, and d, if any, (3) rank the absolute values of the voltages across the diode and the two resistors, and (4) find the output voltage of this circuit. Given that performance on parts 1 – 3 was largely similar to that reported in section 3.2, we limit our discussion to student reasoning about the diodes.

To answer the first question, students have to understand that this diode can only let current flow from point d to c, if it is forward biased. But in this circuit it is inversely biased, so no current can flow from point c to d. Since there is no load attached, then there is no closed loop for the current, so there will be no current anywhere in this circuit. So current through point a is equal to zero.

As we discussed in the first question, there is no current anywhere in this circuit, so the correct answer would be a = b = c = d = 0.

Since there is no current anywhere in the circuit, then the voltage drop across any of the two resistors will be zero. Applying Kirchhoff's loop rule, students should recognize that the input voltage will be applied across the diode (V_{in} = V_{cd}). Thus, the correct answer would be $V_D > V_{R1} = V_{R2} = 0$.

Building on what we discussed in question 3, one should be able to see that - since there is no voltage drop across R_1 - the potential at points a, b, or c will be equal to the input voltage and the output voltage. So, V_{out} = +6.0 V.

The interviews provided considerably more insight into student thinking, particularly with respect to how students are applying Kirchhoff's loop rule to a diode circuit. For example, the interview excerpt below describes the subsequent discussion that resulted after one interview participant correctly indicated that there was no current anywhere in the circuit.

> I: *... Now, please rank the absolute values of the voltages across D_1, R_1, and R_2.*
>
> S.: *Ok, ... (thinks) potential at a is Vin, since the circuit is open and no current…*
>
> I.: *So, what is the voltage drop across R_1 then?*
>
> S.: *hmm… (time) zero, because there is no current…*
>
> I.: *Ok, what about the other two elements now?*
>
> S.: *ehh… For the diode it is 0, because ... no current and also for R_2. So $R_1 = R_2 = D = 0$. D_1 has zero V across it because no current runs through it.*

This student claimed that since no current through the diode, the voltage across it is zero, treating the diode as an ohmic element and incorrectly using Kirchhoff's loop rule.

This belief is so strong that the same student refuses to believe it is wrong when his ideas are conflicted in the final question.

> I: *What, if anything, can you say about the value of V_{out}?*
>
> S.: *It will equal the potential at b, so it will be 6 V…*
>
> I.: *Ok, let's look at this. Why?*
>
> S.: *Voltage is the same at a, b or c. Open circuit, so we get the source voltage there…*
>
> I.: *Ok, I still do not get how you have a 6 V at b and a ground there and you say that the vd across that diode is zero… Can you explain?*
>
> S.: *Yeah, ... I remember that if there is going to be a current, then the voltage drop across d and c has to be 0.7 V… (thinks hard, puzzled) I do not know… This is back-biased right… I get what you are saying… (thinks hard again) it is back-biased ... The voltage drop has to be 6 V, across c and d… (more time)*
>
> I.: *So, before you said that no current means 0 V across this and that end of the diode, are you ok with the 6 V and no current situation now?*
>
> S.: *(silence for long) There is something I do not like ... Yes, probably… It has to be forward biased to allow current. I do not*

know…

Another student arrived to the same conclusion at first.

I: *We will talk about that later. Now, please rank the absolute values of the voltages across D_1, R_1, and R_2.*

S.: *Ok, … (thinks) I said no current, so zero vd across the R1 … ererywhere zero (firmly).*

I.: *Ok.*

In the next question he was able to find the correct answer, with a subtle prompting from the interviewer.

I: *What, if anything, can you say about the value of V_{out}?*

S.: *It will be equal to 6V, as it will come out of that branch (he shows the vertical one)…*

I.: *But, you say 6 V and there is nothing across that diode and that resistor…*

S.: *Yes, I get what you are saying… That is the problem, I have no way to thonk about these, how to divide up the voltages…*

I.: *So, are you sure about the vd across the two Rs?*

S.: *Since there is no current, they have to be zero.*

I.: *Ok, what about the diode then? Why is that zero?*

S.: *Because, as I think of it, since there is no current anywhere in the circuit and so there is no voltage drop, then the potential must be the same at all points! A, b, c, d … Yet, there is the ground and the potential has to drop to zero…!*

I.: *Ok, why do you think that c and d will be at the same potential?*

S.: *Yeah… The diode is not a resistor… then the diode must hold the 6 V voltage… Yet there is something I do not like… I just do not… But it must be…*

I.: *Is that consistent with the IV characteristic of this diode? Can you draw one for me?*

S.: *(easily draws a correct one) This is it.*

I.: *Ok, could you show me the point at which this is functioning in this circuit?*

S.: *I guess it must be this one… (he finds the correct one)… The* *diode could obtain a lot of different voltage values for zero value of current…!*

I.: *So is the voltage axis representing the cd or the dc voltage values?*

S.: *Oh, you want this detail… This is more complex than I thought … I guess the cd… Or… No, it is the cd is +6, so it must -6, then it shows the dc…! Never thought of that…*

I.: *Ok.*

Another student, who believes that no current through the diode means no voltage across it, is totally confused trying to apply Kirchhoff's loop rule to this loop.

I: *Please rank the absolute values of the voltages across D_1, R_1, and R_2.*

S.: *There is no voltage drop across R2 and the diode, so all that branch up o c is zero. Vout is also zero. About R1, we have 6 V to the left and 0 V at right, so there is a vd of 6 V across it. So, R1=6>D=R2=0, yes.*

I: *What, if anything, can you say about the value of V_{out}?*

S.: *It will be zero!*

I.: *Ok, let's look at these potentials on either side of R1 again....*

S.: *Oh, no how can this be without a current? Let me think again…*

I.: *Ok…*

S.: *hmmm… vd … we cannot have a vd across R1, then the potential at a has to be again 6 V…! haha… It must be the same at b and c… Could the current go up from d to c, but it would be going from 0 to 6, then it would not be a v drop, it has to be going from zero to minus… diode… (time) I do not know… I lost it… I cannot find the 6 V…*

I.: *So you say no current anywhere, 6 V at a, b and c, zero at d … but you have to have zero V across the diode, because there is no current through it, right?*

S.: *Yes… maybe the diode is destroyed and current goes through it (laughs) this is the only explanation I can think of!!!*

I.: *The way you describe it, you are having 6 V across the diode, c to d. Why is this so bad?*

S.: *The current cannot go through there down, so it must be destroyed, because I cannot have a pd across the diode and no current…! I have* *really lost it*

I.: *Ok.*

3.4 Difficulties identified

Several difficulties were identified through the use of the tasks described above.

Findings from all populations indicate that students struggle with applying device-specific rules and foundational circuit analysis ideas (e.g., Kirchhoff's rules) simultaneously. In many cases, students tended to apply Ohm's law to diodes, which are non-ohmic, when analyzing diode circuits. Results from introductory students suggest that this latter difficulty may be related to student understanding of open switches in simple dc circuits.

We describe the most prevalent conceptual and reasoning difficulties identified and discuss some of the implications of our findings for electronics instruction.

Tendency to apply Kirchhoff's loop rule inconsistently in diode circuits.

On the questions asking students to rank the absolute values of the voltages across the circuit elements, a significant percentage of the responses included rankings for the sum of the voltages in a closed loop was not equal to zero. Thus, Kirchhoff's loop rule was not correctly applied every time. While it is clear that students in electronics courses have a solid understanding of the loop rule, specific features of these more advanced circuits seem to trigger alternative lines of reasoning that make it more difficult for students to recognize the need to apply Kirchhoff's loop rule in such cases.

Tendency to argue that I = 0 yields V = 0 when considering the voltage across a back-biased diode.

As discussed in the student responses (3.2.3) and the interview excerpts ((3.3), many students claimed that there is no voltage across a diode, if there is no current through it, i.e. if the diode is back-biased. Thus, they treated the diode as an ohmic element, being unable to correctly relate to the *IV* characteristic of the diode and its implications.

Tendency to mistake voltage V_{ab} with V_{ba} across the diode.

As seen in the interview excerpts ((3.3), many students look at the *IV* curve and do not relate to the correct voltage. The curve is referring to the *V*ab voltage (see Fig 3.1), i.e. fro the back of the diode to its front. This is not always clear to the students.

Tendency to mistake the open end of a represented circuit as being loaded.

Many students identify a current through the open end of a circuit, even if an explicit statement of no load is written in the question.

3.5 Implications for Instruction.

In all populations studied, we have found evidence of substantive difficulties associated with the analysis of basic diode circuits after all relevant instruction on diodes. While specific difficulties are described throughout this paper, we strongly believe that our findings may inform instruction in physics courses in a productive manner. Although we are currently in the process of developing and refining laboratory-specific activities and research-based tutorials on diode circuits that employ strategies similar to those in *Tutorials in Introductory Physics,*[37] here we offer some general recommendations based on our multi-year, multi-institutional investigation.

Provide additional support for students as they attempt to apply foundational circuits concepts in these more advanced contexts.

As an electronics instructor, it is sometimes tempting to gloss over the details of the application of Kirchhoff's rules, Ohm's law, *etc*. New device rules, conventions, and representations (*e.g.,* absence of visible closed loops) can make it more challenging for students to apply these concepts successfully. For example, in our questions (Figs. 2 and 4), we found that many students did not appear to consider Kirchhoff's rules when deciding whether or not there is a voltage drop across the diode. Providing students with opportunities to apply foundational concepts in these more challenging contexts and to receive feedback on their efforts to do so can help students recognize the applicability of foundational circuits concepts throughout the course and the subtleties associated with the application of these concepts in many electronic circuits.

Use "perturbed" circuits in instruction, even if they aren't necessarily touchstone circuits.

Given time constraints, there is often a tendency in electronics courses to focus on a small number of touchstone or standard circuits that may serve as building blocks in complex designs. Our research, however, has indicated that slight modifications of standard circuits, even if those modifications seem to be pointless or non-ideal from a circuit design perspective, can be productive in that they force students to reason from foundational concepts and rules. Moreover, analyzing these “perturbed” circuits may be viewed as a first step in thinking about chaining one or more standard circuit chunks together as part of a larger design.

Provide explicit prompts for students to check that their circuit predictions are consistent with foundational circuits concepts.

Over the course of this investigation, we have collected considerable evidence indicating that students make predictions about basic op-amp circuit behavior that are inconsistent with foundational circuits concepts. It may also be beneficial to prompt students explicitly to verify that their predictions are consistent with Ohm’s law or Kirchhoff’s rules. Indeed, these kinds of basic consistency checking strategies may be used productively both in and out of the laboratory.

Examine currents and voltages in one or more circuits.

While students will eventually need to think about a given diode circuit as a single “chunk” in a larger circuit or circuit design, our findings suggest that students often are unable to think about what is actually going on in some of these very basic diode circuits. Providing students with an opportunity to think about the currents and voltages at various locations in a circuit for one or more values of V_{in} can help students reason through the behavior of the circuit on their own and effectively apply Ohm’s law and Kirchhoff’s rules.

Revisit the open end representation of a circuit and its effects on the circuit.

As many students identify a current through the open end of a circuit, this should be revisited. In many cases, instructors should be careful, when assuming that students easily understand that a load of infinite resistance is exactly equivalent to a circuit with no load.

Attend to the implications of the diode’s IV curve on behavior when back biased.

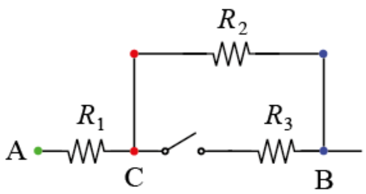
Many students are claiming there is no voltage across a diode, if there is no current through it, i.e. if the diode is back-biased, treating the diode as an ohmic element. The *IV* characteristic of the diode should be revisited, so that students can think of its implications, especially in the case of a backward biased diode.

Chapter 4:

Foundational circuits concepts

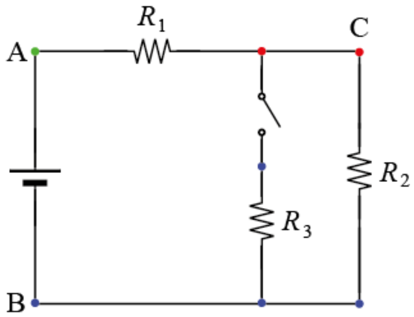
4.1 Introduction

There has been extended work in the literature on electric circuits. Many researchers suggest that students struggle with basic electric circuits concepts, like Ohm's law and Kirchhoff's rules[38].



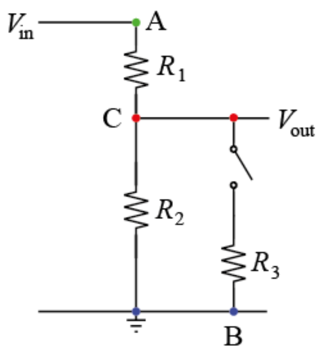
all resistors identical
constant V_{AB}

Fig. 4.1a



voltage source ideal
all resistors identical

Fig. 4.1b



all resistors identical
constant V_{in}

Fig. 4.1c

On top of that, students in upper-division electronics courses face additional challenges due to the increasingly abstract representations employed. As a specific example, a simple dc circuit or network may look very different when different representations are used, as illustrated in Fig. 4.1a, 4.1b and 4.1c.

Figure 4.1a is what students in junior high schools and high schools are used to seeing. Note that the source is not shown, but its presence is implied.

Figure 4.1b is typical for high school students, as well as students in introductory physics courses.

A couple of years later, the same students enter upper-division courses and are presented with figure 4.1c. Again, the source is not shown, and neither is the closed loop.

Figure 4.1c depicts a standard voltage divider and figures 4.1a - 4.1b are the corresponding representations in high school and introductory Physics courses. We know that voltage dividers are an integral part of the analysis of electronic circuits and any issues students face in figures 4.1a - 4.1b will affect their reasoning in and understanding of electronic circuits.

Every year, we have 240 students enrolled in the junior year electronics course in the Physics department in the National and Kapodestrian University of Athens. All these students are Physics majors, with a strong background in Physics, since they have to follow mandatory Physics courses from grade 8 and on.

Basic ideas in electric circuits are taught in Greek high schools in grades 9, 11 and 12. Students in the UA Physics department are also taught Ohm’s law and Kirchhoff’s rules in their sophomore year, as part of the EM course. As a result, students that take the Electronics course in their junior year are considered to be very familiar with basic electric circuits.

More specifically, junior year students have been taught Ohm’s law, as well as the concepts of capacitance and inductance. They also know how to deal with dc and ac electric circuits.

To test student understanding on the above, we administered a pretest / series of conceptual questions at the start of the laboratory session.

Typically, the two-hour laboratory sessions follow lecture instruction. In the laboratory, students work in pairs. The lab holds up to 12 pairs of students, so 10 laboratory sessions are organized weekly, for each of our 240 students to attend once every two weeks.

Because of this setup, we had the opportunity to administer 10 unique pretests, each one slightly different from the other, but all targeting the same conceptual difficulties. Students answered these pretests individually.

Each pretest consisted of a set of four true or false questions. Explanations were asked for every answer.

4.2 The questions

The pretests given to the students are shown in Appendices 3a and 3b.

To answer part A, students had to understand that since there is no current through resistor R_3, then current has no other path to follow than the one from point A to C and then to B through resistor R_2. So resistors R_1 and R_2 are connected in series. Statement A is true.

Based on the above conclusion and since all resistors are identical, then the potential difference across R_1 and R_2 is the same, thus statement B is true.

Closing the switch will affect the resistance of the whole arrangement. Resistor R_2 will now be in parallel with resistor R_3, thus the total resistance between points C and B will be decreased. This will cause the total resistance between points A and C to decrease. Since the potential difference V_{AC} is unchanged, then the total current will increase. The total current is going through resistor R_1, then statement C is true.

Since the current through resistor R_1 is increased, obviously the voltage drop across it will increase, thus statement D is true.

4.3 Results and Discussion

The questions were administered to 3rd year Physics majors, after all relevant lectures and instruction in DC circuits, and one month into the upper division electronics course.

We categorized the questions in two major types. Several question (e.g., questions A and C) on both pretests explicitly focused on currents when the switch was open or closed. Other question (e.g., questions B and D) on both pretests explicitly focused on voltages when the switch was open or closed.

Analysis showed that only 40% of the students were able to answer correctly and give a correct explanation on questions on current with the switch open. The percentage dropped to 20% for questions on current with the closed switch.

The task proved even more difficult when students faced the questions on potential difference. Only 30% of the students were able to answer correctly and give a correct explanation on questions on potential difference with the switch open. Moreover, the percentage dropped to 10% for questions on potential difference with the closed switch.

Table 4.1
(% of students with correct answers and correct explanations)

	Switch open	**Switch closed**
Qs on current	40% (N = 140)	20% (N = 190)
Qs on potential difference	30% (N = 98)	10% (N = 215)

Analyzing the wrong answers we identified the following student difficulties:

(a) Failure to identify that adding a resistor in parallel will *decrease* the total resistance. That led them to wrong answers on questions on current.

One student states:

> "… total resistance is increasing, because we add R_3 to the circuit … so the total current will decrease."

(b) Inappropriate application of Kirchhoff's junction rule to justify that the current through elements in series cannot be affected by changes to the circuit.

As one student claims:

> "… resistors are connected in series, so current through them will not change after we close the switch ..."

(c) Failure to recognize that the currents through two *identical* resistors connected in parallel *will* have the same value.

According to a student:

> "… when resistors are connected in parallel, then currents through them must have different values ..."

(d) Open circuit suggests zero potential difference across open ends.

As a student sees it:

> "… V_{AB} is not equal to V_{BC}, because the latter is across an open switch, so it is zero."

(e) Closing the switch affects only a specific part of the circuit, where the switch is situated (local reasoning[1]).

One student clearly says:

> "… when the switch in BC closes, loop AB will not be affected ..."

(f) Difficulties in applying multi-variable relationships such as Ohm's law.

As s students sees it:

> "… total resistance is decreased … V_{AC} is not changing, so the total current – being proportional to V_{AC} – will not change ..."

The above findings reveal a problem of which the instructor must be aware. Students do not seem to be able to deal with voltage dividers. For the upper-division electronics

[1] Lillian C. McDermott and Peter S. Shaffer, Am. J. Phys. 60 (11), 1992.

course, voltage dividers are an integral part of the analysis of Electronic circuits, as seen in the diagrams below, showing two of the basic circuits studied in any such course.

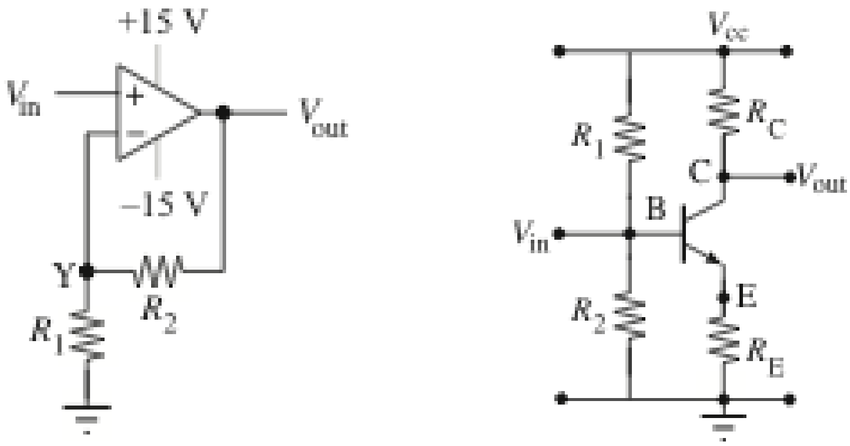


Fig. 4.2

Our findings show that students cannot really analyze the voltage divider, especially when a load is added to the circuit. They learn the rule about the load being huge with respect to the output resistance of the circuit, without grasping the origins of the rule to the simplest of circuits.

Chapter 5:

Conclusions and future work

5.1 Operational Amplifier Circuits

In this multi-year, multi-institutional study, we investigated student conceptual understanding of basic operational-amplifier circuits in the context of upper-division physics courses on analog electronics as well as electrical engineering courses on introductory circuits and analog electronics. After instruction, students in all populations struggled to analyze basic op-amp circuits. As described in this article, tasks involving canonical op-amp circuits (*e.g.,* the inverting amplifier question in which students were asked about various currents and voltages in the circuit) allowed for the identification of several specific difficulties that were prevalent in all populations. Many students failed to demonstrate a basic understanding of the role of the op-amp's power rails, and a significant percentage of students did not apply foundational circuits concepts consistently and systematically when analyzing op-amp circuits. In addition, a large number of students in all populations gave explanations of reasoning and drew conclusions that were inconsistent with the Golden Rules, as discussed in a companion article.

Our findings suggest that, after traditional instruction on op-amp circuits in both physics and engineering courses, many students have not developed the kind of robust understanding required to analyze basic op-amp circuits in a flexible and productive manner. Perhaps more importantly, this investigation has documented several prevalent student difficulties and has also revealed the need for increased emphasis during instruction on a number of specific topics including the role of the power rails in op-amp circuits. Additional instructional recommendations that have emerged from this project are described elsewhere in these paired articles. Although we have provided some general recommendations for instructors, we are actively developing and refining research-based and research-validated instructional materials on op-amp circuits that may be flexibly incorporated into either laboratory or lecture instruction. Future work will examine the effectiveness of such materials.

5.2 Diode Circuits

We also investigated student conceptual understanding of basic diode circuits in the context of upper-division physics courses on analog electronics. After instruction, students in all populations struggled to analyze basic diode circuits. As described in this text, tasks involving diode circuits allowed for the identification of several specific difficulties that were prevalent in all populations. A significant percentage of students did not apply foundational circuits concepts consistently and systematically when analyzing diode circuits.

Our findings suggest that, after traditional instruction on diode circuits in physics, many students have not developed the kind of robust understanding required to analyze basic diode circuits in a flexible and productive manner. Perhaps more importantly, this investigation has documented several prevalent student difficulties and has also revealed the need for increased emphasis during instruction on a number of specific topics including the behavior of the diode when backward biased. Instructional recommendations that have emerged from this project are described in part 3.5. Although we have provided some general recommendations for instructors, we are actively developing and refining research-based and research-validated instructional materials on diode circuits that may be flexibly incorporated into either laboratory or lecture instruction. Future work will examine the effectiveness of such materials.

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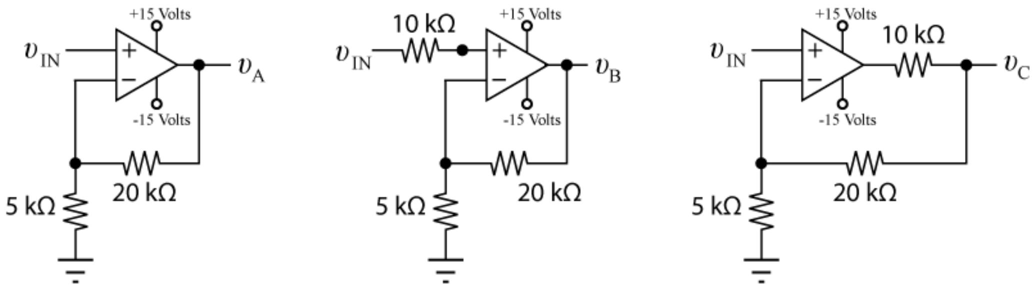
Appendix 1:

Operational Amplifiers Circuits: Interview Protocol

I. Non-Inverting Amplifier

A. What is υ_{OUT} in terms of υ_{IN}? Why? How do you find that out?



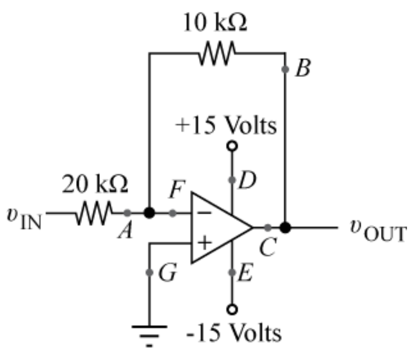
B. Compare υ_A to υ_B.

C. Compare υ_A to υ_C.

D. What is the current in υ_+?

E. What about currents? Is it a junction device? What do you think about the rails? Explain.

II. Inverting Amplifier



Let υ_{IN} = - 4 Volts.

A. What is υ_{OUT}?

What is the DIRECTION of current through F? (to the right, to the left, or zero) Explain.

What is the DIRECTION of current through G? (to the right, to the left, or zero) Explain.

B. Rank the currents through D, C, B, A. Explain.

Note: If they do not know about D, tell them to rank what they can!

C. Compare currents through D and E. Explain.

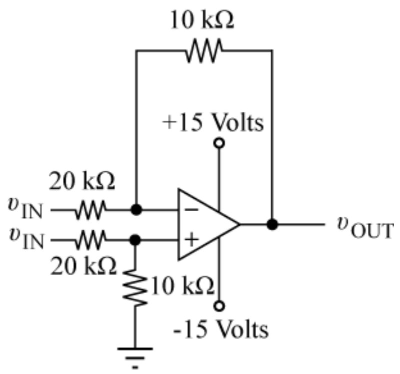
D. What are the voltages at G and A? Explain. (Ask ONLY if they do not mention.)

E. Back-up question after II-A: How many places can current enter the device? What is the role of the rails? What is the output impedance of the op-amp? Reconcile.

F. Follow-up: Suppose $\upsilon_{IN} = 0$ Volts. What is υ_{OUT}? We have measured currents through D and E to be non-zero. (demo?) Rank these currents.

III. Differential amplifier

What is υ_{OUT}? Explain. (Note: use a demo in the final phase.)

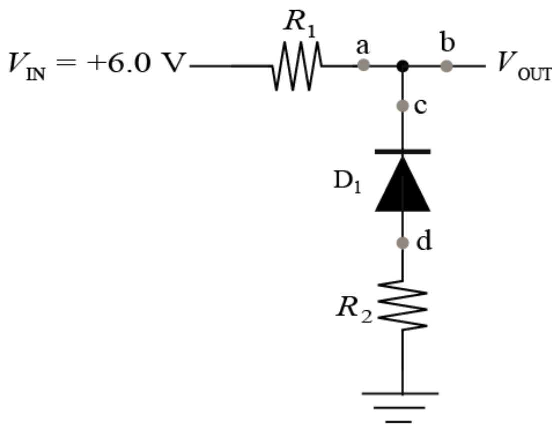


Appendix 2:
Diode Circuits: Interview Protocol

During each interview, we followed a pre-defined protocol, consisting of two parts. This protocol is shown and discussed in the following:

Part 1a (resistors – 1 diode): V_{in} is a positive DC voltage of +6.0 V.

Students were presented with the circuit diagram shown at right (see appendix 1). Attention was drawn to the facts that (a) all sources and diodes should be treated as ideal, (b) no load is attached to the output, and (c) resistors are identical.



The following questions were asked one at a time, providing no further explanations and giving students all the time they needed to think and express themselves.

Q1: Is the current at point a to the right, to the left, or equal to zero?

Q2: Rank the absolute values of the currents at points a-d.

Q3: Rank the absolute values of the voltages across D_1, R_1, and R_2.

Q4: What, if anything, can you say about the value of V_{out}?

Optional:

Q4: Grounding of the output and observations (does not show well – one bulb does not light).

Q5: The case where V_{in} = +0.4 V (or any other positive value less than 0.6 V).

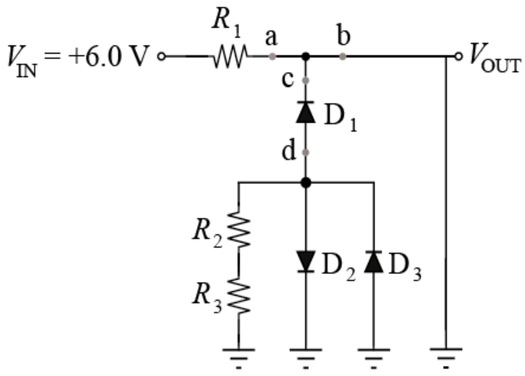


Fig. 4.3

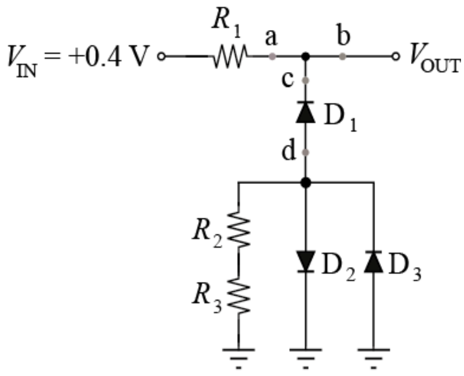


Fig. 4.3

Part 1b (resistors – 1 diode): V_{in} is a negative DC voltage of -6.0 V.

Students were presented with the circuit diagram shown at right (see appendix 2). Attention was drawn to the facts that (a) all sources and diodes should be treated as ideal, (b) no load is attached to the output, and (c) resistors are identical.

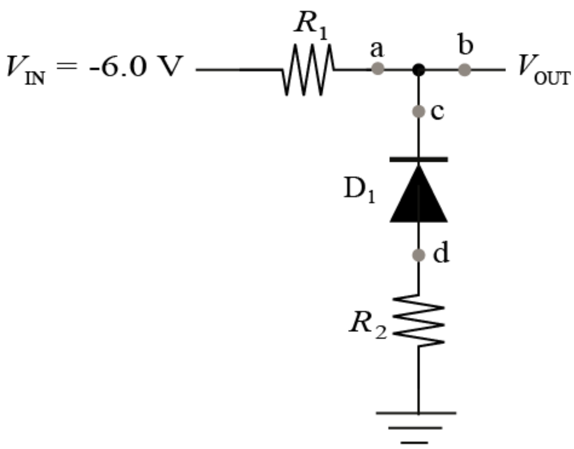


Fig. 4.1

The following questions were asked one at a time, providing no further explanations and giving students all the time they needed to think and express themselves.

Q1: Is the current at point a to the right, to the left, or equal to zero?

Q2: Rank the absolute values of the currents at points a-d.

Q3: Rank the absolute values of the voltages across D_1, R_1, and R_2.

Q4: What, if anything, can you say about the value of V_{out}?

Optional:

Q4: The case where V_{in} = - 0.4 V (or any other positive value greater than 0.6 V).

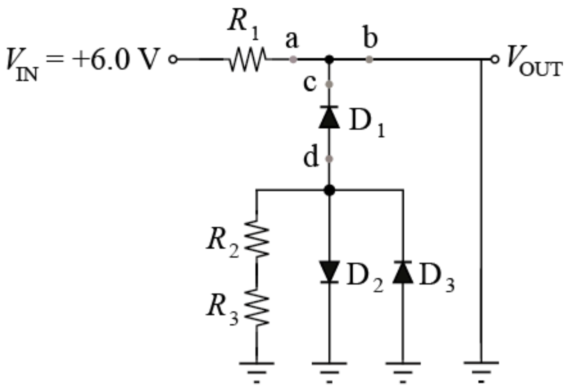


Fig. 4.3

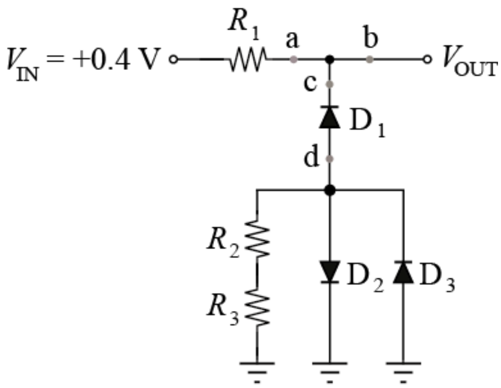


Fig. 4.3

Part 2 (resistors – 3 diodes): V_{in} is a positive DC voltage of +6.0 V.

This part was used only with students who performed very well in part 1.

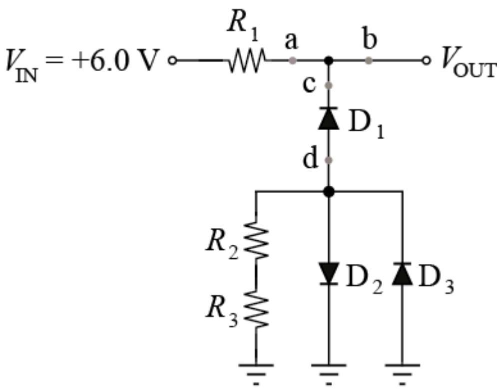


Fig. 4.3

Students were presented with the circuit diagram shown at right. Attention was drawn to the facts that (a) all sources and diodes should be treated as ideal, (b) no load is attached to the output, and (c) resistors are identical.

The following questions were asked one at a time, providing no further explanations and giving students all the time they needed to think and express themselves.

Q1: Is the current at point a to the right, to the left, or equal to zero?

Q2: Rank the absolute values of the currents at points a-d.

Q3: Rank the absolute values of the voltages across D_1, D_2, and D_3.

Part 3: Introduction to bulbs.

As a "calibration task", a simple demonstration was used: students were presented with a circuit consisted of 2 batteries and 3 identical bulbs in series. The circuit was assembled on a rectangular piece of wood to be exactly like the diagrams used.

The circuit is initially disconnected from the source, to allow students time to make a prediction, as they are asked the following question:

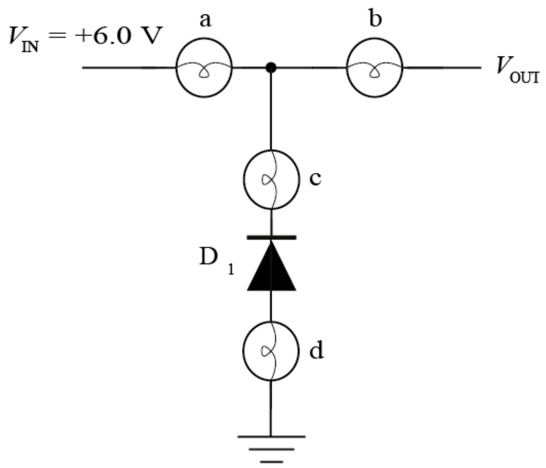
Q1. Will all the bulbs light, if we close the circuit? If so, how are their brightnesses compared? Explain.

Circuit is now connected to the source to test the students' prediction. To reconcile observation and prediction, we are asking the .following questions:

Q2. How do the currents through the bulbs compare? Explain.

Q3. How do the voltages across each of the three bulbs compare? Explain.

Part 4a (bulbs): V_{in} is a positive DC voltage of +6.0 V.

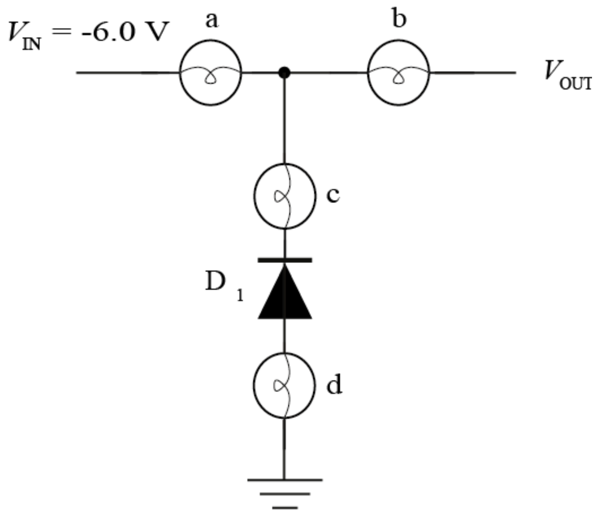


Students are also presented with a breadboard on which we have set up the circuit with the bulbs shown below right – not yet connected to the source. The "names" of the different bulbs and diodes are indicated on the actual circuit.

Attention is drawn to the facts that (a) all diodes should be treated as ideal, (b) no load is attached to the output, and (c) bulbs are used as a current indicator (see part 1).

What are you expecting to observe? In particular, which, if any, of the bulbs will light? If any of the bulbs will light, how are their brightness compared? Explain.

Part 4b (bulbs): V_{in} is a negative DC voltage of -6.0 V.



Students are also presented with a breadboard on which we have set up the circuit with the bulbs shown below right – not yet connected to the source. The "names" of the different bulbs and diodes are indicated on the actual circuit.

Attention is drawn to the facts that (a) all diodes should be treated as ideal, (b) no load is attached to the output, and (c) bulbs are used as a current indicator (see part 1).

What are you expecting to observe? In particular, which, if any, of the bulbs will light? If any of the bulbs will light, how are their brightness compared? Explain.

Part 4c (bulbs): Demonstrations.

Circuit 4a is connected to the source to test prediction.

Reconcile observation and prediction.

Circuit 4b is connected to the source to test prediction.

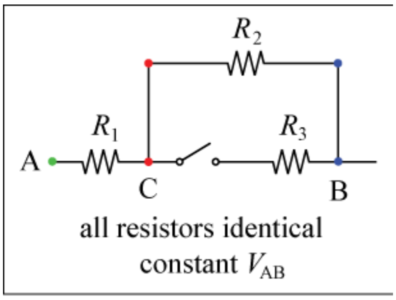
Reconcile observation and prediction.

Part 3 maybe followed by the case where V_{in} = –0.4 V (or any other negative value greater than –0.6 V).

Part 5 maybe followed (or replaced) by the case where V_{in} is a low-frequency sinusoidal signal of 6.0 V amplitude. A plot of Vin as a function of time would be provided and the student should come up with a plot of V_{out} as a function of time (difficult).

Appendix 3a:

Foundational Circuits Concepts: Pretests in UA



In the circuit at right, voltage V_{AB} is constant and positive. All resistors are identical. The switch is initially open.

Characterize each of the following statements as True or False. Explain your reasoning.

A. While the switch is <u>open</u>, current through resistor R_1 is the same as current through resistor R_2.

B. While the switch is <u>open</u>, voltage V_{AC} equals voltage V_{CB}.

C. If we <u>close</u> the switch, then current through resistor R_1 will increase.

D. If we <u>close</u> the switch, then the potential difference V_{AC} will increase.

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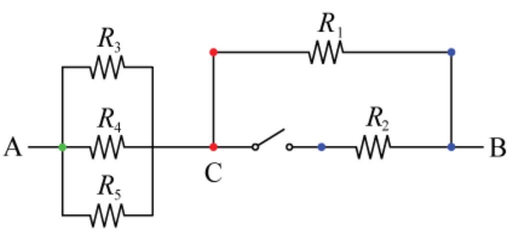
Appendix 3b:

Foundational Circuits Concepts: Pretests in UA

In the circuit at right, voltage V_{AB} is constant and positive. All resistors are identical. The switch is initially open.



all resistors identical
constant V_{AB}

Characterize each of the following statements as True or False. Explain your reasoning.

A. While the switch is <u>open</u>, currents through resistors R_1 and R_4 are equal.

B. While the switch is <u>open</u>, the potential difference V_{AC} is equal to potential difference V_{CB}.

C. If we <u>close</u> the switch, then the total current through dipole AB will decrease.

D. If we <u>close</u> the switch, then voltage V_{AC} will increase.

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Appendix 4a:

Tutorial 1 - Voltage Dividers and loading

Voltage divider with no load.

Consider the circuit at right. The input voltage V_{in} is from an ideal DC voltage source. $R_1 = R_2 = R$.

What is V_{out}? Show your work and/or briefly explain your reasoning.

Suppose a student wants to increase the value of V_{out}. What change(s) could he or she make to the circuit? Explain.

Loading the voltage divider.

A load of resistance R_L is now added to the circuit from part I as shown. $R_1 = R_2 = R$.

Suppose $R_L = R$. What is V_{out}? Show your work and/or briefly explain your reasoning.

How does V_{out} for this circuit compare to $V_{out,\ no\ load}$? Why is this the case?

What is I_L (the current through the load)? Explain.

Stop here for a brief class discussion.

Suppose $R_L = 0$? What is V_{out}? (*Hint:* Sketch an equivalent circuit.) Explain.

What is I_L? Explain.

Suppose $R_L >> R$? What is V_{out}? (*Hint:* Sketch an equivalent circuit.) Explain.

What is I_L? Explain.

Without performing a calculation, what, if anything, can you say about the value of V_{out} when $R_L = 10R$? Briefly explain.

Calculate V_{out} when $R_L = 10R$. Is the value consistent with your previous response?

Under what conditions will the addition of a load not significantly impact the value of V_{out}?

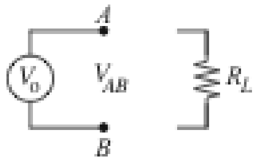
Stop here for a brief class discussion.

We will revisit and refine these conditions on the load resistance when we study Thevenin equivalent circuits.

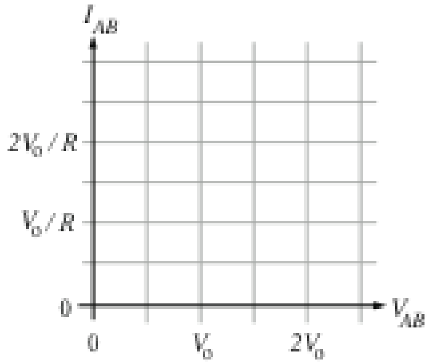
Appendix 4b:
Tutorial 2 - Equivalent Circuits

I. Ideal voltage and current sources.

A. Consider the two-terminal circuit network at right. The ideal DC voltage source (or ideal battery) maintains a constant voltage of V_o between open terminals A and B.

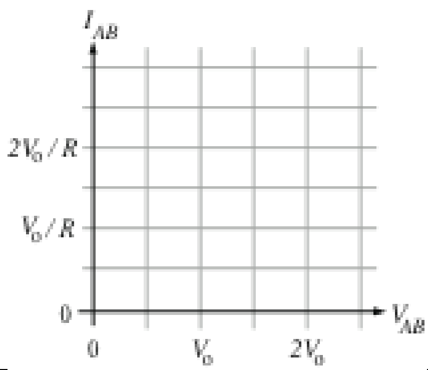


A1. Suppose a load resistor of $R_L = R$ were attached between open terminals A and B. Let I_{AB} be the absolute value of the current that flows out of terminal A and into terminal B. (Note that I_{AB} is equivalent to I_L, the absolute value of the current through the load.) Let V_{AB} be the absolute value of the voltage between terminals A and B. In the space at right, draw a qualitatively correct graph of I_{AB} vs. V_{AB} for the two-terminal circuit network when $R_L = R$. Briefly explain.



A2. Suppose the load resistor were replaced such that $R_L > R$. How, if at all, would your plot from part A change? Explain.

A3. Now suppose that R_L is varied from 0 to ∞. Draw a qualitatively correct graph of I_{AB} vs. V_{AB} for the two-terminal circuit network. Such a graph is typically referred to as the *load line* of the two-terminal circuit network.

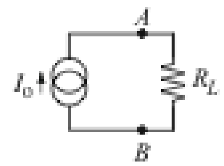


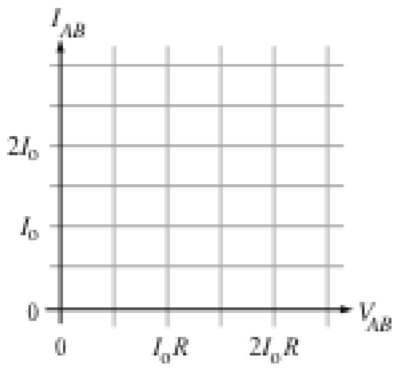
How does your graph reflect the fact that the DC voltage source is ideal? Explain.

A4. On your graph from part 0, draw a qualitatively correct *IV* characteristic curve for a load resistor with $R_L = R$. Briefly explain and describe how the two graphs may be used to determine the *operating point* of the circuit.

B. Consider the two-terminal circuit network at right. The ideal DC current source maintains a constant current of I_o when a load is connected between terminals *A* and *B*. Let R_L represent the resistance of the load.

B1. Suppose that R_L is varied from 0 to ∞. Draw a qualitatively correct graph of I_{AB} vs. V_{AB} (or load line) for the two-terminal circuit network. Explain.

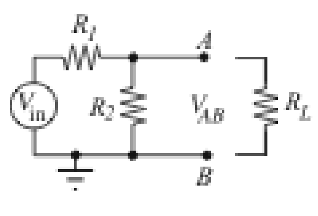




B2. Briefly describe how you could use your plot from part 0 in order to determine the operating point of the circuit for a load resistor $R_L = R$.

Stop here for a brief class discussion.

II. The voltage divider as a two-terminal circuit network



Consider the circuit at right. The input voltage V_{in} is from an ideal DC voltage source. $R_1 = R_2 = R$. A load of resistance R_L is added to the circuit between terminals *A* and *B* as shown. *(Note:* You examined this circuit in the tutorial *Voltage dividers and loading.)*

A. In this part of the tutorial, you will be asked to draw the load line for this two-terminal circuit network. To help you plot the load line for this network, consider the following questions.

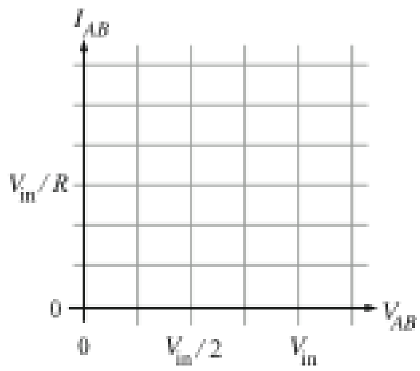
A1. Suppose $R_L \to \infty$. What is V_{AB}? What is I_{AB} (or I_L) in this limit? Show your work and/or briefly explain your reasoning.

A2. Suppose $R_L = 0$. What is V_{AB}? What is I_{AB} in this case? Show your work and/or briefly explain your reasoning.

A3. Suppose $R_L = R$. What is V_{AB}? What is I_{AB} in this case? Show your work and/or briefly explain your reasoning.

B. Plot the three data points from part 0 on the graph of I_{AB} vs. V_{AB} at right.

C. On the same graph, draw the load line for the two-terminal voltage divider circuit. Briefly explain.



Is the load line for the two-terminal voltage divider circuit linear or non-linear?

Does the voltage divider circuit behave like an ideal voltage source? Does the voltage divider circuit behave like an ideal current source? Explain how you can tell.

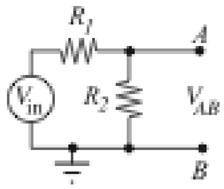
How, if at all, does the load line for the voltage divider circuit differ from the *IV* characteristic curve for a load resistor of constant resistance *R?* Explain.

Stop here for a brief class discussion.

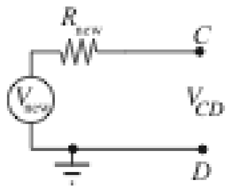
For two-terminal circuit networks containing only voltage source, current sources, and linear elements such as resistors, the relationship between voltage and current is linear, as you have seen in part 0. Mathematically, this relationship is given by $V_{AB} = aI_{AB} + b$, where *a* and *b* are the two constant parameters that characterize the two-terminal network.

III. Thevenin equivalent circuits

In section 0, you examined the load line characterizing the simple voltage divider circuit reproduced at right and labeled as circuit 1. Consider the two-terminal circuit below right, circuit 2, which consists of a linear resistor of resistance R_{new} in series with an ideal DC voltage source with a voltage V_{new}. In this section, you will be asked to identify an appropriate voltage source and linear resistor so that circuit 2 behaves identically to circuit 1 for all loads.



Circuit 1



Circuit 2

A. If two different two-terminal circuit networks behave identically for any given load resistance, how must their load lines compare? Explain.

B. In order for circuit 2 to behave identically to circuit 1 under open-circuit conditions ($R_L \to \infty$), V_{new} must be equal to what value? Explain.

Under open-circuit conditions ($R_L \to \infty$), what, if anything, can be said about the value of I_{CD} (or the current through the load) for the new circuit? Explain.

C. Under short-circuit conditions ($R_L = 0$), what, if anything, can be said about the value of V_{CD}? Briefly explain.

In order for the new circuit to behave identically to the original voltage divider, I_{CD} must have what value under short-circuit conditions? Explain.

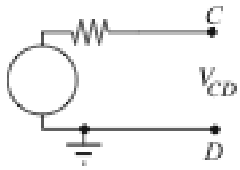
Based on your work in part B, what, if anything, can you say about the value of V_{new} for this circuit, when R_L=0?

What, if anything, does this imply about the value of R_{new}? Explain your reasoning and show any work.

D. Equivalently to your work in part C, you could find the value of R_{new} with the following procedure: Eliminate the function of the independent voltage source, i.e. short-circuit it. What is now the value of R_{new}? Briefly explain. (*Hint.*: Draw the circuit.)

Is the value you just arrived at the same as the one you came up in part C?

E. On the diagram of circuit 2 reproduced at right, indicate the values of V_{new} and R_{new} that you obtained in parts 0 and 0.



Circuit 2

For these voltage and resistance values, does circuit 2 behave identically to circuit 1 when $R_L = R$? Show your work and explain your reasoning.

F. In what sense is circuit 2 from part 0 equivalent to circuit 1?

Stop here for a brief class discussion.

As you have seen, it is possible to choose the voltage and resistance values for circuit 2 such that its function is identical to that of circuit 1 for all loads. This observation is consistent with *Thevenin's Theorem,* which states that any two-terminal network of sources and resistors - generalizing: a linear network circuit - is equivalent to a single ideal voltage source, V_{TH}, in series with a single resistor, R_{TH}. The steps for determining the Thevenin equivalent circuit that you used in parts 0 and 0 or D are summarized as follows:

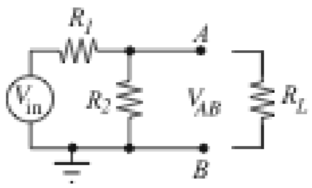
(1) Determine the open-circuit ($R_L \rightarrow \infty$) voltage across the terminals. $V_{TH} = V_{open\ circuit}$.

(2) Determine the short-circuit ($R_L = 0$) load current, $I_{L,\ short\ circuit}$. $R_{TH} = V_{TH}/I_{L,\ short\ circuit}$. Alternatively, you might eliminate the function of the independent voltage and current

sources (short-circuit the voltage sources and open-end the current sources) and calculate the resistance.

IV. Loading the voltage divider: Revisited

A. In the tutorial *Voltage dividers and loading*, you identified the conditions under which the addition of a load R_L will not significantly impact the value of V_{AB} (or V_{out}) in a circuit like that shown at right. Assume $R_1 \neq R_2$. What are those conditions?



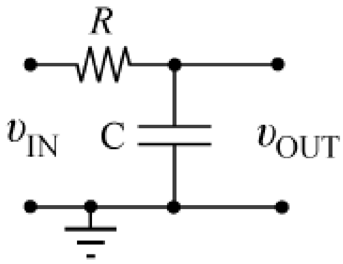
B. Apply Thevenin's theorem in order to determine an appropriate equivalent circuit.

How might you refine your statement of the conditions on the load resistance (from part 0) by considering this Thevenin equivalent circuit? Explain.

Appendix 4c:

Tutorial 3 - RC Circuits: Filters and AC Voltage Dividers

I. DC signals



Consider the circuit at right.

A. Let the input be a DC voltage $\upsilon_{IN} > 0$ and assume the circuit is closed for a long time.

Is the capacitor fully charged or not? Explain.

Which plate of the capacitor is positively charged? Explain.

Is there current through the circuit? Explain.

In this case, does the capacitor behave like an open switch or like a wire with no resistance? Briefly explain.

What is the voltage across the resistor? Explain.

What is the output voltage of this circuit (voltage across the capacitor)? Explain.

B. In the original circuit above, what changes can we make in order for the capacitor to charge to the same potential difference more quickly? Explain.

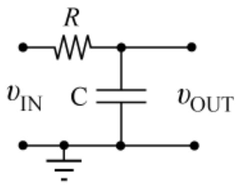
For a given υ_{IN}, what determines how quickly a capacitor gets fully charged?

C. For the original circuit, assume that the capacitor is uncharged again. Let the input be a DC voltage $\upsilon_{IN} < 0$ and assume the circuit is closed for a considerable time.

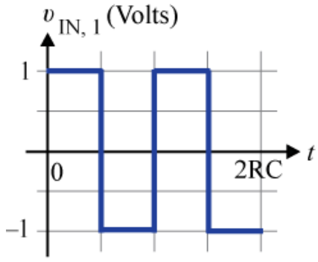
Which plate of the capacitor is positively charged? Explain.

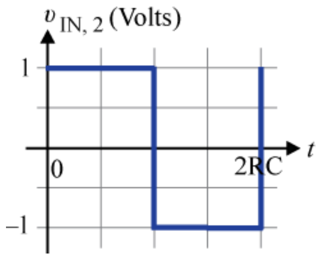
What is the output voltage of this circuit (voltage across the capacitor)? Explain.

D. Consider the circuit at right. The capacitor is uncharged at t = 0 s. Assume the input signal is either of the two square waves shown below right.



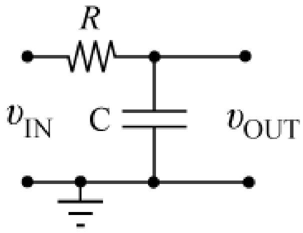
For which of the two input voltage signals will the absolute value of υ_{OUT} attained be greater? Explain.





II. AC signals

A. Let the input be a sinusoidal AC voltage υ_{IN}, of very-very <u>low frequency</u> $(T\,? \quad \tau = RC)$.



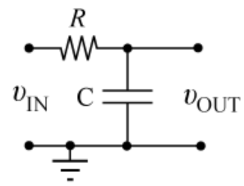
How, if at all, is this situation similar to the cases you examined in part IA-C? Explain.

Does the capacitor spend more time fully charged of uncharged? Explain.

In this case of very-very low frequency signal, does the capacitor behave more like an open switch or more like a wire with no resistance?

In this limit, what happens to the peak-to-peak output voltage of this circuit? Explain.

B. Let the input be a sinusoidal AC voltage υ_{IN}, of very-very <u>high frequency</u>.



Does the capacitor spend more time fully charged of uncharged? Explain.

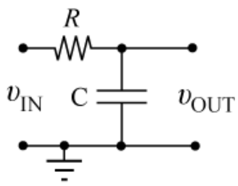
In this case of very-very low frequency signal, does the capacitor behave more like an open switch or more like a wire with no resistance?

In this limit, what happens to the peak-to-peak output voltage of this circuit? Explain.

C. Recall that the magnitude of the impedance of the capacitor is equal to $\frac{1}{\omega C}$.

Explain why this is consistent with your answers above.

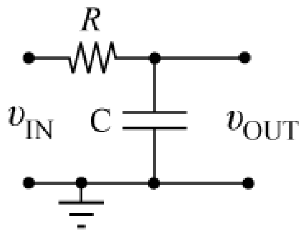
D. Examined in the frequency domain, this circuit is called a "*Low-Pass filter*". Is this name justified? Explain.



What could happen if we measured υ_{OUT} across the resistor? Explain. (Hint: Draw the new circuit.)

III. Extending the idea of the voltage divider

A. Let the input be an AC voltage υ_{IN} (do not assume very low or very high frequencies). This circuit can be considered as a voltage divider.



Suppose the frequency of the input voltage is increased. Does the magnitude of the impedance of the capacitor *increase, decrease* or *remain the same*? Explain.

As the frequency of the input voltage increases, does the peak-to-peak voltage across the capacitor *increase, decrease* or *remain the same*? Explain.

What would the voltage across the capacitor be, if a DC voltage was used as input? Explain.

Is your answer consistent with the ones in part IA and IB? If not, resolve any inconsistencies.

B. As the frequency of the input voltage increases, is the peak-to-peak voltage <u>across the resistor</u> *increased*, *decreased* or *remains the same*? Explain.

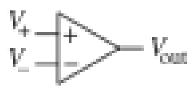
What would the voltage across the resistor be, if a DC voltage was used as an input? Explain.

Is your answer consistent with the ones in part IA and IB? If not, resolve any inconsistencies.

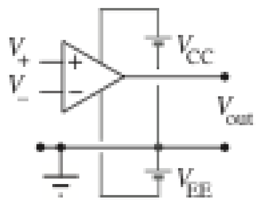
Appendix 4d: Tutorial 4 - Introduction to Operational Amplifiers and Negative Feedback

Σε αυτό το tutorial, εισάγουμε τον *τελεστικό ενισχυτή* (ή *τ.ε.*) ως ένα στοιχείο κυκλώματος, και ερευνούμε τον τρόπο με τον οποίο μπορεί να χρησιμοποιηθεί σε συγκεκριμένα κυκλώματα. Ακόμα, δείχνουμε τη σημασία και διατυπώνουμε τους δύο *χρυσούς κανόνες* για την συμπεριφορά ενός τ.ε. με αρνητική ανάδραση.

Τελεστικοί ενισχυτές: Εισαγωγικό κείμενο

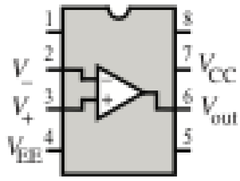


Ένας τελεστικός ενισχυτής είναι ένας διαφορικός ενισχυτής με υψηλό συντελεστή απόδοσης που αποτελείται από ένα μεγάλο αριθμό τρανζίστορ ενσωματωμένων σε ένα ολοκληρωμένο κύκλωμα. Το σύμβολο ενός τ.ε. φαίνεται στα δεξιά. Κάθε τ.ε. έχει δύο εισόδους σημάτων: μία μη-αναστρέφουσα είσοδο (V_+) δείχνεται με ένα "+" και μία αναστρεφουσα είσοδο ($V_–$) δείχνεται με ένα "–". Ο τ.ε. έχει επίσης μία έξοδο σήματος, που αναγνωρίζεται στο διάγραμμα ως V_{out}.



Όπως και τα ττρανζίστορ που τον αποτελούν, ο τ.ε. είναι ένα ενεργό στοιχείο κυκλώματος και για αυτό απαιτεί μία εξωτερική (DC) τροφοδοσία για τη σωστή λειτουργία του. Μπορούμε να τροφοδοτήσουμε τον τ.ε. χρησιμοποιώντας το κύκλωμα που φαίνεται στα δεξιά. Συχνά, όπως και σε αυτό το tutorial, $V_{CC} = V_{EE} = V_{supply}$ (τυπικά 10 – 12 V), και έτσι and thus the op-amp's power-supply inputs are connected to $+V_{supply}$ and $–V_{supply}$. (In many circuit diagrams, the power-supply circuit is not explicitly shown.) The output of the op-amp is limited by the external power source, so $–V_{EE} \leq V_{out} \leq V_{CC}$.

A diagram of the chip itself, including the pins for V_{CC} and V_{EE}, is shown at right.



The op-amp is a differential amplifier, meaning that the voltage difference between the two inputs is amplified. The behavior of the op-amp may be represented mathematically by $V_{out} = G(V_+ - V_-)$, where G is the amplifier's open-loop voltage gain.

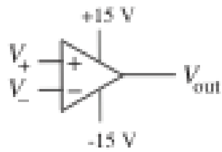
Typical op-amps have the following characteristics for DC and low frequency (<100 Hz) inputs:

- The impedance of the inverting and non-inverting op-amp inputs is very large ($10^9\ \Omega - 10^{12}\ \Omega$);
- The output impedance of the op-amp is very small (typically less than 50 Ω); and
- The open-loop voltage gain G of the op-amp is a very large positive number (approximately $10^5 - 10^8$).

An *ideal op-amp* is characterized by infinite input impedance, zero output impedance, and an infinite voltage gain. Given the typical op-amp characteristics described above, we can often treat the op-amps we use in this course as being ideal (or nearly ideal).

Ένας διαφορικός ενισχυτής με μεγάλο συντελεστή ενίσχυσης

Θεωρήστε το κύκλωμα τ.ε. στα δεξιά. $V_+ = 1.3$ V και $V_- = 1.2$ V. Υποθέστε $G = 10^5$.



Θεωρήστε την επόμενη δήλωση ενός μαθητή:

> "Εφόσον γνωρίζουμε τον συντελεστή ενίσχυσης τάσης G για τον τ.ε., η τάση εξόδου πρέπει να είναι 10,000 V."

Συμφωνείτε ή διαφωνείτε με τον μαθητή; Εξηγήστε.

Ποια είναι η μέγιστη τάση εξόδου που μπορεί να προσφέρει αυτός ο τ.ε.; Εξηγήστε.

Πώς θα άλλαζε, αν θα άλλαζε καθόλου, η τάση εξόδου αν η V_+ αυξανόταν σε 1.4 V ενώ η V_- παρέμενε ίδια; Εξηγήστε.

Πώς θα άλλαζε, αν θα άλλαζε καθόλου, η τάση εξόδου αν η V_+ παρέμενε 1.4 V ενώ η V_- αυξανόταν σε 1.7 V; Εξηγήστε.

Για ποια περιοχή τιμών της διαφορικής τάσης εισόδου ($V_+ - V_-$) θα παίρνει η τάση εισόδου τιμές κάπου μεταξύ των +15 V και των –15 V? Εξηγήστε.

Με βάση αυτό, τι συμπέρασμα μπορούμε να βγάλουμε για τη σχέση των V_+ και V_- όταν ο τ.ε. παράγει τάσεις εξόδου στην περιοχή αυτή;

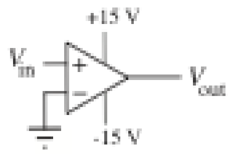
Έστω $V_- = 0$ V. Ποια τιμή της V_+ θα δίνει τάση εξόδου +5 V;

Η μη-αναστρέφουσα είσοδος

Consider the circuit at right. As before, assume $G = 10^5$.

Suppose $V_{in} = -20\ \mu V$.

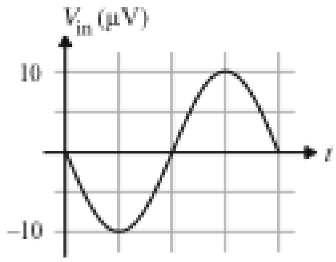
What is the voltage at the inverting input of the op-amp? Explain.

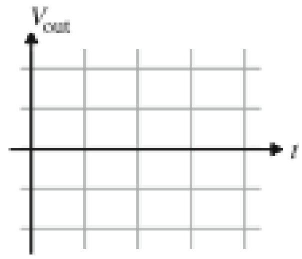


What is the output voltage of this circuit? Show your work and briefly explain.

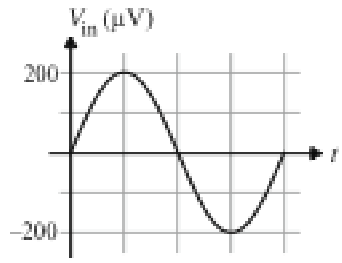
How does V_{out} compare to V_{in}? Consider both sign and absolute value.

Suppose V_{in} were the AC sinusoidal signal shown at right. In the space provided, sketch V_{out} as a function of time. Scale your V-axis appropriately. Briefly explain.

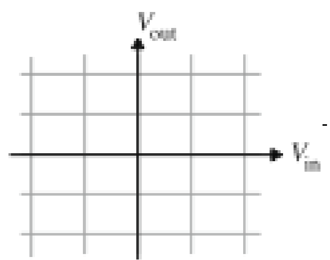




Why is the name *non-inverting input* appropriate for V_+?



Now suppose V_{in} were the sinusoidal signal shown at right. Note that the amplitude is 20 times larger. In the space provided, plot a qualitatively correct graph of V_{out}



as a function of V_{in}. Explain.

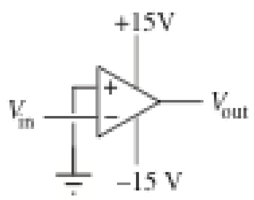
How, if at all, would your graph differ if V_{in} were a sinusoidal signal of amplitude 2 V (rather than 200 μV)? Explain.

The inverting input

Consider the circuit at right. As before, assume $G = 10^5$.

Suppose $V_{in} = -20$ μV.

What is the output voltage of this circuit? Show your work and briefly explain.



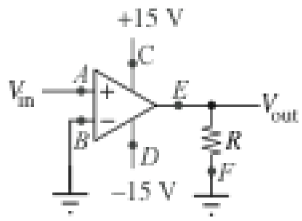
How does V_{out} compare to V_{in}? Consider both sign and absolute value.

Suppose V_{in} is an AC sinusoidal signal. What is the phase difference between V_{out} and V_{in}?

Why is the name *inverting input* appropriate for V_-?

Ρεύματα σε κυκλώματα τ.ε.

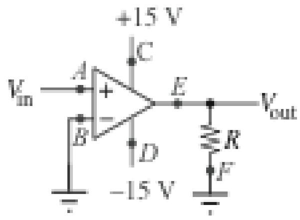
Consider the circuit at right. Assume V_{in} = 20 μV.

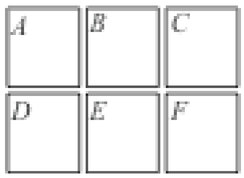


What is the approximate value of the current drawn by the non-inverting input? (*Hint:* Consider the value of the impedance of the input.) Explain.

If V_{in} had been connected to the inverting input instead, what could you say about the approximate value of the current drawn by the inverting input? Explain.

For each of the labeled points, *A*–*F*, indicate the direction of the current in the corresponding boxes. If there is no current at any point, state so explicitly. Explain.





Rank, from largest to smallest, the absolute values of the currents through points *C, E,* and *F*. Explain.

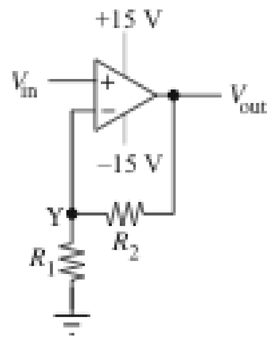
Suppose V_{in} were –20 μV. Rank, from largest to smallest, the absolute values of the currents through points *D*, *E,* and *F*. Explain.

In general, what is the relationship between the total current into the op-amp and the total current out of the op-amp?

Μη-αναστρέφων ενισχυτής

Consider the circuit at right. Assume V_{in} is a constant DC voltage.

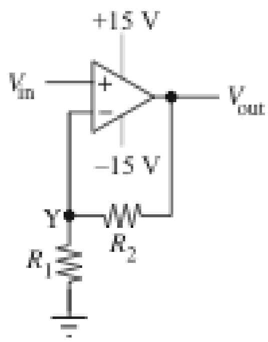
Write an expression for V_{out}. Express your answer in terms of V_{in}, V_Y, and G (the open-loop voltage gain of the op-amp). Explain.



Is there current through resistors R_1 and R_2? (*Hint:* In order for both resistors to have no current, what values would V_Y and V_{out} need to have? Are these values possible for this circuit?) Explain.

How must the current through R_1 compare to that through R_2? Explain.

Write an expression for V_Y in terms of V_{out}, R_1, and R_2. Briefly explain why this expression makes sense. (*Note:* The circuit is reproduced at right.)



Use your expressions from parts A and D to determine the relationship between V_Y and V_{in}.

Recall that $G = 10^5$. In most applications, R_1 and R_2 are greater than or equal to 100 Ω, and $R_2/R_1 \leq 10^3$. Under such conditions, is V_Y (V_-) *significantly greater than, significantly less than,* or *approximately equal to* V_{in} (V_+)? Briefly explain.

Consider the following student discussion:

Student 1: "MBased on what we have done, it seems reasonable to treat the inverting and non-inverting input voltages as being essentially the same in this circuit."

Student 2: "I disagree. If the input voltages are the same, the output voltage must be zero. But we've argued that the output cannot be zero, so the inputs must be different."

With which student, if either, do you agree? Explain.

Use the approximation you have made for V_- above in order to express V_{out} in terms of V_{in}, R₁, and R₂.

What is the *voltage gain* (V_{out}/V_{in}) for this circuit? Explain.

Does the voltage gain of the circuit depend on the open-loop gain G of the op-amp? Explain.

In the non-inverting amplifier circuit above, we have seen an example of *feedback* in an op-amp circuit. In general, the idea of feedback is that a part of the output signal is sent back to one of the two op-amp inputs, thereby producing a subsequent change in the output. (In the non-inverting amplifier circuit, the R_1 and R_2 voltage divider chain ensured that a specific fraction of the output was sent to the inverting input.) If this change in the output is of the same sign as the change in the feedback signal, then it is called *positive feedback.* When the sign of the resulting change in the output is opposite to that of the change in the feedback signal, it is called *negative feedback.*

Consider the non-inverting amplifier circuit discussed above. Suppose V_- were to increase slightly while $V_{in} = V_+$ remained unchanged. Would V_{out} *increase, decrease,* or *remain the same?* Explain. (*Hint:* Recall that the op-amp is a differential amplifier.)

Is the feedback in the non-inverting amplifier circuit *positive* or *negative?* Explain.

In general, feedback in an op-amp circuit is positive when the signal is returned to the non-inverting input of the op-amp; feedback is negative when the signal is returned to the inverting input of the op-amp. Explain why this is the case.

As we have seen, the behavior of an (ideal) op-amp in a circuit with appropriate negative feedback may be described by two *Golden Rules*[2]*:*

1. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero.

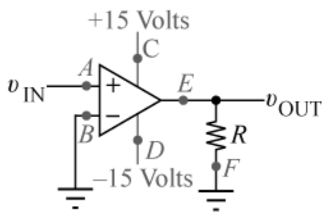
2. The inputs draw no current.

[2] P. Horowitz and W. Hill, *The Art of Electronics, Second Edition* (Cambridge University Press, NY, 1991).

Appendix 4e: Tutorial 5 - Currents, Negative Feedback and the non-Inverting Amplifier

Ρεύματα σε κυκλώματα τ.ε.

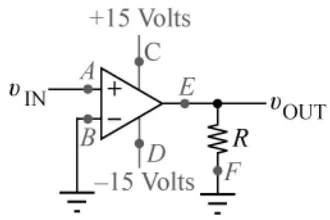
Θεωρήστε το κύκλωμα στα δεξιά. Υποθέστε ότι V_{in} = 20 μV.

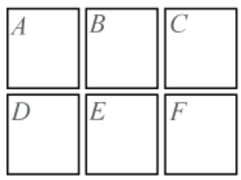


Ποια είναι η προσεγγιστική τιμή του ρεύματος που διαρρέει την μη-αναστρέφουσα είσοδο; (*Υπόδ:* Σκεφτείτε την τιμή της εμπέδησης της εισόδου.) Εξηγήστε.

Αν η V_{in} είχε συνδεθεί στην αναστρέφουσα είσοσο, τι θα λέγατε σχετικά με την προσεγγιστική τιμή του ρεύματος που θα διέρρεε την αναστρέφουσα είσοδο; Εξηγήστε.

Για καθένα από τα σημεία *A*–*F*, δείξτε τη φορά του ρεύματος στα αντίστοιχα κουτιά. Αν για κάποιο σημείο το ρεύμα είναι μηδενικό, να το δηλώσετε καθαρά. Εξηγήστε.





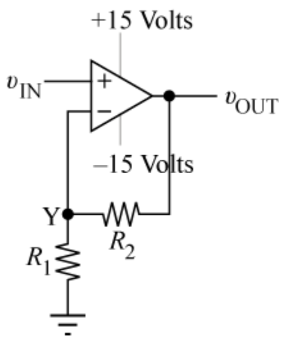
Ταξινομήστε, από τη μεγαλύτερη προς τη μικρότερη, τις απόλυτες τιμές των ρευμάτων που περνούν από τα σημεία *C*, *E*, και *F*. Εξηγήστε.

Υποθέστε ότι η V_{in} ήταν –20 μV. Ταξινομήστε, από τη μεγαλύτερη προς τη μικρότερη, τις απόλυτες τιμές των ρευμάτων που περνούν από τα σημεία *D, E,* και *F*. Εξηγήστε.

Γενικεύοντας, ποια είναι η σχέση μεταξύ του συνολικού ρεύματος που εισέρχεται στον τ.ε και του συνολικού ρεύματος που εξέρχεται από αυτόν;

Μη-αναστρέφων ενισχυτής

Θεωρήστε το κύκλωμα στα δεξιά. Υποθέστε ότι η V_{in} είναι μία σταθερή DC τάση.



Γράψτε μία σχέση για την V_{out}. Εκφράστε την απάντησή σας ως προς τα V_{in}, V_Y, και *G* (ο συντελεστής ενίσχυσης τάσης ανοιχτού βρόχου του τ.ε.). Εξηγήστε.

Διαρρέονται από ρεύμα οι αντιστάσεις R_1 και R_2? (Υπόδ*:* Για να μην διαρρέονται από ρεύμα και οι δύο αντιστάσεις, ποια τιμή θα έπρεπε να έχουν οι V_Y and V_{out}; Είναι αυτό δυνατό για το κύκλωμά μας;) Εξηγήστε.

Πώς συγκρίνεται το ρεύμα που περνά από την R_1 με αυτό από την R_2; Εξηγήστε.

Γράψτε μία έκφραση για την V_Y με όρους των V_{out}, R_1, και R_2. Εξηγήστε σύντομα γιατί αυτή η έκφραση έχει νόημα. (*Σημείωση:* Το κύκλωμα αναπαράγεται στα δεξιά.)

Χρησιμοποιήστε τις εκφράσεις σας από τα μέρη A και D για να βρείτε μία σχέση μεταξύ των V_Y και V_{in}.

Θυμηθείτε ότι $G = 10^5$. Στις πιο πολλές εφαρμογές, οι R_1 και R_2 είναι μεγαλύτερες από ή ίσες με 100 Ω, και $R_2/R_1 \leq 10^3$. Κάτω από αυτές τις συνθήκες, είναι η V_Y (V_-) *σημαντικά μεγαλύτερη από, σημαντικά μικρότερη από,* ή *προσεγγιστικά ίση με* την V_{in} (V_+)? Εξηγήστε σύντομα.

Σκεφτείτε την ακόλουθη συζήτηση μεταξύ μαθητών:

Student 1: "Βασιζόμενος σε ό,τι έχουμε κάνει, φαίνεται λογικό να θεωρούμε τις τάσεις στην αναστρέφουσα και στην μη-αναστρέφουσα είσοδο σαν να είναι ουσιαστικά οι ίδιες στο κύκλωμα αυτό."

Student 2: "Διαφωνώ. Αν οι τάσεις εισόδου είναι ίδιες, η τάση εξόδου πρέπει να είναι μηδενική. Αλλά έχουμε υποστηρίξει ότι η έξοδος δεν μπορεί να είναι μηδενική, συνεπώς οι τάσεις εισόδου πρέπει να είναι διαφορετκές."

Με ποιον μαθητή συμφωνείτε, αν συμφωνείτε με κάποιον; Εξηγήστε.

Χρησιμοποιήστε την προσέγγιση που έχετε κάνει παραπάνω για την V_{-} ώστε να εκφράσετε την V_{out} με όρους των V_{in}, R_1, και R_2.

Ποιος είναι ο *συντελεστής ενίσχυσης τάσης* ($V_{\text{out}}/V_{\text{in}}$) για το κύκλωμα αυτό; Εξηγήστε.

Εξαρτάται ο συντελεστής ενίσχυσης τάσης του κυκλώματος από τον συντελεστή ενίσχυσης τάσης ανοιχτού βρόχου *G* του τ.ε.; Εξηγήστε.

Στο παραπάνω κύκλωμα In the non-inverting amplifier circuit above, we have seen an example of *feedback* in an op-amp circuit. In general, the idea of feedback is that a part of the output signal is sent back to one of the two op-amp inputs, thereby producing a subsequent change in the output. (In the non-inverting amplifier circuit, the R_1 and R_2 voltage divider chain ensured that a specific fraction of the output was sent to the inverting input.) If this change in the output is of the same sign as the change in the feedback signal, then it is called *positive feedback.* When the sign of the resulting change in the output is opposite to that of the change in the feedback signal, it is called *negative feedback.*

Θεωρήστε το κύκλωμα του μη-αναστρέφοντος ενισχυτή που συζητήθηκε παραπάνω. Υποθέστε ότι η V_{-} αυξάνεται ελάχιστα ενώ η $V_{in} = V_{+}$ παραμένει ίδια. Τότε η V_{out} *αυξάνεται, μειώνεται,* ή *παραμένει ίδια*; Εξηγήστε. (*Υπόδ:* Θυμηθείτε ότι ο τ.ε. είναι ένας διαφορικός ενισχυτής.)

Είναι η ανάδραση στο κύκλωμα του μη-αναστρέφοντος ενισχυτή *θετική* ή *αρνητική*; Εξηγήστε.

Γενικά, η ανάδραση σε ένα κύκλωμα τ.ε. είναι θετική όταν το σήμα επιστρέφει στην μη-αναστρέφουσα είσοδο του τ.ε. και αρνητική όταν το σήμα επιστρέφει στην αναστρέφουσα είσοδο του τ.ε. Εξηγήστε γιατί ισχύει αυτό.

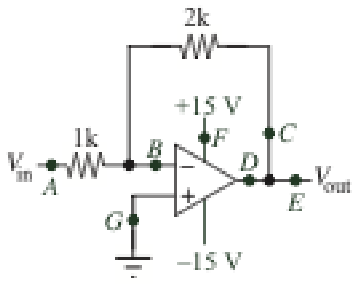
Όπως έχουμε δει, η συμπεριφορά ενός (ιδανικού) τ.ε. σε ένα κύκλωμα με κατάλληλη αρνητική ανάδραση μπορεί να περιγραφεί από δύο Χρυσούς Κανόνες (*Golden Rules*[3]):

1. Η έξοδος προσπαθεί να κάνει οτιδήποτε είναι απαραίτητο ώστε η διαφορά δυναμικού μεταξύ των τάσεων των εισόδων να είναι μηδενική.
2. Οι είσοδοι δεν διαρρέονται από ρεύμα.

[3] P. Horowitz and W. Hill, *The Art of Electronics, Second Edition* (Cambridge University Press, NY, 1991).

Appendix 4f:
Tutorial 6 - The Inverting Amplifier

Ρεύματα και Τάσεις στο κύκλωμα του αναστρέφοντος ενισχυτή



Θεωρήστε το κύκλωμα τ.ε. στα δεξιά. Υποθέστε ότι δεν υπάρχει φορτίο συνδεδεμένο στο κύκλωμα. $V_{in} = -2$ V.

Θεωρήστε την παρακάτω δήλωση ενός μαθητή:

> "Πρέπει να υπάρχει αρνητική ανάδραση εφόσον ένα κλάσμα του σήματος εξόδου επιστρέφει στην αναστρέφουσα είσοδο."

Συμφωνείτε ή διαφωνείτε με τν μαθητή;

Τι θα μπορούσατε να πείτε, αν θα μπορούσατε να πείτε κάτι, σχετικά με το ρεύμα που περνά από το σημείο *B*? Εξηγήστε.

Τι θα μπορούσατε να πείτε, αν θα μπορούσατε να πείτε κάτι, σχετικά με το ηλεκτρικό δυναμικό στο σημείο *B*? Εξηγήστε.

Τι θα μπορούσατε να πείτε, αν θα μπορούσατε να πείτε κάτι, σχετικά με το ηλεκτρικό δυναμικό στο σημείο *A*? Εξηγήστε.

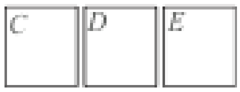
Is there current through the 1-kΩ resistor? If yes, find its value and show its direction. If not, why not? In any case, explain your reasoning.

Is there current through the 2-kΩ resistor? If yes, find its value and show its direction. If not, why not? In any case, explain your reasoning.

What is V_{out}? Briefly explain.

Σταματήστε εδώ για μία σύντομη συζήτηση.

In the boxes at right, show the direction of current through points D, D, and E. If there is no current through any of these points, state so explicitly. Explain.



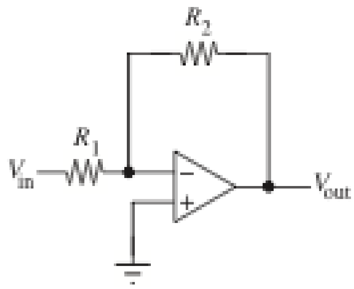
Πώς θα άλλαζε, αν θα άλλαζε καθόλου, η απάντησή σας αν η V_{in} ήταν +2 V?

For V_{in} = -2 V, rank, from largest to smallest, the absolute values of the currents through points A, D, and F. If there is no current through any of these points, state so explicitly. Explain.

Σχέση μεταξύ των V_{out} και V_{in} για τον αναστρέφοντα ενισχυτή

Θεωρήστε το κύκλωμα τ.ε. στα δεξιά. Υποθέστε ότι στο κύκλωμα δεν έχει συνδεθεί φορτίο.

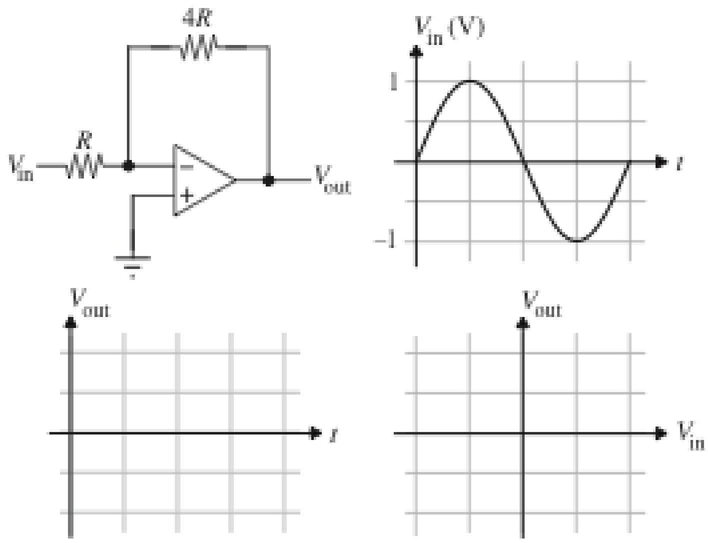
Ποια η V_{out}; Εκφράστε την απάντησή σας σε σχέση με τα V_{in}, R_1, και R_2. Εξηγήστε συνοπτικά.



Ποιος είναι ο συντελεστής ενίσχυσης του κυκλώματος αυτού;

Θεωρήστε το κύκλωμα στα δεξιά. Προσέξτε ότι δεν υπάρχει φορτίο συνδεδεμένο στο κύκλωμα. Υποθέστε ότι η V_{in} είναι το ημιτονικό σήμα που φαίνεται.

Στο κενό που διατίθεται, σχεδιάστε την V_{out} ως συνάρτηση του χρόνου. Εξηγήστε συνοπτικά.



Γιατί είναι το όνομα *αναστρέφων ενισχυτής* κατάλληλο για το το κύκλωμα αυτό;

In the space above, plot V_{out} as a function of V_{in}. Explain briefly.

Consider the following statement from a student:

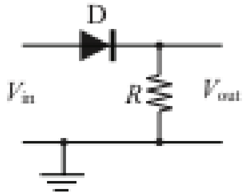
> "Κοιτάζοντας το διάγραμμά μου για την V_{out} vs. V_{in}, βλέπω ότι μία θετική V_{in} δίνει μία αρνητική V_{out}. Συνεπώς, γνωρίζω ότι υπάρχει αρνητική ανάδραση."

Do you agree or disagree? Explain.

Σταματήστε εδώ για μία σύντομη συζήτηση.

Appendix 4g:
Tutorial 7 - Diode Circuits

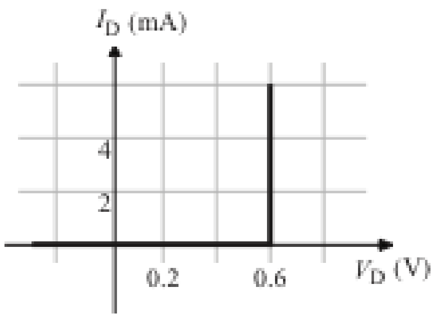
Κύκλωμα ημιανόρθωσης



Θεωρήστε το κύκλωμα στα δεξιά. Θεωρήστε την δίοδο ιδανική. Η χαρακτηριστική I-V για μία ιδανική δίοδο φαίνεται δεξιά κάτω. Έστω V_D η τάση στα άκρα της διόδου και V_R = V_{out} η τάση στα άκρα της αντίστασης.

A. Υποθέστε V_{in} = +5 V.

A1. Ποια η τιμή της V_D? Εξηγήστε.



A2. Διαρρέεται η δίοδος από ρεύμα; Διαρρέεται η αντίσταση από ρεύμα; Εξηγήστε.

A3. Είναι οι απαντήσεις σας στις ερωτήσεις 0 και 0 συνεπείς με την χαρακτηριστική I-V της ιδανικής διόδου; Εξηγήστε γιατί ή γιατί όχι.

A4. Ποια η τιμή της V_{out}; Εξηγήστε. (Υπόδ: Είναι συνεπής η απάντησή σας με τον κανόνα τω βρόχων του Kirchhoff;)

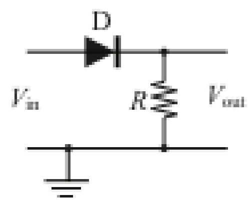
B. Τώρα υποθέστε V_{in} = –5 V.

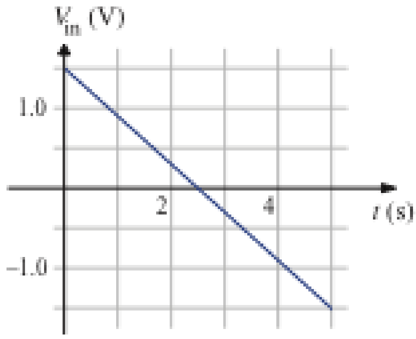
B1. Ποια η τιμή της V_D; Εξηγήστε.

B2. Διαρρέεται η αντίσταση από ρεύμα; Εξηγήστε πώς οι απαντήσεις σας είναι συνεπείς με την χαρακτηριστική I-V της ιδανικής διόδου.

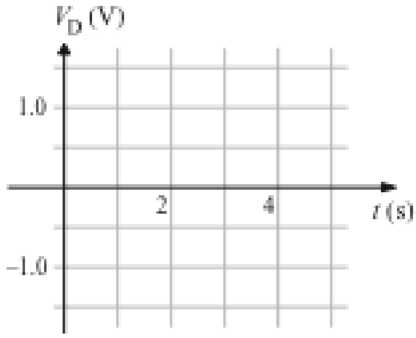
B3. Ποια η τιμή της V_{out}; Εξηγήστε.

C. Τώρα υποθέστε ότι η V_{in} για το κύκλωμα παρακάτω μεταβάλεται με τον χρόνο όπως φαίνεται στα δεξιά.

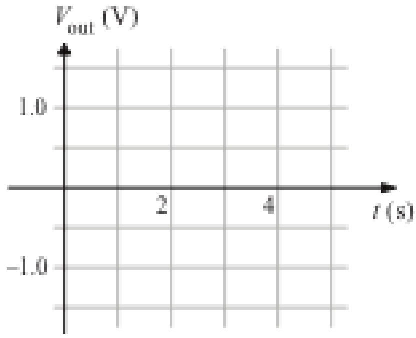




C1. Περιγράψτε σύντομα τι συμβαίνει στην V_{in} με τον χρόνο. Παίρνει η τάση εισόδου αρνητικές τιμές;



C2. Στα διατιθέμενα κενά, σχεδιάστε τις V_D και V_{out} ως συναρτήσεις του χρόνου. Εξηγήστε σύντομα.

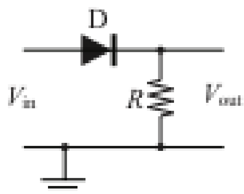


C3. Σε οποιαδήποτε χρονική στιγμή, είναι τα διαγράμματά σας συνεπή με τον κανόνα των βρόχων του Kirchhoff; Εξηγήστε πώς μπορείτε να το πείτε αυτό.

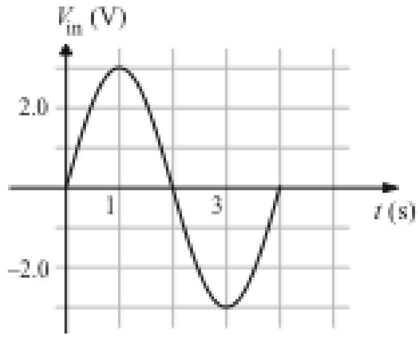
C4. Είναι ποτέ αρνητική η τάση στα άκρα της αντίστασης (V_{out});

Σταμτήστε εδώ για μία σύντομη συζήτηση.

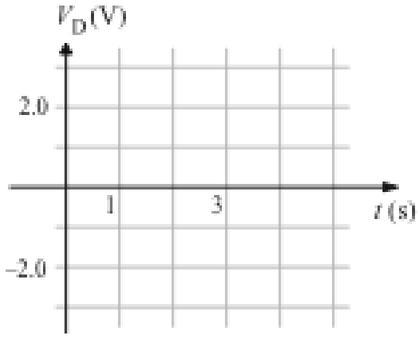
D. Τώρα υποθέστε ότι η V_{in} για το παρακάτω κύκλωμα είναι το AC σήμα τάσης που φαίνεται στα δεξιά.



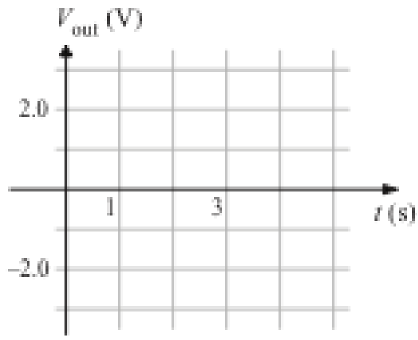
D1. Στα κενά που διατίθενται παρακάτω, σχεδιάστε τις V_D και V_{out} ως συναρτήσεις του χρόνου. Εξηγήστε συνοπτικά.



D2. Σχολιάστε το πώς διαφέρουν οι V_{out} και V_{in}. (*Υπόδ*: Τι έχει αλλάξειει το κύκλωμα στο AC σήμα εισόδου?)



D3. Στο κύκλωμα αυτό αναφερόμαστε συνήθως με το όνομα *κύκλωμα ημιανόρθωσης*. Εξηγήστε γιατί αυτό το όνομα είναι ταιριαστό.

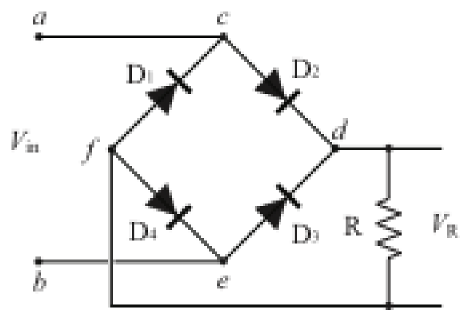


Κύκλωμα πλήρους ανόρθωσης

Θεωρήστε το κύκλωμα κάτω δεξιά. Έστω V_{D1}, V_{D2}, V_{D3} και V_{D4} οι τάσεις στα άκρα της καθεμιάς από τις τέσσερις διόδους και $V_R = V_{out}$ η τάση στα άκρα της αντίστασης.

A. Υποθέστε V_{in} = +6 V. (Έστω V_a = +6 V and V_b = 0 V.)

A1. Στον κόμβο *c*, ποιον δρόμο ακολουθεί το ρεύμα; Εξηγήστε.



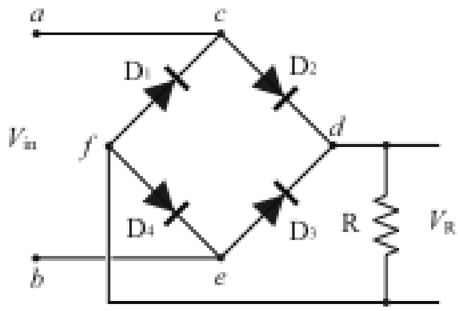
A2. Στον κόμβο *f*, ποιον δρόμο ακολουθεί το ρεύμα? *(Υπόδ:* Πώς συγκρίνεται το δυναμικό στο *f* με αυτό στο *c?)*

Εξηγήστε.

A3. Στο κύκλωμα παραπάνω, σχεδιάστε τον δρόμο που ακολουθεί το ρεύμα στο κύκλωμα (όταν V_{in} = +6 V).

A4. Έστω $V_R = V_d - V_f$. Ποια η τιμή της V_R? (*Υπόδ:* Ποια η τάση στα άκρα μίας ορθά πολωμένης διόδου;) Εξηγήστε.

Σταμτήστε εδώ για μία σύντομη συζήτηση.



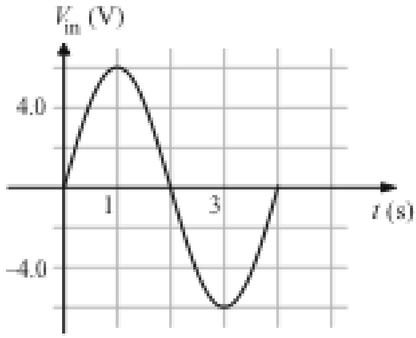
B. Υποθέστε V_{in} = –6 V. (Έστω V_a = –6 V and V_b = 0 V.)

B1. Στο κύκλωμα δεξιά, σχεδιάστε τον δρόμο που ακολουθεί το ρεύμα στο κύκλωμα (όταν V_{in} = –6 V).

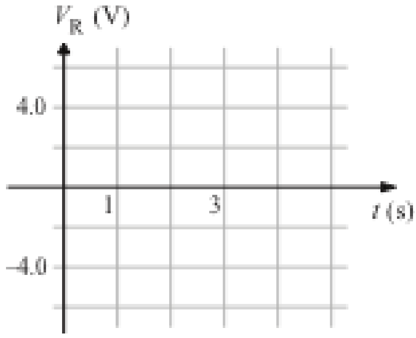
B2. Ποια η τιμή της V_R στην περίπτωση αυτή; Εξηγήστε.

C. Τώρα υποθέστε ότι η V_{in} είναι ένα ημιτονικό σήμα με πλάτος 6 V όπως φαίνεται στα δεξιά.

C1. Στα κενά που διατίθενται, σχεδιάστε την V_R ως συνάρτηση του χρόνου. Εντοπίστε πιθανά χρονικά διαστήματα κατά τα οποία V_R = 0 V.



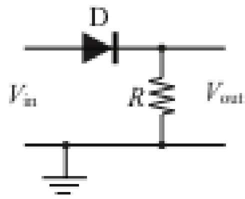
C2. Το κύκλωμα αυτό είναι γνωστό ως *κύκλωμα πλήρους ανόρθωσης*. Πώς διαφέρει η V_R από την V_{out} για το κύκλωμα ημιανόρθωσης;



C3. Ποιο(α) στοιχείο(α) κυκλώματος θα μπορούσατε να προσθέσετε ώστε να «εξομαλύνετε» το σήμα της τάσης εξόδου (δηλ., να κάνετε την V_R να μοιάζει περισσότερο με μία σταθερή DC τάση); Εξηγήστε.

Σταμτήστε εδώ για μία σύντομη συζήτηση.

II. Κύκλωμα ημιανόρθωσης

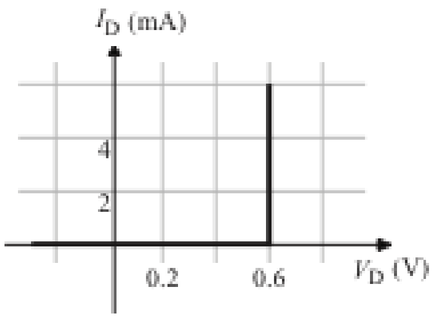


Θεωρήστε το κύκλωμα στα δεξιά. Θεωρήστε την δίοδο ιδανική. Η χαρακτηριστική *I-V* για μία ιδανική δίοδο φαίνεται δεξιά κάτω. Έστω V_D η τάση στα άκρα της διόδου και V_R = V_{out} η τάση στα άκρα της αντίστασης.

A. Υποθέστε V_{in} = +5 V.

A1. Ποια η τιμή της V_D? Εξηγήστε.

A2. Διαρρέεται η δίοδος από ρεύμα; Διαρρέεται η αντίσταση από ρεύμα; Εξηγήστε.



A3. Είναι οι απαντήσεις σας στις ερωτήσεις 0 και 0 συνεπείς με την χαρακτηριστική *I-V* της ιδανικής διόδου; Εξηγήστε γιατί ή γιατί όχι.

A4. Ποια η τιμή της V_{out}; Εξηγήστε. (*Υπόδ:* Είναι συνεπής η απάντησή σας με τον κανόνα τω βρόχων του Kirchhoff;)

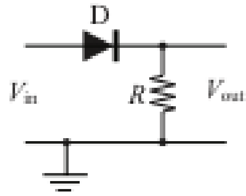
B. Τώρα υποθέστε V_{in} = –5 V.

B1. Ποια η τιμή της V_D; Εξηγήστε.

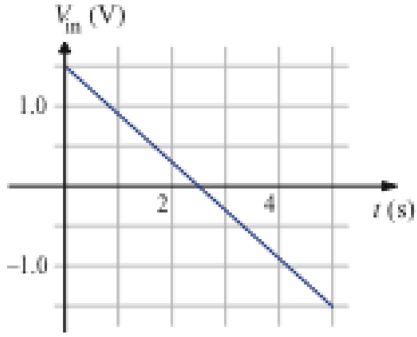
B2. Διαρρέεται η αντίσταση από ρεύμα; Εξηγήστε πώς οι απαντήσεις σας είναι συνεπείς με την χαρακτηριστική *I-V* της ιδανικής διόδου.

B3. Ποια η τιμή της V_{out}; Εξηγήστε.

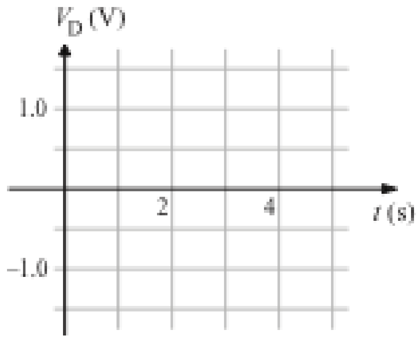
C. Τώρα υποθέστε ότι η V_{in} για το κύκλωμα παρακάτω μεταβάλεται με τον χρόνο όπως φαίνεται στα δεξιά.



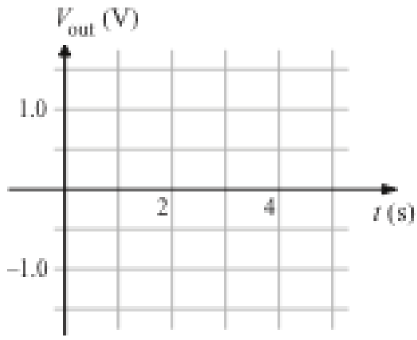
C1. Περιγράψτε σύντομα τι συμβαίνει στην V_{in} με τον χρόνο. Παίρνει η τάση εισόδου αρνητικές τιμές;



C2. Στα διατιθέμενα κενά, σχεδιάστε τις V_D και V_{out} ως συναρτήσεις του χρόνου. Εξηγήστε σύντομα.

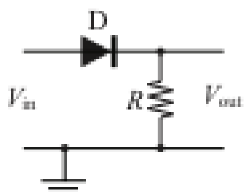


C3. Σε οποιαδήποτε χρονική στιγμή, είναι τα διαγράμματά σας συνεπή με τον κανόνα των βρόχων του Kirchhoff; Εξηγήστε πώς μπορείτε να το πείτε αυτό.



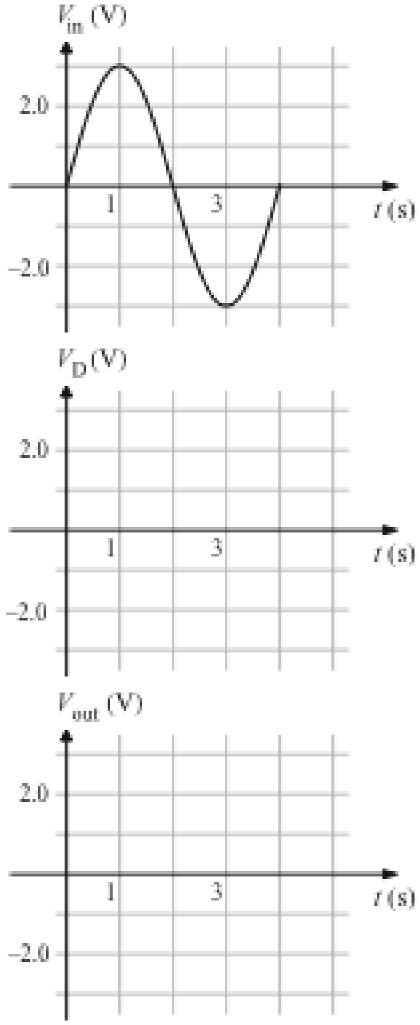
C4. Είναι ποτέ αρνητική η τάση στα άκρα της αντίστασης (V_{out});

Σταμτήστε εδώ για μία σύντομη συζήτηση.



D. Τώρα υποθέστε ότι η V_{in} για το παρακάτω κύκλωμα είναι το AC σήμα τάσης που φαίνεται στα δεξιά.

D1. Στα κενά που διατίθενται παρακάτω, σχεδιάστε τις V_D και V_{out} ως συναρτήσεις του χρόνου. Εξηγήστε συνοπτικά.

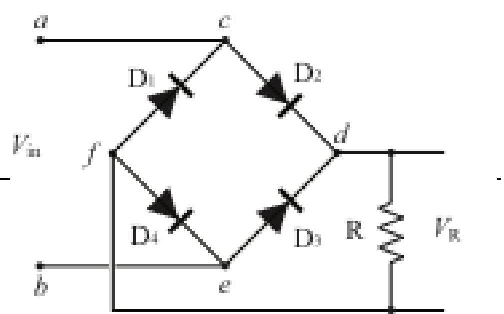


D2. Σχολιάστε το πώς διαφέρουν οι V_{out} και V_{in}. (*Υπόδ*: Τι έχει αλλάξειει το κύκλωμα στο AC σήμα εισόδου?)

D3. Στο κύκλωμα αυτό αναφερόμαστε συνήθως με το όνομα *κύκλωμα ημιανόρθωσης*. Εξηγήστε γιατί αυτό το όνομα είναι ταιριαστό.

III. Κύκλωμα πλήρους ανόρθωσης

Θεωρήστε το κύκλωμα κάτω δεξιά. Έστω V_{D1}, V_{D2}, V_{D3} και V_{D4} οι τάσεις στα άκρα της



καθεμιάς από τις τέσσερις διόδους και $V_R = V_{out}$ η τάση στα άκρα της αντίστασης.

A. Υποθέστε V_{in} = +6 V. (Έστω V_a = +6 V and V_b = 0 V.)

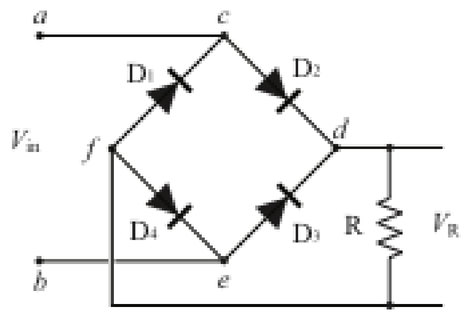
A1. Στον κόμβο *c,* ποιον δρόμο ακολουθεί το ρεύμα; Εξηγήστε.

A2. Στον κόμβο *f*, ποιον δρόμο ακολουθεί το ρεύμα? *(Υπόδ:* Πώς συγκρίνεται το δυναμικό στο *f* με αυτό στο *c?)* Εξηγήστε.

A3. Στο κύκλωμα παραπάνω, σχεδιάστε τον δρόμο που ακολουθεί το ρεύμα στο κύκλωμα (όταν V_{in} = +6 V).

A4. Έστω $V_R = V_d - V_f$. Ποια η τιμή της V_R? (*Υπόδ:* Ποια η τάση στα άκρα μίας ορθά πολωμένης διόδου;) Εξηγήστε.

Σταμτήστε εδώ για μία σύντομη συζήτηση.



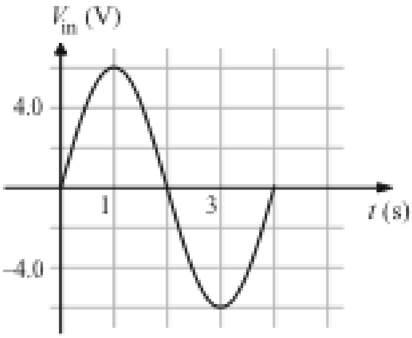
B. Υποθέστε V_{in} = –6 V. (Έστω V_a = –6 V and V_b = 0 V.)

B1. Στο κύκλωμα δεξιά, σχεδιάστε τον δρόμο που ακολουθεί το ρεύμα στο κύκλωμα (όταν V_{in} = –6 V).

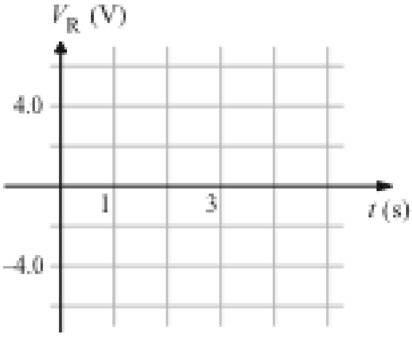
B2. Ποια η τιμή της V_R στην περίπτωση αυτή; Εξηγήστε.

Β. Τώρα υποθέστε ότι η V_{in} είναι ένα ημιτονικό σήμα με πλάτος 6 V όπως φαίνεται στα δεξιά.

Β1. Στα κενά που διατίθενται, σχεδιάστε την V_R ως συνάρτηση του χρόνου. Εντοπίστε πιθανά χρονικά διαστήματα κατά τα οποία V_R = 0 V.



Β2. Το κύκλωμα αυτό είναι γνωστό ως *κύκλωμα πλήρους ανόρθωσης*. Πώς διαφέρει η V_R από την V_{out} για το κύκλωμα ημιανόρθωσης;



Β3. Ποιο(α) στοιχείο(α) κυκλώματος θα μπορούσατε να προσθέσετε ώστε να «εξομαλύνετε» το σήμα της τάσης εξόδου (δηλ., να κάνετε την V_R να μοιάζει περισσότερο με μία σταθερή DC τάση); Εξηγήστε.

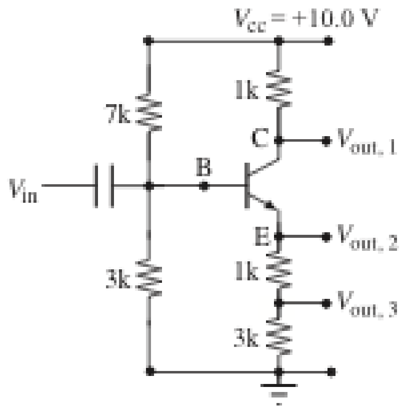
Σταμτήστε εδώ για μία σύντομη συζήτηση.

Appendix 4h:
Tutorial 8 - Transistor Circuits: Followers and Amplifiers

Θεωρήστε το κύκλωμα στα δεξιά. Υποθέστε β = 100 για το *npn* διπολικό τρανζίστορ επαφής.

Υποθέστε $V_{in} = -4.0$ V.

Ποια η V_B; Εξηγήστε. (*Υπόδ*: Πώς θα διέφερε, αν θα διέφερε καθόλου, η απάντησή σας αν η V_{in} ήταν + 4.0 V;)



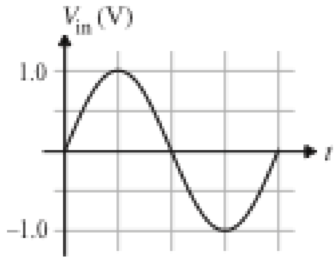
Είναι η $V_{out,3}$ *μεγαλύτερη από*, *μικρότερη από,* ή *ίση με την* $V_{out,2}$; Εξηγήστε.

Ποια η $V_{out,3}$? Εξηγήστε συνοπτικά.

Ποια η προσεγγιστική τιμή της $V_{out,1}$? Εξηγήστε συνοπτικά. (*Υπόδ*: Διαφέρει σημαντικά το I_C από το I_E?)

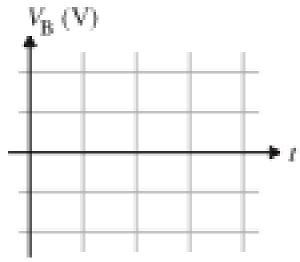
Σταματήστε εδώ για μία σύντομη συζήτηση.

Τώρα υποθέστε ότι η V_{in} για το κύκλωμα είναι το 1 kHz AC σήμα τάσης που φαίνεται δεξιά.



Στο κενό που διατίθεται, σχεδιάστε την V_B ως συνάρτηση του χρόνου. Εξηγήστε συνοπτικά και διαλέξτε κατάλληλη κλίμακα για τον V-άξονα.

Είναι το peak-to-peak πλάτος της $V_{out,\,2}$ *μεγαλύτερο από*, *μικρότερο από*, ή *ίσο με* αυτό της V_{in}; Εξηγήστε.



Είναι το peak-to-peak πλάτος της $V_{out,\,3}$ *μεγαλύτερο από*, *μικρότερο από*, ή *ίσο με* αυτό της $V_{out,\,2}$; Εξηγήστε.

Όσο η $V_{out,\,2}$ αυξάνεται, τότε το I_E *αυξάνεται*, *μειώνεται*, ή *παραμένει ίδιο*; Εξηγήστε.

Όσο το I_E αυξάνεται, τότε η $V_{out,\,1}$ *αυξάνεται*, *μειώνεται*, ή *παραμένει ίδια*; Εξηγήστε.

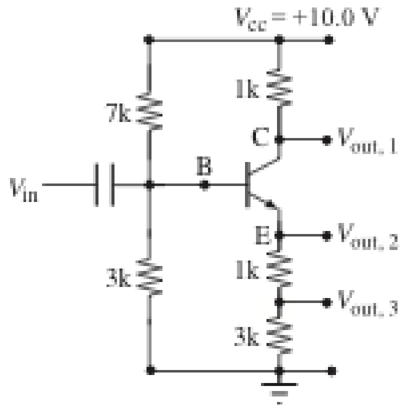
Για μία δεδομένη μεταβολή του I_E (δI_E), είναι η απόλυτη τιμή της μεταβολής της $V_{out,\,1}$ ($|\delta V_{out,\,1}|$) *μεγαλύτερη από*, *μικρότερη από*, ή *ίση με* αυτήν της $V_{out,\,2}$ ($|\delta V_{out,\,2}|$); Εξηγήστε.

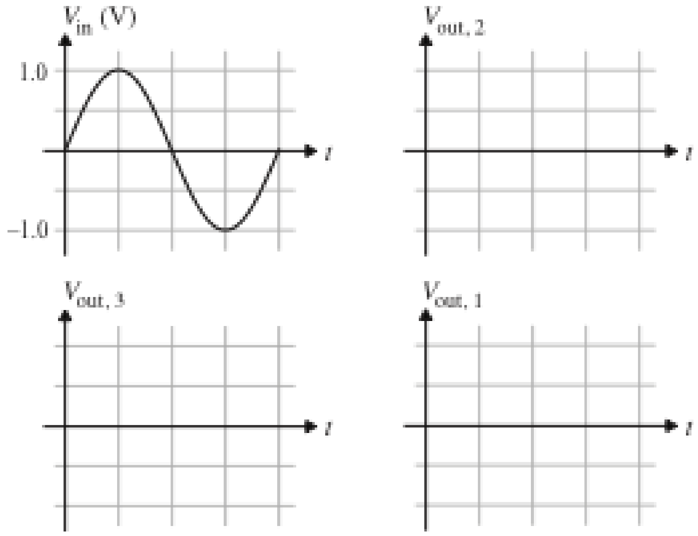
Είναι το peak-to-peak πλάτος της $V_{out,\,1}$ *μεγαλύτερο από*, *μικρότερο από*, ή *ίσο με* αυτό της $V_{out,\,2}$? Εξηγήστε.

Σταματήστε εδώ για μία σύντομη συζήτηση.

Τώρα θα επικεντρωθούμε σε μερικές γραφικές αναπαραστάσεις των σημάτων εισόδου και εξόδου.

Στα κενά που διατίθενται, σχεδιάστε τις $V_{out,\ 2}$, $V_{out,\ 3}$, και $V_{out,\ 1}$ ως συναρτήσεις του χρόνου. Εξηγήστε συνοπτικά και επιλέξτε κατάλληλες κλίμακες για τους V-άξονες.



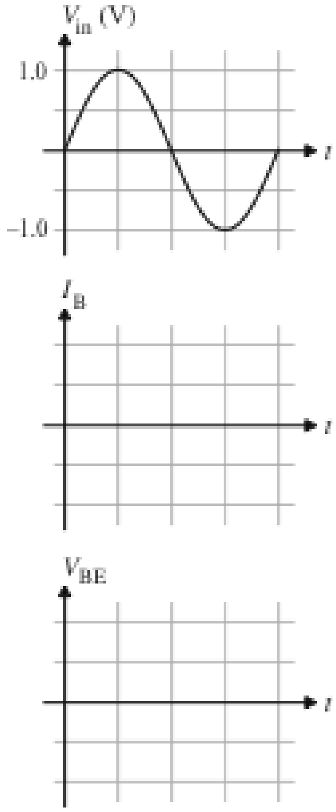


Υπάρχει διαφορά φάσης μεταξύ των $V_{out,\ 1}$ και V_{in}; (*Υπόδ*: Σκεφτείτε την απάντησή σας στην ερώτηση B4.)

Δικαιολογείται το όνομα *αναστρέφων ενισχυτής τάσης* όταν αναφερόμαστε στην $V_{out,\ 1}$; (*Υπόδ*: Ποια η ενίσχυση τάσης αυτού του ενισχυτή;) Εξηγήστε.

Για το χρονικό διάστημα που φαίνεται, σχεδιάστε ποιοτικά το I_B ως συνάρτηση του χρόνου.

Για το χρονικό διάστημα που φαίνεται, σε πόσες χρονικές στιγμές είναι το I_B ίσο με 0 A? Εξηγήστε. (*Υπόδ*: Μπορεί να σας φανεί χρήσιμο να σχεδιάσετε την V_{BE} ως συνάρτηση του χρόνου.)



Σταματήστε εδώ για μία σύντομη συζήτηση.

Endnotes

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