STUDYING A COMMUNITY OF SECONDARY MATHEMATICS TEACHERS WHO NEGOTIATE NEW RESOURCES FOR THE TEACHING OF STATISTICS

by

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To my son Dimitris

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Table of Contents

ΠΕΡΙΛΗΨΗ

Η διατριβή αφορά στη μελέτη της ανάπτυξης της διδασκαλίας της στατιστικής μέσα από τη συνεργασία 11 εκπαιδευτικών μαθηματικών της δευτεροβάθμιας εκπαίδευσης. Η συνεργασία στοχεύει στο να αναπτύξουν οι μαθητές στατιστική σκέψη και συλλογιστική και γίνεται μέσα από μια σειρά δράσεων που αναπτύσσονται στην ομάδα των εκπαιδευτικών μέσα από την υποστήριξη της ερευνήτριας. Οι δράσεις αυτές περιλαμβάνουν τη διερεύνηση στατιστικών προβλημάτων από τους ίδιους τους εκπαιδευτικούς, μελέτη καταστάσεων μάθησης και διδασκαλίας της στατιστικής, καθώς και σχεδιασμό και εφαρμογή δικών τους διδασκαλιών. Μέσα από τρία στοιχεία που περιγράφουν τη διδασκαλία (διδακτικοί πόροι, επιδιωκόμενα μαθησιακά αποτελέσματα και χαρακτηριστικά της διδασκαλίας) μελετάται η εξέλιξη της διδακτικής πρακτικής της ομάδας ιδιαίτερα σε σχέση με την αξιοποίηση ενός ψηφιακού εργαλείου που εξειδικεύεται στη διδασκαλία της στατιστικής που ήταν κεντρικός διδακτικός πόρος στις συζητήσεις της ομάδας.

Το θεωρητικό και μεθοδολογικό πλαίσιο της εργασίας είναι η κοινότητα πρακτικής του Wenger από όπου αξιοποιούνται θεωρητικές έννοιες για να μελετηθεί πώς συσχετίζονται οι πόροι, τα χαρακτηριστικά της διδασκαλίας και τα επιδιωκόμενα μαθησιακά αποτελέσματα. Τα δεδομένα αναλύονται αρχικά με μεθόδους της Θεμελιωμένης Θεωρίας (Glaser 1998, Charmaz, 2006) και αφορούν στις συζητήσεις των ομάδων των εκπαιδευτικών κατά τη διάρκεια ενός έτους. Η φάση αυτή της ανάλυσης οδηγεί στην ταξινόμηση των διδακτικών πόρων, των επιδιωκόμενων μαθησιακών αποτελεσμάτων και χαρακτηριστικών της διδασκαλίας σε σχέση με τις κύριες δράσεις της ομάδας. Η θεματική ανάλυση που ακολούθησε αφορά στο πώς οι εκπαιδευτικοί διαπραγματεύονται το νόημα και την αξιοποίηση του συγκεκριμένου διδακτικού πόρου, του ψηφιακού εργαλείου. Από την ανάλυση αυτή αναδείχτηκαν τρεις φάσεις, η ανάδυση, διερεύνηση και διείσδυση που χαρακτηρίζουν την πορεία ενσωμάτωσης του διδακτικού πόρου στη διδακτική πρακτική των εκπαιδευτικών. Το επόμενο επίπεδο ανάλυσης εμβαθύνει

στον τρόπο διαμόρφωσης της διδακτικής πρακτικής μέσα από τα εργαλεία της «συμμετοχής» (participation) και των «παραγομένων» (reifications) του θεωρητικού της πλαισίου. Ιδιαίτερα εστιάζει στο είδος των συνδέσεων των τριών στοιχείων της διδασκαλίας στις φάσεις της πορείας ενσωμάτωσης του πόρου. Τα αποτελέσματα αυτού του επιπέδου ανάλυσης μας δείχνουν ότι οι συνδέσεις που δημιουργούνται ανάμεσα στα τρία στοιχεία είναι πιο ισχυρές και ρητές στην επικοινωνία των εκπαιδευτικών κατά τη φάση της διείσδυσης του πόρου στην πορεία ενσωμάτωσης του στη διδακτική πρακτική. Στο σύνολο τους τα αποτελέσματα δείχνουν ότι οι εκπαιδευτικοί αναπτύσσουν τη διδακτική τους πρακτική σχετικά με τη στατιστική που δίνει έμφαση στην ανάπτυξη της στατιστικής σκέψης και συλλογιστικής των μαθητών.

Η έρευνα προσθέτει στην κατανόηση μας για την ανάπτυξη της διδασκαλίας της στατιστικής κάνοντας τα μέχρι τώρα ερευνητικά αποτελέσματα σχετικά με την ανάπτυξη της στατιστικής σκέψης και συλλογιστικής καθώς και τα χαρακτηριστικά της διδασκαλίας που την υποστηρίξουν υλοποιήσιμα στην εκπαίδευση των εκπαιδευτικών. Διακρίνεται από πρωτοτυπία α) στην εννοιολογική σύνδεση που προσφέρει ανάμεσα στους διδακτικούς πόρους, στα επιδιωκόμενα μαθησιακά αποτελέσματα και στα χαρακτηριστικά της διδασκαλίας που υποστηρίζουν τη στατιστική σκέψη και συλλογιστική, β) στα μεθοδολογικά εργαλεία που αξιοποιεί για να αναλύσει τη συνεργασία εκπαιδευτικών που επιτρέπουν τη σύνδεση της μορφή της συνεργασίας με ζητήματα που αφορούν στο μαθηματικό περιεχόμενο, τη μάθηση και τη διδασκαλία του, και γ) στη μελέτη αξιοποίησης των διδακτικών πόρων και ιδιαίτερα των ψηφιακών εργαλείων που εξειδικεύονται στη διδασκαλία της στατιστικής σε συνεργατικά πλαίσια εκπαιδευτικών.

ABSTRACT

In the context of a community of practice (CoP), eleven secondary mathematics teachers collaborated in exploring various resources that aim to support teaching statistics that fosters learners in developing statistical thinking and reasoning (DSTR). Through successive cycles of (a) exploring statistical tasks, (b) discussing educational material, (c) designing for their students, (d) implementing in the classroom and (e) reflecting on the implementation, the teachers interact with various resources which are other times get into the teaching repertoire of the community and other times not. Viewing the teaching practice in a dynamic triangle of resources, learning potentials and teaching features, we are interested in how the interactions among these elements form the teachers' community shared repertoire. Our analysis makes use of the data produced in the five-steps cycle as described above, i.e. 10 meetings last about three hours each. The data are analyzed into three levels of analysis. The first level is based on the principles of Grounded Theory (Glaser 1998, Charmaz, 2006) aiming to capture particular elements of the teaching practice, namely resources, learning potentials and features, that emerged in teachers' work. This level gave us an insight into elements that seems to be crucial for the DSTR teaching practice and also of the appearance of the emerged elements in the five 4 stages of the inquiry cycle (see above).The next two levels focus on a particular resource that appears to have a crucial role for DSTR both in our data analysis and in the statistics education literature, which is the dynamic software tools for statistics instruction (STSI). On a second level, the tracking of this certain material resource, through thematic analysis, revealed three phases in the process of the resource's integration in the community's shared repertoire. These three phases, namely the emersion phase, the exploration phase, and the immersion phase, are further qualitatively analyzed in a search of the characteristics and particularities of each phase in terms of the negotiation of meaning with respect to the practice of the community. Aspects that seem to frame the transition among the various phases of this process and implications for the re-sourcing process of the DSTR practice are also considered in our research.

The originality and novelty of this study stem from the following: First, the use of the idea of links among the elements of the practice as both a theoretical and a methodological tool. Particularly, it provided a concept to connect theoretically resources with the actual teaching by acknowledging the underlying context and also a methodological concept to study the development of the teaching. Second, the use of the community's practice framework in the re-sourcing of STSI tools. This framework helped to get insight on both global and local aspects that are related to the use of STSI tools and it let socio-cultural aspects to be acknowledged and studied.

Further, it highlighted the important role of STSI tools not only as a tool for the learning of statistics but also as a tool for the designing of the teaching of statistics. And last, it brought to light in the role of human resources not only in the STSI re-sourcing process but in the DSTR teaching generally.

1. Introduction

1.1. Rationale Of The Study

When I finished my graduate studies in Mathematics, I continued my post-graduate studies in Financial Mathematics. In this post-graduate program, there was a large part of the curriculum dedicated to statistics, econometrics, stochastic processes, and risk management. Thus, through my studies in this program, I had the opportunity to understand deeper, not only tools and methods significant to decision and prediction making but also the importance of familiarizing with the stochastic context and mathematical content related to it in order to meet the challenges of modern societies. Just after finishing my studies in Financial Mathematics I decided to enter the Mathematics Education community and learn more about teaching and learning mathematics at the school level. In my very first steps in mathematics education research, I realized two things regarding the development of stochastic thinking at the school level. The first thing was that probability and statistics constituted a very small part of the secondary mathematics curriculum, and they also appear in the curriculum as two separate subjects with no conceptual connection between them. The second thing was that, although there were some sporadic studies in relation to the teaching of theoretical probability, studies that focus on stochastic thinking and reasoning with statistical and probabilistic tools in making decisions under uncertainty were almost scarce if existing at all in the Greek mathematics education research landscape.

These two observations were the starting point to begin an exploration of the specificities of the statistical and probabilistic problems that stem from the stochastic context, and the idea of randomness inherent to them. This exploration brought to the fore epistemological differences between mathematics and statistics as well as different ways of thinking and reasoning when engaging in mathematical and statistical activity. Moreover, it revealed a plethora of studies worldwide, investigating the teaching and learning of statistics at the secondary level. These studies provide rich evidence of students’ understandings, misconceptions, and difficulties, and they also provide innovative teaching tools and strategies in order to support students in understanding statistical activity and develop ways of thinking that support problem-solving in contexts with uncertainty.

The awareness of the existing richness in the research findings with regard to the teaching and learning of statistics, combined with the reality in the secondary mathematics classrooms motivated my interest to study how interactions between the statistics education research and the actual teaching can be facilitated and developed. This aim was particularly important in the Greek context where statistics education research and mathematics teaching constituted two fields with absolutely no overlap between them. Supportive of this aim was also the fact that the period that the idea of this study was born, a new mathematics curriculum that reinforces the role and the content of statistics was designed and pilot implemented.

1.2. The Aim Of The Study

The research interest to the bridging between the statistics education research and the secondary mathematics teaching was rested on three theoretical pillars. The first pillar was to develop a conceptualization of what is statistical activity and what defines a practice to statistical teaching. How this teaching can support learners to develop statistical tools and ways of thinking, namely what resources and teaching strategies seem to assist learners in developing deep statistical understandings and reasoning. The second pillar is the specificities of this practice in terms of particular resources and learning potentials in relation to those employed in the other subjects of the mathematics curriculum. We are particularly interested to see these resources in-context-in-practice, namely to see how the context influences the use of the resources-in-practices and how the specific use of the statistical resources provided by statistics education literacy can influence and develop the teaching practice (Adler, 1998). The third pillar is the theoretical lens that can allow the study of the teaching resources-in-context-in-practice in a context that will illuminate aspects that shape the statistics teaching as a social practice. The main aim of this study is to merge the three theoretical pillars to a consolidated theoretical structure that will allow us to investigate how innovative resources from statistics education research immerse in the teaching of secondary mathematics teachers as they collaborate to develop statistics teaching that facilitates statistical reasoning and thinking. Next, we will argue on the importance of this aim, not only in the Greek context but generally for mathematics and statistics education research.

The last years, the focus on statistical reasoning and thinking has become central in the discussion among statistics educators and researchers, as well as, in the current reform efforts. Ben-Zvi and Garfield (2004) highlighted some notable factors that set the development of statistical reasoning and understanding as a central goal in statistics teaching. These factors included: (a) changes in the field of statistics, including new techniques of data exploration; (b) changes and increases in the use of technology in the practice of statistics, and its growing availability in schools and at home; (c) increased awareness of students' inability to think or reason statistically, despite good performance in statistics courses; and (d) concerns about the preparation of statistics teachers at the K-12 and college levels, many of whom have never studied applied statistics or engaged in data analysis activities (Ben-Zvi & Garfield, 2004, p.5). Responding to the challenge of developing statistical reasoning and thinking, many researchers explored a variety of teaching tools and strategies that seem to support students' learning (e.g. Ben-Zvi, 2006; Gourgey, 2000; Groth, 2006).

All the above considerations resulted in a considerable amount of research studies and in an International Collaboration for Research on Statistical Reasoning, Thinking and Literacy (SRTL), which began in 1999, and, until today it counts nine international research forums. Responding to this open field of inquiry, many researchers provide various teaching approaches and explored a variety of teaching tools that seem to support students in developing statistical reasoning and thinking. Many of these studies emphasize the importance of students' engagement with statistical investigations and the use of real data (e.g. MacGillivray & Pereira-Mendoza, 2011; Makar, 2008); data simulations in generating sampling distributions, exploring probabilities, and making inferences (e.g. Burrill, 2002; Gourgey, 2000; Stohl Lee, Angotti, & Tarr, 2010); media extracts and real contexts to acknowledge contextual aspects and improve students' decision making skills (e.g. Merriman, 2006; Pfannkuch, 2011; Watson, 1997); and dynamic software tools that support data explorations (e.g. Ben-Zvi, 2006; Fitzalen & Watson, 2014).

These innovative teaching approaches introduce to statistics teaching a plethora of new material (e.g. technological tools, physical objects, and statistical objects) and human resources (e.g. experiences with data, interpretations, knowledge about concepts and procedures). Our study examines how secondary mathematics teachers integrate such resources into their teaching. In mathematics education, the study of resources is a field of research that has gained increased attention in the last few years (Adler & Reed, 2002; Pepin, Trouche, & Gueudet, 2012; Remillard, Herbel-Eisenmann, & Lloyd, 2009). However, there are surprisingly few studies that focus on resources in the area of statistics education (e.g. Visnovska, Cobb, & Dean, 2012). The study of teaching resources is especially important in the case of statistics because of the: (a) multiplicity of existing resources that can support the teaching and learning of statistics as described above; (b) fact that statistical resources constitute a great challenge for mathematics teachers (Bakogianni, Potari & Paparistodemou, 2013; Bakogianni, 2015; Burgess, 2011; Groth, 2007; Lee & Hollebrands, 2011; Skott & Østergaard, 2016); and (c) epistemological differences between statistics and mathematics that differentiate the teaching and learning in each area (delMas, 2004; Meletiou-Mavrotheris, 2007; Moore & Cobb, 2000; Scheaffer, 2006). In our work, we use the term resource in accordance with Adler's (2000a) conceptualization "as both noun and verb, as both object and action" (p. 207). I, therefore, use the verb re-source to describe the action of integrating a new resource into teaching or using an old resource in a new way.

Although the studies on resources mainly concentrate on the individual and focus on the interplay between teachers and resources (Adler, 2012; Choppin, 2011; Gueudet & Trouche, 2009; Remillard, 2012), very recently there has been growing interest in teachers' collective work with resources (Gueudet, Pepin, Sabra, & Trouche, 2016; Visnovska & Cobb, 2013). Such an approach aims to advance the study of resources and shed light on collective aspects of teaching that eventually can help in understanding the role of resources in teachers' collective work and their professional learning (Gueudet, Pepin & Trouche, 2013).

We adopted a social perspective and utilized the communities of practice (CoP) framework (Wenger, 1998) to investigate the re-sourcing process in a group of secondary mathematics teachers who had been working systematically and collectively for many years to form statistics teaching practice that promotes statistical reasoning and thinking We are particularly interested in how resources are entering in the community and immerse in the community's practice. We selected this framework for its potentiality to shed light on the development of statistical teaching (Shaughnessy, 2014) and also for its potentiality to promote teachers' collaboration and interaction which is proved to be important in teachers' education especially regarding the content of statistics (Ponte & Noll, 2018).

1.3. The Structure Of The Thesis

The thesis is divided into five main chapters. Chapter 1 constitutes the introduction of the thesis. In this Chapter, I present in brief the author's academic and research background as well as the rationale for this study. Then I illustrate the topic of our research and the particular research aims as well as the importance of the study. I finish this chapter with an overview of the thesis.

In Chapter 2 I present the theoretical background of the study and a brief review of the relative literature. This Chapter consists of five sections. In the three first sections, namely 2.1, 2.2 and 2.3 I discuss the literature in relation to the three core theoretical pillars of our study as mentioned above, that is (a) the social practice theory and specifically the Communities of Practice framework (Wenger, 1998) as well as concepts and ideas of this theory utilized in our study, (b) the study of teaching resources in mathematics education and the location of our study in this open field of inquiry, and (c) the characteristics and specificities of statistics teaching and learning as well as issues of teachers professional development in this subject. The last two sections summarize how the three theoretical pillars of our study, (viz., the communities of practice, the teaching resources and the teaching and learning of statistics), forms our conceptualization for the development of the new practice, to wit the teaching and learning of statistics for developing statistical thinking and reasoning. This synthesis and summary aim also to illustrate the potentiality and importance of our study for both statistics education and mathematics education research communities.

In Chapter 3 I illustrate the methodology of our research. Particularly, I describe the context of the study and our specific research questions. I then present the participants of our study, our role as researchers in the newly established community of the teachers and the main agenda of our work with them. Next, I describe the data and the method of the study as well as the three-level process of our data analysis.

Chapter 4 consists of three main sections. Each section presents the main results produced in each level of the data analysis. The findings of each level are further divided into subsections that assist the clarity and precision in the presentation of the results. The last subsection in each section summarized the main findings of the associated level of analysis.

The last chapter of this dissertation, Chapter 5, reflects on the findings of the study. This chapter is divided into 4 sections. In the first section (5.1) I discuss the results of the study in relation to the research questions I formulated in the beginning, integrating them with our theoretical background and the relevant existing literature. Next, I clarify the contribution of the study for the research in secondary mathematics teachers' professional development with respect to the teaching and learning of statistics for developing statistical reasoning and thinking. In the last section of this Chapter, I refer to the limitations of this study and I also discuss possibilities for further research. I end this dissertation with a final conclusion that summarizes the essence of the study.

2.1. Teachers’ Professional Development In Communities Of Practice

2.1.1. Main Constructs Of Communities Of Practice Theory

The research on mathematics education and on mathematics teachers’ education have been influenced by two metaphors for learning: the acquisition metaphor, with an individual focus of learning as gaining knowledge and, the participation metaphor, with a social conceptualization of learning as participation in social practices (Sfard, 1998). Central to this participatory perspective is the social practice theory of Lave & Wenger (1991) who developed the idea of situated learning. In situated learning theory, learning is situated within social interactions and learners are involved in communities of practice that embody certain beliefs and behaviors which affect the development of professional identities. Rooted in this expanded conceptualization of practice, Wenger (1998), in Communities of Practice (CoP), developed a new model that bridges theories of social structure with theories of the situated learning experience. This model theorizes practice in social systems and locates learning in the relationships between the person and the social context in which the person acts. According to Wenger, practice is about “doing in a historical and social context that gives structure and meaning to what I do. In this sense, practice is always social practice” (Wenger, 1998; p. 47) and knowledge of practice is about participating in this practice in its social setting. The CoP model for theorizing learning in practice integrates four components:

The Meaning

In the social theory of learning, practice is not about action but rather about both action and interpretation of the action. It is thus about meaning which is produced through the engagement in the practice and reflects the accumulated experience of everyday life. Our engagement with a practice is a continuous and dynamic process by which we produce meanings that negotiate anew the history of meanings of which they are part. In this sense, the meaning is the core and the essence of practice and it is located in the negotiation of meaning. The negotiation of meaning is a powerful process by which all involved elements (namely the meaning, the situations to which the meaning is given and those who participate in the process) interact and are mutually affected. The process of the negotiation of meaning contains two distinct and complementary processes, participation and reification. Participation refers to "the social experience of living in the world in terms of membership and active involvement in social enterprises" and thus to the recognition of ourselves in each other. Thus participation reflects mutuality. On the other hand, reification refers to "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" and thus to the projection of ourselves onto the world. Participation and reification can't be considered in isolation but contrary, they form a duality which is fundamental to the nature of the practice.

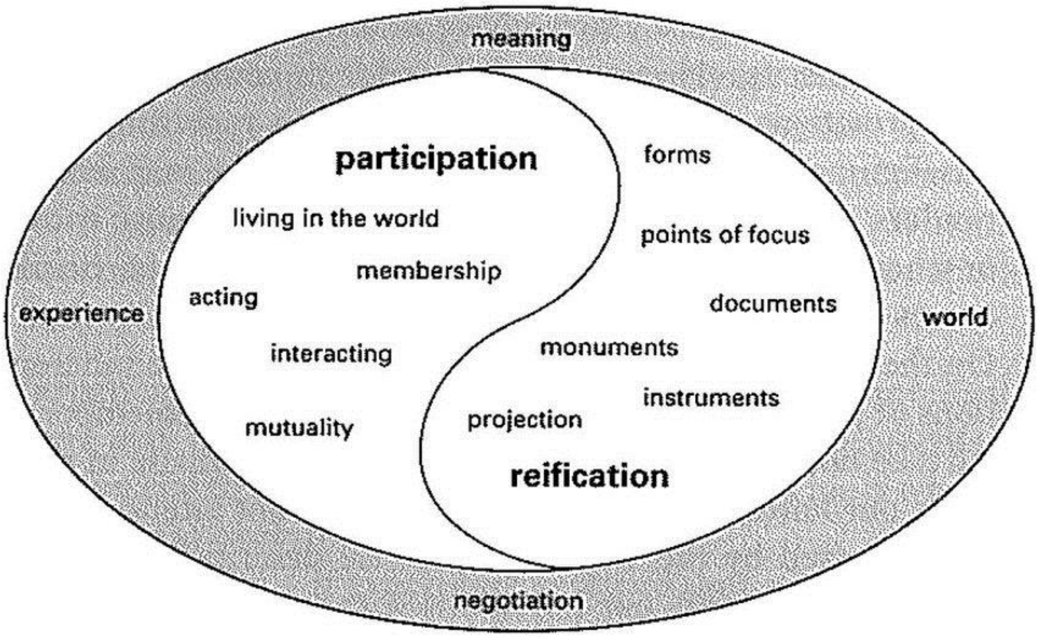


Figure 2-1- The duality of participation and reification (Wenger, 1998; p. 63)

The Community

In social practice theory, community and practice are associated in a coherent entity. Particularly, the community shapes the practice in which its members are engaged, and the practice becomes a source of coherence for the community that is defined by this practice (Wenger, 1998). In a community of practice, the practice becomes a property of the community and this special type of community is characterized by three dimensions of practice: a) *mutual engagement* in terms of belonging, of role recognition and of mutuality of relationships. According to Wenger, “membership in a CoP is a matter of mutual engagement. It draws on what we do and what we know, as well as, on our ability to connect meaningfully to what we don’t do and what we don’t know – that is to the contributions and knowledge of others” (p.76). b) *a joint enterprise* which refers to a negotiated and indigenous enterprise as well as to a regime of mutual accountability to this enterprise. Illustrating the idea of the joint enterprise Wenger mentioned: “the enterprises reflected in our practices, include the instrumental, the personal and the interpersonal aspects of our lives… It is joint not in that everybody believes the same thing or agrees with everything, but in that it is communally negotiated… Even when specific members have more power than others, the practice evolves into a communal response to that situation” (pp. 77-80). And c) *a shared repertoire* which refers to a shared history of mutual engagement that can become part of the practice and constitutes resources for negotiating the meaning. Wenger explains further that “resources of mutual engagement are not always an outcome of an agreement, mismatched interpretations or misunderstandings are not merely problems to resolve but occasions for the production of new meanings” (pp. 82-84).

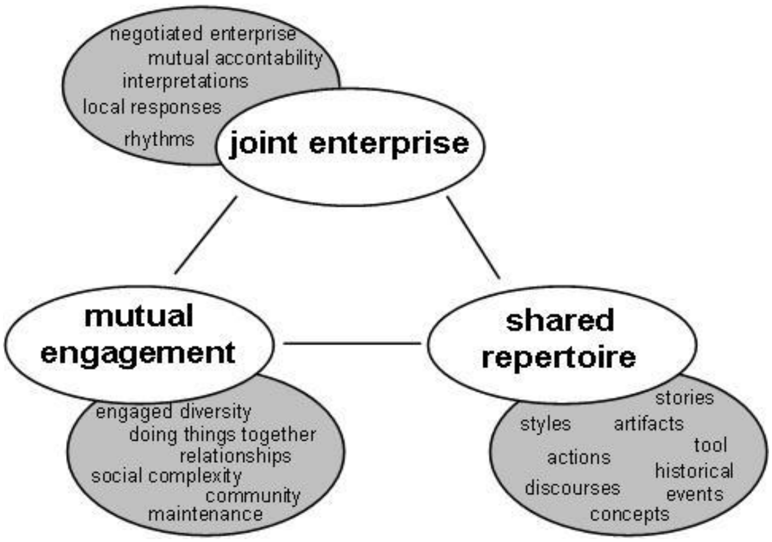


Figure 2-2 - Dimensions of practice as the property of a community

The Learning

Learning is the engine of practice and practice is the history of learning (Wenger, 1998; p. 96). In the social practice theory, learning becomes a characteristic of the practice and the practice becomes the outcome of the participation and reification processes intertwined over time. Learning in practice thus includes three core processes: (i) *evolving forms of engagement* which refers to the discovery of how to engage, to develop mutual relationships, to define identities, to establish who is who, who is good at what, who is easy or hard to get along with, (ii) *understanding and tuning the enterprise* in terms of defining the enterprise and reconciling conflicting interpretations of what the enterprise is about, aligning their engagement with the enterprise and learning to hold each other accountable to it, and (iii) *developing the repertoire, styles and discourse*s *of the community,* namely renegotiating the meaning of various elements; producing or adopting tools, artifacts, representations; recording and recalling events; inventing new terms and redefining or abandoning old ones; telling and retelling stories; creating and breaking routines (Wenger, 1998; p. 95). Learning is an ongoing process that changes our ability to engage in a practice, the understanding of why we engage in it, and the resources we have at our disposal to do so.

Learning has to do both with the development of our practices, and our ability to negotiate the meaning. It is not the mere acquisition of memories, habits, and skills, but rather is about developing personal histories and about forming an identity. In order to learn in practice, someone need not only to experience meaning but also to be able to negotiate this meaning. The interaction of mutuality of engagement, accountability to the enterprise and negotiability to the repertoire can result in learning, and to a change in the alignment between experience and competence.

Boundary

Joining a community of practice involves entering not only its internal configuration but also its relations with the rest of the world. In this sense, the practice creates boundaries that are not simple lines of demarcation between inside and outside but form a complex social landscape of boundaries and peripheries. Participation and reification act as sources of social discontinuity and as connections that create continuities across boundaries. Particularly, two types of connections among CoPs are identified: (a) *boundary objects*, namely the products of reification that can cross boundaries and enter different practices (e.g. artifacts, documents, terms, concepts etc) and (b) *brokering*, namely connections provided by people who can introduce elements of one practice into another through experiences of multi membership.

The interactions among various CoP can have several types of boundary encounters such as meetings, conversations, and visits. Wenger (1998) identifies three types of boundaries encounters between different communities of practice and discusses strong and weak points in each one (Wenger, 1998; p. 112): (a) the *one-to-one* which refers to a conversation between two members of two different communities, (b) the *immersion*, when a member of a community becomes an observer for another community by visiting the practice and (c) the *delegations*, when delegations of a number of participants from each community are involved in an encounter.

In addition to being a source of boundary for outsiders and insiders, a practice can also become a form of connection and there are three ways that practice itself can become such a connection. The first type of connection provided by a practice is the *boundary practices* (if a bounder encounter –especially of the delegation variety- becomes established and provides an ongoing forum for mutual engagement, then a practice is likely to start emerging and its enterprise is to deal with boundaries and sustain a connection between a number of other practices and by addressing conflicts, reconciling perspectives and finding resolutions; p. 114). The second type is the *overlaps* when a particular enterprise of one community constitutes a direct and sustained overlap between two practices (p.115). The last type is the *peripheries*, where “a periphery of a practice is a region that is neither fully inside nor fully outside and surrounds the practice with a degree of permeability” (p. 117). As communities of practice differentiate themselves and also interlock with each other, they constitute a complex social landscape of shared practices, boundaries, peripheries, overlaps, connections, and encounters.

The Locality

A practice constitutes a form of locality in that is always located in a particular time and in a particular space. Moreover, aspects of the repertoire of practice are exportable to other communities. Although mutual engagement is a matter of locality, in a constellation of practices the global and the local coexist and shape each other. Communities of practice negotiate meanings, learn, develop their practice and form their identities in a local base. However, the results of the negotiation of meaning can have a global impact and also can be affected by meanings and resources that come from the broader societal context. In these terms, local and global have a continuous interaction in the development of the practice of the community.

In Wenger’s conceptualization of social theory of learning issues of identity are omnipresent and the concept of identity “serves as a pivot between the social and the individual, so that each can be talked about in terms of the other” (p. 145). Identity and practice are in a constant interplay as on the one hand, our identities include elements from the practices we engage in or the practices we come into contact even if we don’t belong to them and, on the other hand, the practices gain specific meanings through engagement in the process of negotiation of meaning among participants. In this sense, participation or non-participation into a practice constitutes both sources of identity in the context of a CoP. Particularly, Wenger identifies four forms of participation: (a) full participation (insider); (b) full non-participation (outsider); (c) peripherality (participation enabled by non-participation, which leads to a full participation or remains on a peripheral trajectory and (d) marginality (participation restricted by non-participation, which leads to non-membership or to a marginal position) (p.166).

2.1.2. Studying Teachers' Learning In A Cop Context

The framework of CoP has gained increasing attention in the last few years (Llinares & Krainer, 2006). Among the several studies that utilize the idea of CoP to conceptualize learning to teach mathematics, Goos (2014) discerns three main categories with regard to their focus. The first category concerns studies that investigate preservice teacher education or the professional learning and development of practicing teachers. For example, Lachance & Confrey (2003) investigated the professional development of secondary mathematics teachers as regards the extension and deepening of both their content knowledge and their technological skills in the context of an emerging community of practice. The results showed on the one hand, how the development of professional communities can be encouraged by researchers and teacher educators and, on the other hand, provided evidence that the context of the community can support curriculum reforms and teaching innovations. Similarly, Gomez & Rico (2007) used Wenger's theory to explore the learning processes of mathematics teachers who worked in collaboration in a methods course. In their work CoP proved to be a useful concept both for structuring and studying mathematics teachers' professional development.

The second category considers studies that concern face-to-face or online interaction or a combination of both. An example of such studies is Barab, MaKinster & Rebecca Scheckler's (2003) work in which the researchers supported the development of an online community of practice consisted of mathematics and science teachers who are creating, reflecting upon, sharing, and improving inquiry-based pedagogical practices. They further explore variables that characterize the dynamics of such networks and they discuss methodological tools for meeting the challenges of designing for a CoP. A further study is that of Dalgarno & Colgan (2007). In their work, they reveal the potentialities of on-line communities in supporting elementary teachers' professional development.

The third category as discussed by Goos (2014) concerns questions about how a community of practice is formed and sustained compared with questions about the effectiveness of a CoP in promoting teacher learning. This category may also contain theoretical studies which criticize Wenger' s framework or add new perspectives in the social theory of practice. An example of such studies is Lerman's (2000) publication in which he explores theoretical frameworks that have social roots as regards knowledge and learning processes and he also investigates the impact of such frameworks in mathematics education research. At the end of the discussion, Lerman proposed the person-in-practice-in-person for the study of individuals in social practices. According to Lerman (2000), this unit of the analysis incorporates individual aspects as well as social interactions and contextual features. A further example is that of Adler (2000) who is discussing central ideas of social practice theory that facilitate mathematics teacher education as long as emphasizing a number of issues as regards teachers' professional development that the communities of practice framework seem to encounter as potential challenges. Jaworski's (2006) work for communities of inquiry is also rooted in communities of practice. Her conceptualization of professional learning sets inquiry at the core of mathematics teachers' education, inquiry as both a tool and a "way of being". Jaworski highlights the importance that teachers need to keep a critical stance on their practices, questioning them and explore alternative approaches to teaching and modifies Wenger's (1998) concept of *alignment* to *critical alignment* (Jaworski, 2006). Last in a very recent research approach, Gueudet & Trouche (2012) drawing on Wenger's conceptualization of Communities of Practice (Wenger, 1998), expanded their scheme for an individual *documentational genesis* (Gueudet & Trouche, 2009), which encounters the interplay between a teacher and a set of resources leading to a document, to a *community documentational genesis* which encounters the interplay between a teacher's CoP and sets of resources, mobilized for achieving common goals (Gueudet & Trouche, 2012, pp. 308-309). As Llinares & Krainer (2006) pointed out, the interest of researchers in using the CoP framework to conceptualize learning in teachers' professional development constitutes a growing field of research in the mathematics education community.

Nevertheless, in statistics education literature the utilization of social practice theory in designing or studying practicing teachers' development is still in its infancy. An approach that utilized this framework is Meletiou-Mavrotheris & Paparistodemou's (2011) study, which explored the forms of collaboration and shared knowledge building in a multinational group of elementary and secondary teachers who participated in an online professional program. Another example is Zapata-Cardona's (2014) study that structured and investigated a professional development program in statistics following the CoP principles with an aim to support teachers in bridging between theory and practice.

Although there are not many studies that have employed the CoP framework in the statistics education community, such approaches are considered crucial in establishing links between research and practice in statistics education since teachers in this context are key stakeholders in the formation and transformation of their own teaching practice (Shaughnessy, 2014).

The community of practice framework has a dual role in our study. Firstly, I designed our study in a way that promoted the emergence of a CoP among secondary mathematics teachers who were working on developing their teaching practice. Secondly, I employed concepts and structures of Wenger's (1998) theory as tools to investigate and interpret the secondary mathematics teachers' work within the newly established CoP of our study.

Particularly, I focus on the development of teaching statistics for the development-of- statistical-thinking-and-reasoning (DSTR) practice. This practice exists on the boundaries of three well-established CoPs, namely the Mathematics Teaching CoP, the Mathematics Education CoP, and the Statistics Education CoP. The DSTR is a currently developed practice which is gradually shaping its own CoP (see Figure 2-3). In Greece, this practice is in its infancy. Particularly, this study coincided with a reform in the curriculum which first introduced secondary mathematics teachers with teaching tools and approaches aligned to this practice. Thus, DSTR practice forms a newly developed community in the international terrain, and I aim to investigate the local evolvement of this practice in a newly established community in Greece. Participants in this community, except the teachers, are also two researchers who are members of mathematics education and statistics education community and act as brokers in the formation of the DSTR CoP.

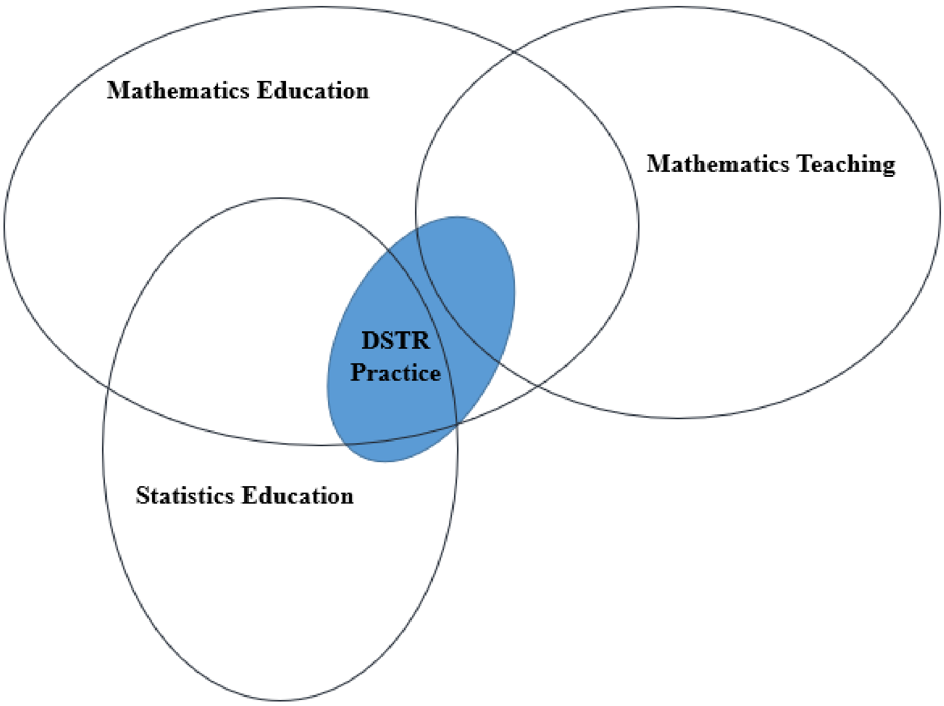


Figure 2-3 - The emergence of a boundary practice

The emergence of a community of practice is not something that usually happens but rather it is something that is nurtured and needs time to occur. In my study, I worked with 11 secondary mathematics teachers for two academic years in a regular and volunteer base. I view this group as a newly established community of practice by recognizing this the three characteristics identified by Wenger as follows:

(a) Mutual engagement: the group was gathered by the researchers but, although the teachers worked voluntarily, continued to work (except a few losses in the second year) for a period of two years. The attempts made by the researchers and the willingness and interest showed by the teachers, both resulted in the active engagement of the teachers in the group meetings. Throughout the two years, the teachers built relationships among them and between them and the researchers, they gained their place within the group and they recognized the specific contribution of the others in the group's work. For all these reasons we believe that the group members gradually built a mutual engagement within the group and gain their unique role inside the community.

(b) Joint enterprise: The main enterprise for the group was the teaching of statistics-for-developing-statistical-thinking -and- reasoning. This enterprise was not joint because everybody agreed or felt confident to teach statistics in this way, but more because the group, negotiated openly the elements and the characteristics of the desired teaching and the collective outcome of this negotiation formed the content of the enterprise.

(c) Shared repertoire: Throughout the two years of the group's work, the teachers negotiated various resources and ideas; they experimented with various tools and created various forms of reification (teaching interventions, worksheets, examples, histories, research reports, etc). By engaging with and reflecting on new resources, the teachers started to develop a new meaning related to the reified objects of their work. Thus, these reified objects are not just points of reference, but rather meaningful products that reflect a history of mutual engagement and because of that, they consist of a shared repertoire among the group's teachers.

In the next parts of this Chapter, I will discuss in detail what are the specificities and the nature of DSTR teaching and what are the guidelines and the challenges with respect to this practice. Moreover, I will try to illustrate why I consider it to exist on the boundaries of mathematics teaching and not inside mathematics teaching. Last, I will synthesize the main theoretical constructs of our study to a concise scheme and I will further explain how the concepts of CoP theory influence the dynamics of this scheme.

2.2. The Study Of Teaching Resources In Mathematics Education

The role of the resources in the teaching and learning process, as well as teachers' work with resources, has been widely discussed in mathematics education literacy and it gains an increasing attention the last few years (Adler & Reed, 2002; Remillard, Herbel-Eisenmann, & Lloyd, 2009; Pepin, Trouche, & Gueudet, 2012). However, the conceptualization of a resource differs in the various theories and forms a different starting point for the study of mathematics teaching resources and their use.

For example, Remillard (2005) focuses on curriculum materials, namely "printed, often published resources designed for use by teachers and students during instruction" (p. 213). In her work, she sees teachers as active designers that interact in a dynamic way with curriculum and she analyses teachers' interactions with resources in terms of *interpretation of* and *participation with* them. The interpretation of resources refers to the meaning teachers attribute on the resource-based on their knowledge and their previous experiences, while the participation with resources refers to the way teachers shape and transform the material they use.

In another approach, Gueudet & Trouche's (2009) theory of documentational genesis focused on teachers' design process. Particularly, they see teachers' products as *documents*, the outcome of teachers' activity, which are consisted of a set of resources saturated with teachers' experiences – *scheme of utilization* (see Figure 2-4). In this sense, a resource is never isolated but rather it always belongs to a set of resources. The documentational process includes two interrelated processes, the process of *instrumentation* which refers to the influence of the resource into the teachers' practice, and the process of *instrumentalization* which refers to the reshaping and transformation of the resources as the teachers draw on them. Gueudet's & Trouche's conceptualization of resources exceed from curricular resources to include all the resources that are developed and used by teachers in their design activity, including for example colleagues discussions or electronic communications.

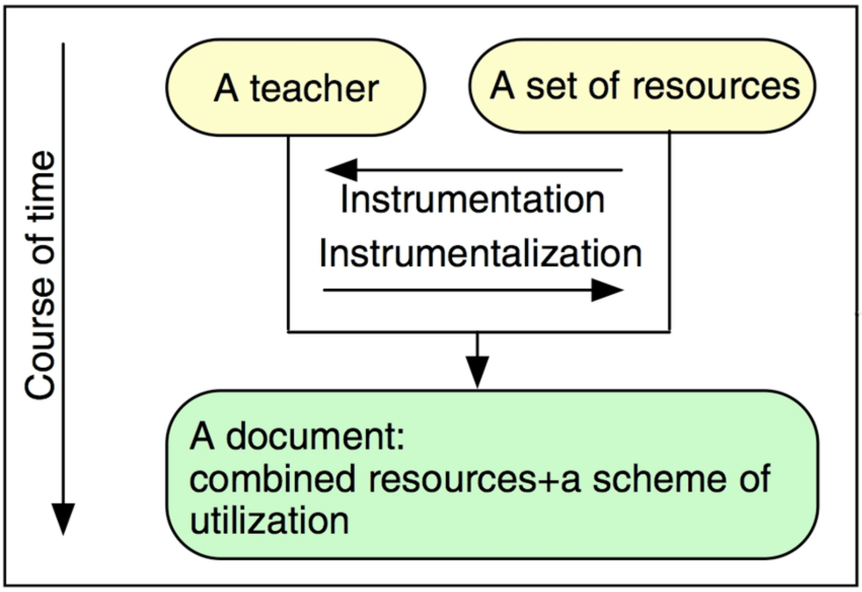


Figure 2-4 - Schematic representation of a documentational genesis (Gueudet & Trouche, 2009)

Adler (2000a) sees resources-in-practice-in-context and her conceptualization for resources includes: (a) a range of other material resources (i.e. technologies like computers or the internet, school mathematics materials, such as specialized software tools or geoboards ,and mathematical objects such as theorems, graphical representations; (b) everyday objects such as calculators, newspapers, stories; (c) other human resources (i.e. persons such as parents or colleagues, processes such as collegiality, and knowledge base resources such as everyday knowledge, mathematical knowledge, curricular knowledge or professional knowledge); and (d) socio-cultural resources (e.g. language, time) (Adler, 2000a). Adopting a social perspective, Adler (1998) acknowledges complex contexts and practices in which resources are used and changed over time (Adler, 1998; p. 34), and she uses the term *re-source* as both a noun and a verb to describe teachers' use of resources to *re-source* their practice, namely to source again or differently about the teaching-learning process (Adler, 2000a). Moreover, she builds on the notion of transparency (Lave & Wenger, 1991) to tackle the complexity of functioning a resource in and for school mathematics. Transparency with its duality of visibility and invisibility is not "property of the resource, but a function of how the resource is used and understood within the practice in context" (Adler, 2000a, p.217).

The concept of resources is not central to the CoP theory. However, one of a CoP's central feature is the *shared repertoire*, which according to Wenger:

> *…includes routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice. The repertoire combines both reificative and participative aspects (Wenger, 1998; p. 83).*

In this sense, a CoP can constitute a context where the interactions between members and resources can be acknowledged and studied. Furthermore, the study of resources under the lens of a CoP can also move the research focus from an individual perspective to a social one regarding the use and transformation of mathematics teaching resources. However, the research on resources mainly concentrates on the individual, focusing on the interplay between teachers and resources (Ruthven, 2009; Gueudet & Trouche, 2009; Choppin, 2011; Adler, 2012; Remillard, 2012). Yet, very recently there is a growing interest in teachers' collective work with resources (Gueudet, Pepin, Sabra, & Trouche, 2016; Visnovska & Cobb, 2013). Such an approach aims to proceed the study of resources and shed light on collective aspects of teaching that eventually can help in understanding the role of resources in teachers' collective work and in teachers' professional learning (Gueudet, Pepin & Trouche, 2013).

In our study, *resources* constitute the driving force for the process of negotiation of meaning. The members of the community interact with the resources in their attempt to form the joint enterprise. Through the participation and the reification, while the members negotiate the meaning of a resource, they conclude to practices that align with the aims and the expectations of DSTR. These shared practices constitute a consensus among the members, link the resource to the actual teaching and informs the shared repertoire of the community. I describe this process as a re-sourcing process in accordance with the use of the term re-source in Adler's work (Adler, 2000a). Concentrating on the re-sourcing process of the DSTR practice, our lens moves from the use of the resource in an individual base to the collective use of the resource in the context of the community.

Although frameworks from mathematics education community, that concentrate on the study of resources are still unexploited by statistics education community, the research topic of resources is rather important in the case of DSTR practice because of the: (a) multiplicity of existing resources that can support the teaching and learning of statistics (see 2.3.1); (b) fact that teaching resources for statistics instruction constitute a great challenge for mathematics teachers (see 2.3.3); and (c) epistemological differences between statistics and mathematics that differentiate the teaching and learning in each area (see 2.4.1.). In the following sections of this Chapter, I will explain in more details how I view resources in DSTR and what resources seem to have a prominent role in this practice according to the existing research.

2.3. The Statistics Education Research Field

2.3.1. Statistics And Statistical Activity

In his article titled "Statistics among the liberal arts" David Moore (1998) mentioned:

> *"Statistics is a general intellectual method that applies wherever data, variation, and chance appear. It is a fundamental method because data, variation, and chance are omnipresent in modern life." (Moore, 1998, p. 1254).*

To capture the nature and sense of statistical activity I will refer to the statistical thinking in statistical inquiry framework (Wild & Pfannkuch, 1999) which was built upon the existing literature and on intensive interviews with 11 students of statistics and 6 professional statisticians. In their thorough work, Wild and Pfannkuch (1999) presented four dimensions of statistical thinking in empirical inquiry.

The first dimension according to the researchers is described by the PPDAC model (see Figure 2-5). This model includes the main steps in which statistical thinking occurs in a statistical inquiry activity. The steps involved in this model are: *problem* (P), namely the transformation of the general problem that drive the inquiry to a precise statistical question that can be answered by using data – *plan* (P), namely making all the necessary decisions for the strategy to follow in order to answer the defined question including what to measure, who to ask, what data to gather and what to do with these– *data* (D), namely the actual data production, storage as well as the primary data, organization and cleaning – *analysis* (A), namely the data exploration, the implementation of the planned analysis method, redefining of analysis method if necessary and developing hypothesis for the problem – *conclusion* (C), namely interpret and communicating the conclusions of the investigative cycle and respond whether and how the statistical question has been answered or not.

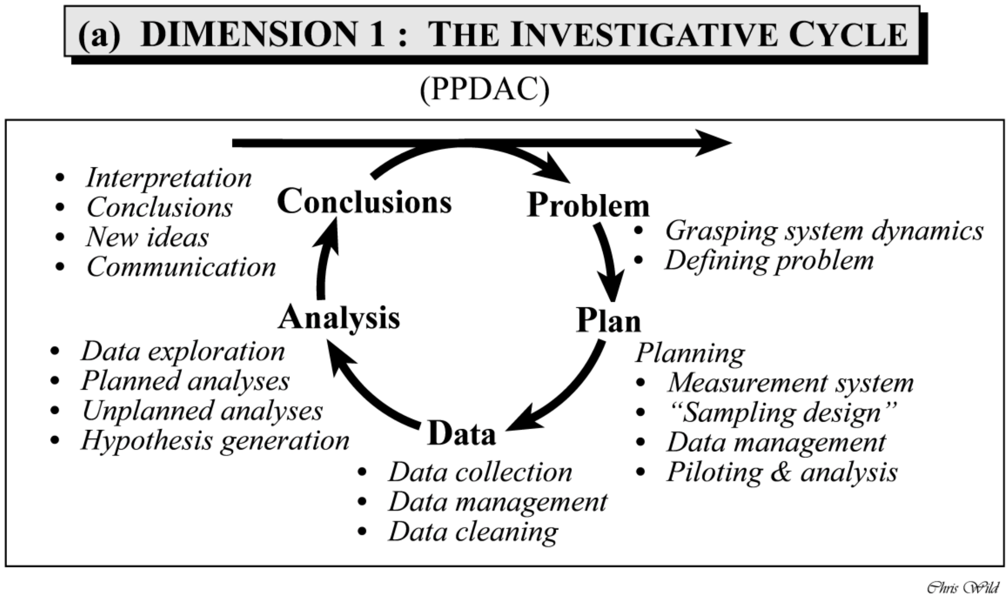
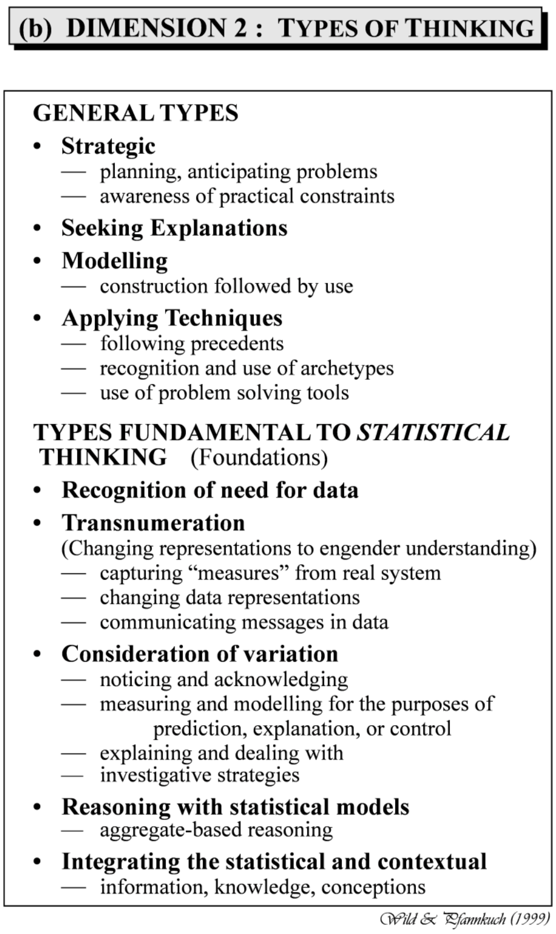


Figure 2-5 - The PPDAC investigative cycle (Wild & Pfannkuch, 1999; p.226)



All the five steps of the investigative cycle require statistical skills and types of thinking that are relative to the general activity of problem-solving. However, in a statistical investigative cycle, the data environment and the stochastic nature of the problems brought to the fore other types of thinking closely relative to the statistical activity itself. These types of thinking constitute the second dimension in statistical inquiry (Wild and Phannkuch, 1999) and are summarized in Wild & Pfannkuch's (1999) types of thinking model (see Figure 2-6).

Figure 2-6 - The Types of Thinking model (Wild & Pfannkuch, 1999; p.226)

As we can see in Figure 2-6, among the general types of thinking that are related to problem-solving situations are the strategic thinking, seeking explanations, modelling and techniques' application. On the other hand, there are types of thinking that are closely connected to the statistical activity itself. For example, data manipulation can be an objective in many problem situations but being aware of the need for data and being able to define specific informational goals that can be addressed using data constitute thinking processes inherent in the statistical inquiry. Another process that pervades all statistical analysis is the process of *transnumeration*.

This term was coined by Wild and Pfannkuch (1999) to refer to the change of our way to look at the data in the hope that this will convey new meanings to us and more particular to numeracy transformations made to facilitate understanding (p 227). The process of transumeration constitutes a primary process in statistical activity since the changes in the data representations can allow a deeper understanding of different aspects of the underlying system as well as of the system itself. Other fundamental processes in the Types of Thinking model are the consideration of variation, reasoning with statistical models, and integrating the statistical and the contextual parameters of the problem.

The statistical thinking in statistical inquiry framework (Wild & Pfannkuch, 1999) involves two more dimensions which are closely related to the inner process that is underlying the five steps of the PPDAC cycle. These dimensions are described by the interrogative cycle and the dispositions model (see Figure 2-7).

The interrogative cycle refers to a cycle of critical questions and considerations that need to be addressed in every step of the PPDAC cycle. As we can see in Figure 2-7, every step in this cycle constitutes a synthesis of aims, possibilities, achievements, and projections to the world outside. The interrogative cycle captures the attempts of the interplay of internal and external sources of information, ideas, validation of methods and results, etc. On the other hand, the fourth dimension refers to the individuals' inherent qualities of mind and character which seem to be important in the statistical activity.

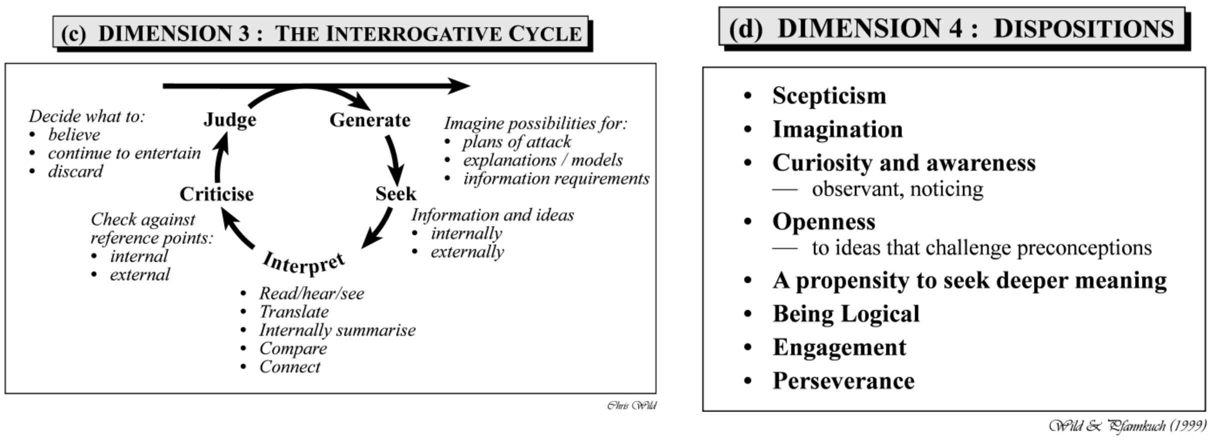


Figure 2-7 - The thinking models for the 3rd and 4th dimensions in the statistical thinking framework (Wild & Pfannkuck, 1999; p. 226)

The framework of Wild and Pfannkuch (1999) manages to capture all the complexities and specificities of the statistical activity in statistical investigations as well as of the nature of statistical problems. The investigative cycle model (see Figure 2-5) may seem quite similar to models of mathematical problem solving and modelling (e.g. OECD, 2013; Blum & Borromeo Ferri, 2009), but as the other 3 dimensions reveal, every step in the statistical investigative cycle entails thinking processes and qualities that are specific to statistical activity (e.g. transnumeration or consideration of variation) as well as challenges and complexities that are inherent to stochastic context of statistical problems. Despite the distinctions between statistical and mathematical problem-solving activity, statistical investigations, and especially as regards the stages of plan and analysis in the PPDAC cycle, requires an extensive use of mathematical theories and particularly a rich range of abstract ideas regarding randomness as well as models aim to make sense and describe the behavior of stochastic structures. The branch of mathematics which is concerned with the study of random phenomena is the probability, and since uncertainty and variability is omnipresent in statistics, probabilistic concepts and thinking are inherent in the statistical activity. Furthermore, phenomena characterized by chance and uncertainty are very often addressed by misleading strategies and false intuitions (Fishbein & Schnarch, 1997; Falk & Konold, 1992; Tversky & Kahneman, 1974) and data-driven approaches and real contexts can facilitate the comprehension of formal probabilistic concepts. Thus, chance and data constitute two interrelated and intertwined notions in identifying and solving stochastic problems. In this manner, statistics and probability constitute two branches that need to coexist in harmony both in problem-solving as in teaching and learning process (Batanero, & Borovcnik 2016; Chernoff, Paparistodemou, Bakogianni & Petocz, 2016; Wild, Utts & Horton, 2018).

2.3.2. The research on teaching and learning statistics

Reflecting on the nature and the objectives of statistics, as the science of learning from data, Wild, Utts and Horton (2018) refer to the mission of statistics education as to “provide conceptual frameworks (structured ways of thinking) and practical skills to better equip our students for their future lives in a fast-changing world”(p. 6). The development of statistical skills for the purpose of handling the challenges of a fast-changing data-oriented world is not a simple task for statisticians let alone for mathematics teachers who have little experience with data themselves.

The teaching of statistics in its infant stages was mainly intended to the training of professionals and it was restricted to demographics, collection of information and presentation of quantitative data. It is only at the beginning of the twentieth century that statistical methods and notions of inference enter in a formal manner at the tertiary level, and as information technology and data processing dominate the evolution of modern societies the need for developing abilities to understand data and make decision in data-driven environments became an important objective of many university departments worldwide. Despite the growing importance of statistics in modern societies, probability and statistics teaching constitutes a rather recent inclusion in secondary mathematics education curriculums and we rarely find any statistical content in mathematics curriculum before 1940 (Zieffler, Garfield, & Fry, 2018).

The history of the development of statistics as a discipline resulted in the late establishment of the statistics education community. Particularly, the statistics education community was initially part of statistical organizations (e.g. Education Committee of International Statistical Institute (ISI), 1948; Section on Statistical Education of American Statistical Association (ASA), 1974) facing issues of teaching and learning statistics among students of different disciplines and considered by researchers of diverse professional identities, for example disciplines such as applied statistics, mathematics, engineering, psychology, medical science, business or financial science etc. The International Association for Statistical Education (IASE) was established in 1993, while the first International Conference on the Teaching of Statistics (ICOTS) was first held in 1982. Despite its quite recent establishment, the statistics education community is now producing a considerable volume of research aiming to support the work of teacher educators, curriculum designers, education researchers, teachers, and other professionals.

Similarly, to the epistemology of statistics and mathematics, the statistics education and mathematics education communities still share their tensions and cooperation. In particular, research focusing on statistics education is not often published in mathematics education research journals, while qualitative and sociocultural methodologies which are very popular in the mathematics education community, are quite scarce in the statistics education community. The latter point will be further discussed in the following section. As regards the representativeness of statistics education studies in mathematics education journals I will refer to the review article for statistics education research by Petocz, Reid & Gal (2018) who reported that:

> *A search through several mathematics education research journals for the period 2010–2014 revealed only a small number of papers on statistics education research. In seven leading journals, we found a total of 20 such papers, including nine in a special issue of a single journal (see Makar & Ben-Zvi, 2011). (p.g. 76)*

The diversity in the scientific origins of statistics teachers and statistics education researchers resulted in a plethora of learning goals regarding statistical teaching as well as in various definitions regarding what constitutes statistical literacy, reasoning, and thinking. In an attempt to synthesize the existing views Garfield, delMas, & Chance (2003) presented the following definitions that aimed to capture the essence of these terms.

• Statistical literacy includes basic and important skills that may be used in understanding statistical information or research results. These skills include being able to organize data, construct and display tables, and work with different representations of data. Statistical literacy also includes an understanding of concepts, vocabulary, and symbols, and includes an understanding of probability as a measure of uncertainty.

• Statistical reasoning may be defined as the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, representations of data, or statistical summaries of data. Statistical reasoning may involve connecting one concept to another (e.g., center and spread), or it may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes and being able to fully interpret statistical results.

• Statistical thinking involves an understanding of why and how statistical investigations are conducted and the “big ideas” that underlie statistical investigations. These ideas include the omnipresent nature of variation and when and how to use appropriate methods of data analysis such as numerical summaries and visual displays of data. Statistical thinking involves an understanding of the nature of sampling, how we make inferences from samples to populations, and why designed experiments are needed in order to establish causation. It includes an understanding of how models are used to simulate random phenomena, how data are produced to estimate probabilities, and how, when, and why existing inferential tools can be used to aid an investigative process. Statistical thinking also includes being able to understand and utilize the context of a problem in forming investigations and drawing conclusions and recognizing and understanding the entire process (from question-posing to data collection to choosing analyses to test assumptions, etc.). Finally, statistical thinkers are able to critique and evaluate the results of a problem solved or a statistical study (Garfield, delMas, & Chance 2003; pp. 7-8).

The growing need for statistical literate citizens and the omnipresence of data in almost all professions and facets of everyday life created a flourishment in the statistical content in secondary education curriculums worldwide. Particularly, the abilities and understandings that are related to statistical reasoning and thinking became among the central aims of the statistics education community. In this respect, the reform movements for the teaching and learning of statistics in the secondary level, focus on the development of statistical reasoning and move away from the formalist and procedural approaches that are followed by the traditional curriculums (NCTM, 2000; Franklin et.al., 2007; New Zealand Ministry of Education, 2007).

Capturing the new guidelines for fostering of statistical reasoning and the relative literature review for teaching tools and strategies that seem to support this reform direction, Pfannkuch (2018) defines the learning experiences that students need to engage in order to be initiated into statistical reasoning and thinking. These essential learning experiences include data-rich environments, probability modelling activities, visualizations for conceptual understanding, designing of statistical investigations and evaluating statistical arguments and facilitating statistical reasoning (Pfannkuch, 2018). A curriculum that supports the bridging between statistics and probability has been considered as essential by many researchers in the literature (Chernoff et.al., 2016; Eichler & Vogel, 2014; Ben-Zvi & Garfield, 2004). This bridging requires environments of uncertainty, existence, and acknowledgment of various sources of variability, controlling and modelling chance behavior, drawing inferences about random situations. Such requirements are related to the notion of Informal Inferential Statistical Reasoning (IIR) which has gained great interest in statistics education research and which according to Gil and Ben-Zvi (2011) "refers to the cognitive activities involved in informally drawing conclusions (generalizations) from data (samples) about a wider universe (the population), while attending to the strengths and limitations of the sampling and the drawn inferences (Ben-Zvi, Gil, & Apel, 2007) and "articulating the uncertainty embedded in an inference" (Makar & Rubin, 2009, p. 85). Rubin, Hammerman, and Konold (2006) considered IIR as statistical reasoning that involves consideration of numerous dimensions: properties of data aggregates, the idea of signal and noise, various forms of variability, ideas about sample size and the sampling procedure, representativeness, controlling for bias and tendency" (p. 88).

The current guidelines for the teaching and learning of statistics as is reflected on the reformed curriculum and statistics education literature bring to the fore teaching approaches and tools that constitute a great challenge for mathematics teachers who are invited to teach statistics in secondary education (Batanero, Burril & Reading, 2011; Eichler & Zapata – Cardona, 2016; Groth & Meletiou - Mavrotheris, 2018). In the next section, I focus on the field of statistics education research that discusses issues related to teachers' preparation and professional development with regard to the teaching of statistics in line with the current guidelines.

2.3.3. Research In Teachers' Professional Development

The case of teachers' education in the field of statistics teaching is quite demanding and challenging since both the content and the teaching tools that aim to support this content are quite new and unfamiliar to mathematics teachers. However, as the statistical content is reinforced among reformed curriculum worldwide, the development of teacher education programs that aim to support teachers to promote statistical inquiry in their classroom is gaining increasing attention in the statistics education community. A central consideration for statistics education research and especially for the researchers who are working on developing professional development programs for teachers is "what should a teacher, who teaches statistics, know?". To this end, there is a growing number of research studies that explore the characteristics and specificities of teachers' knowledge and attitudes towards statistics.

Knowledge-Based Discussions

The frameworks developed to characterize mathematical knowledge for teaching (Shulman, 1987; Hill, Schilling & Ball, 2004; Ball, Thames & Phelps, 2008) constituted building blocks in models developed to characterize and capture the specificities of statistical knowledge for teaching. For example, Groth (2007), building on the notions of *common knowledge* (related to the mathematical content, e.g. correct use of formulas and mathematical notions or making correct statements or computations) and *specialized knowledge* (related to the teaching practice, e.g. provide appropriate examples or good explanations, understand the source of students' mathematical errors) suggested by Hill, Schilling and Ball (2004), described a hypothesized structure of statistical knowledge for teaching (see Figure 2-8).

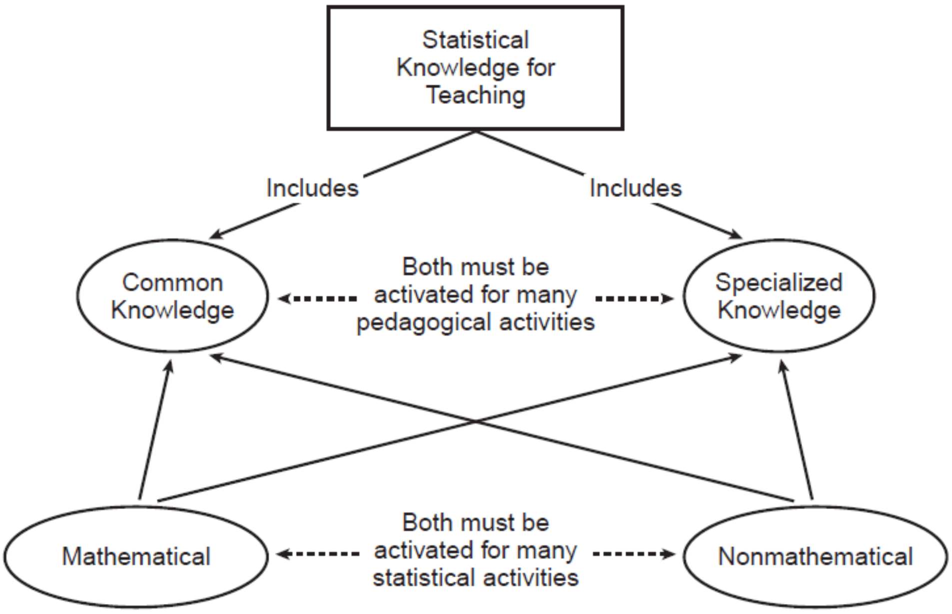


Figure 2-8 - A hypothesized structure of statistical knowledge for teaching (Groth, 2007; p. 429

The model developed by Groth (2007) exceeds the conceptualization of Hill et.al. (2004) by acknowledging in both common and specialized statistical knowledge for teaching mathematical and nonmathematical aspects. In his work, he uses the four components of statistical investigation identified in GAISE framework (Franklin et.al., 2007), namely (a) formulating questions, (b) collecting data, (c) analyzing data, and (d) interpreting results, to illustrate mathematical and nonmathematical aspects in teachers' knowledge landscape. For example, as regards the formulating questions component, it is a matter of mathematical common knowledge to read accurately a box plot and formulate questions based on data but understanding the difference between a deterministic and a stochastic question, although essential in statistical investigation constitutes a skill-based on nonmathematical cognition. Similarly, it is a matter of mathematical specialized knowledge to understand the difference between how students understand box plots and dot plots, but the acknowledgment of the potential fruitfulness of students' posed statistical questions requires nonmathematical insights.

Further to Groth's (2007) model, the research developed to describe mathematical knowledge for teaching has also been used to study and describe statistical knowledge for teaching in particular statistical topics. One such work is the work of Burgess (2011) who suggested a two-dimensional framework to describe teachers' knowledge needed for teaching through statistics investigation. This framework used the concepts of *content knowledge*, with its dimensions of common and specialized knowledge, and *pedagogical knowledge*, with its dimensions of knowledge of content and students as well as of knowledge of content and teaching (Ball, Thames, & Phelps, 2008; Hill, Schilling, & Ball, 2004), in relation to six components of statistical thinking from Wild and Pfannkuch (1999) (namely, the four fundamental thinking types of transnumeration, variation, reasoning with models, and integration of statistical and contextual knowledge, along with general thinking linked to the investigative cycle and the interrogative cycle) (see Figure 2-9).

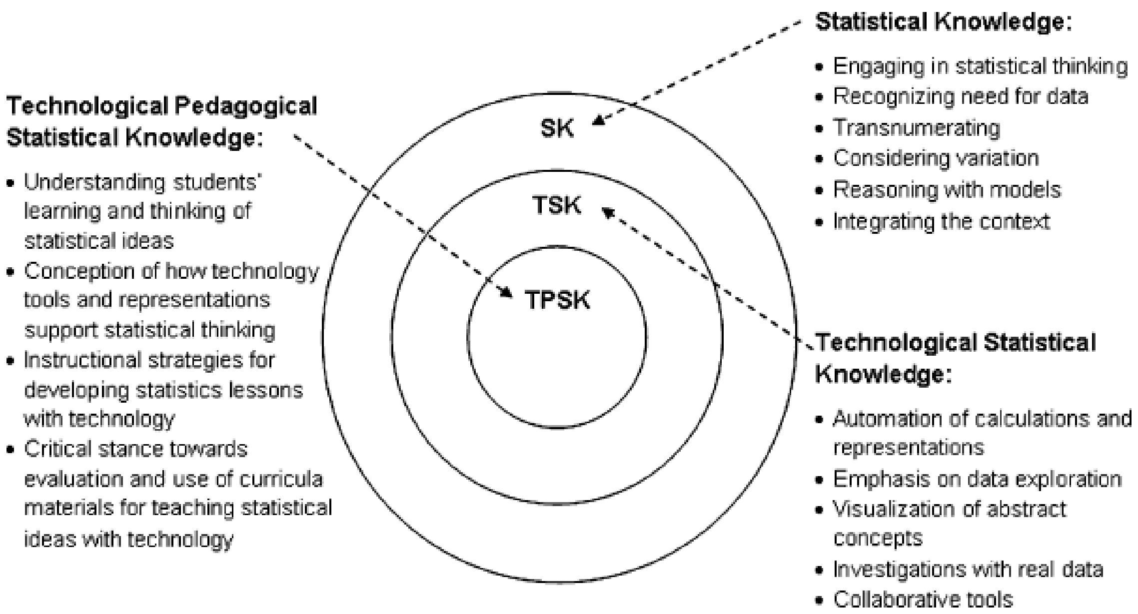
		Statistical knowledge for teaching			
		Content knowledge		Pedagogical content knowledge	
		Common knowledge of content (CKC)	Specialized knowledge of content (SKC)	Knowledge of content and students (KCS)	Knowledge of content and teaching (KCT)
Thinking	Transnumeration				
	Variation				
	Reasoning with models				
	Integration of statistical and contextual				
Investigative cycle					
Interrogative cycle					

Figure 2-9 - Framework for teacher knowledge to teach statistics through investigations (Burgess, 2011; g. 264)

Building on the three core components of Ball et.al. (2008) framework, namely common content knowledge (CCK), specialized content knowledge (SCK) and knowledge of content and students (KCS), Noll (2011) developed a framework specialized in sampling notions. In her study with graduate teaching assistants, she investigated teachers' knowledge about samples and data distributions as well as on their knowledge about students' attitudes towards sampling notions. She concluded that it is hard to distinguish between SCK and KCS and she aimed to identify distinctions among the three components in Ball et.al. (2008) framework in the particular context of teachers' working with variability and sampling notions.

Leavy (2010) explored the knowledge demands entailed when teachers engage in informal inferential reasoning (IIR). Working on a lesson study methodology, Leavy (2010) explored various aspects of prospective teachers' content and pedagogical knowledge and their components (Ball et.al., 2008). She highlighted particular issues related to the teaching of informal statistical inference, and specific difficulties related to teachers' attempts to promote such reasoning in their primary school classrooms (e.g. positioning of opportunities for IIR, excessive focus on procedures, time management issues). Moreover, she further discussed factors that seemed to support the development of inferential reasoning in teacher education (e.g. modelling inferential reasoning, focusing on students' understandings, observe and reflect on the implemented lesson).

Another framework acknowledges a further component that has been considered as essential for statistical learning, namely the use of technological tools (Lee & Hollebrands, 2011). Technological tools that aim to support statistics instruction seem to have a prominent role in statistics teaching and constitute a point of focus of this study. Thus, a more detailed reference to technological tools will be made in the following sections. Except for statistics, technological tools have been considered as significant for many mathematical topics. Many researchers have focused on aspects of teachers' knowledge that encounter effective pedagogical use of such tools in the teaching of mathematics and they used the term technological pedagogical content knowledge (TPCK) (Koehler & Mishra, 2005; Niess, 2005, 2006). In an attempt to describe the cognitive demands that influence the effectiveness of the use of technology in the statistics teaching, Lee &



Hollebrands (2011) suggest a framework that encounters three core components of teachers' knowledge that forms teachers' technological pedagogical statistical knowledge (TPSK) (see Figure 2-10).

Figure 2-10 - Framework for TPSK (Lee & Hollebrands, 2011; p. 362)

In their work, Lee & Hollebrands (2011) emphasize both that the development of TPSK and the effective use of technology in the classroom requires a strong background in statistics, and furthermore, that in order to support teachers to develop deeper statistical understandings we need to engage them in opportunities to develop TSK and TPSK.

All the existing frameworks that aim to characterize statistical knowledge for teaching (SKT) adapt many components of the frameworks that conceptualize mathematical knowledge for teaching (MKT). However, the different epistemologies of mathematics and statistics (Moore, 1992; delMas, 2004; Gattuso & Ottaviani, 2011;) as well as the different thinking processes entailed in mathematical and statistical activity, as has been discussed in section 2.3.1, require modifications and adaptations so that the suggested conceptualizations of statistical knowledge for teaching can capture the particularities of the statistical content. These particularities and differences between SKT and MKT reflect the new demands and qualities that mathematics teachers are challenged with in order to teach statistics, and consequently poses new queries and research considerations for teacher education and professional development programs design.

Teaching Tools And Teaching Practices

In contrast to the rapidly growing field on aspects of teachers' statistical knowledge for teaching, the studies that explore potential practices that could facilitate teachers to meet the current challenges of teaching statistics are still not very richly represented in the existing literature. Chadjipadelis, Meletiou-Mavrotheri, and Paparistodemou (2010) emphasize that in order for mathematics teachers to be able to engage students in statistical reasoning, some special training is not enough but rather they need to develop a different identity and get deeper into the statistical culture. Next, I will discuss tools, experiences, and contexts that have been nominated as essential for supporting the "statistics teacher of the new era" (characterization given by Chadjipadelis, Meletiou-Mavrotheris and Paparistodemou, 2010).

Statistical reasoning and thinking have frequently been associated to authentic contexts, real data and media extracts (Gal et.al., 1997; Watson, 1997; Watson & Callingham, 2003; Chick & Pierce, 2012). Reading and interpreting graphs in media texts have become a central issue in developing the dispositions subsumed under statistical literacy and it constitutes a quite complex and challenging domain for mathematics teachers. In a survey of more than 240 instructors of introductory statistics (Garfield, 2000) less than 25% said they "frequently used "discussions of statistics in the media, and roughly half indicated they never asked students to critique news articles in classroom assessments. It seems that many instructors' teaching of statistics is neither oriented towards statistical literacy nor assess media extracts for teaching purposes. Monteiro and Ainley (2007) investigated 218 prospective teachers and explored elements and processes involved in the interpretation of graphs in media. In their study they noted that, in order for participants to make their interpretations, they mobilized not only their statistical knowledge but also other forms of knowledge, their experience about the context and their feelings, and they suggested critical sense as an important element which can help to mobilize and balance a range of factors presented in the interpretation of graphs process. Other studies show that when teachers are engaged in activities that involve graph interpretations in real contexts, although they deepen their statistical understanding, issues regarding the value and comprehensibility of the represented data and teachers' familiarity with the underlying context can be obstacles for teachers to understand and link such resources to the teaching and learning of statistics (Pierce, Chick & Wander, 2014; Bakogianni, Paparistodemou & Potari, 2014).

Enriching teachers' learning experiences has become of great attention among researchers and statistics teachers' educators. Opportunities to engage teachers in investigations, experiments and data explorations have become central in various professional development programs. The main body of research which focuses on such learning experiences for teachers pays also great attention to the use of appropriate digital tools and especially on the use of TinkerPlots[1]. For example,

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[1] TinkerPlots is a data visualization and modeling tool for use with students (grades 4 through university level) (Konold & Miller, 2005).

Lee et.al. (2016) worked with 27 teachers in developing models to approach inference using a repeated sampling approach. The examination of the teachers' visual representations of the repeated sampling approach to inference brought to the fore conceptualizations that seem to be central in the use of simulations and repeated sampling for drawing inferences. Teachers' engagement with chance-experiments and the randomization process using the TinkerPlots was at the core of the learning trajectory created by Frischemeier and Biehler (2014). In their work, they studied how 23 prospective teachers conducted a randomization test and they identified difficulties and complexities that underlie teachers' ability to formulate a null hypothesis as well on drawing conclusions from a given p-value. The potentiality of Tinkerplots, was also indicated in their further work (Frischemeier and Biehler, 2018) with eight prospective teachers. In their later work, Frischemeier and Biehler study how the teachers addressed a comparing two groups task using the software Tinkerplots. Issues regarding teachers' understandings and interpretations of the statistical content as well as teachers' technological knowledge with respect to the software, were emerged and discussed providing feedback for teachers' professional development. In some further approaches, Cypriot researchers developed frameworks to enhance teachers' Technological Pedagogical Content Knowledge (TPACK) which were based on teachers' engagement with mobile devices (Tsouccas & Meletiou-Mavrotheris, 2017) and digital games (Meletiou-Mavrotheris, Paparistodemou & Tsouccas, 2018). These studies showed that teachers appreciated the potentiality of using tablet technologies and digital games as an instructional tool. However, they also highlighted that the productive use of such tools in the statistical learning process requires the strengthening of teachers' competence to select, evaluate and integrate these tools meaningfully in their teaching practice.

Professional Learning In Collaborative Contexts

All the learning experiences discussed above, engage primary teachers and secondary mathematics teachers in tools and processes that aim to move teaching practices from a formalist and procedural approach to statistics learning, to an inquiry approach in which ideas of probability, randomness, and variability play a central role. As shown in the relative studies, the creation of a learning environment that supports statistical investigations constitutes a quite demanding task for teachers. Moreover, teachers' professional learning to support statistical inquiry in the classroom is also a rather challenging task that concerns researchers and mathematics teachers' educators. An overview of the characteristics and the results of two longitudinal projects, namely the Early Statistics Project (Meletiou-Mavrotheris, Paparistodemou, Mavrotheris, & Stav, 2008) and the Developing Expertise in Teaching Statistical Inquiry (DETSI) Project (Makar, 2008), revealed some key recommendations for teachers' professional development (Makar & Fielding-Wells, 2011).

Among the characteristics that seemed to be crucial for teachers, Makar & Fielding-Wells (2011) emphasize the importance of providing teachers with opportunities to collaborate with their colleagues and with researchers in developing teaching, as well as, to provide opportunities to link their learning experiences with their classrooms' reality and reflect on both their learning and their teaching. Although the benefits from the collaboration among teachers and between teachers and researchers have been considered as crucial in developing statistical teaching (Ponte & Noll, 2018; Eichler & Zapata-Cardona, 2016), there are few empirical studies to provide information of how such collaborative contexts can be encouraged and managed, as well as possible dimensions and tensions inherent in the role of the participants in a collaboration for the development of statistical teaching.

An example of such a study is the work of Zapata-Cardona (2014). She structured and studied a teachers' professional development program following the principles of a community of practice. The aim of this program was to support teachers in bridging between theory and practice in their statistics teaching and Zapata-Cardona (2014) explored the strengths and weaknesses of the program as well as implications for the teacher education. A further example (Meletiou-Mavrotheris & Paparistodemou, 2013) explored the forms of collaboration and shared knowledge building in a multinational group of elementary and secondary teachers who participated in an online professional program for the teaching of statistics. Implications for teacher education and professional development as long as for the design and structure of on-line courses intended for teachers' are emphasized and discussed through the lens of Communities of Practice (Wenger, 1998).

The relative review in this area, although quite scarce, gives us some insight of the benefits and of the complexities that exist in contexts where teachers participate as co-producers of learning environments and provide some valuable directions for professional development programs. Shaughnessy (2014) highlighted the need for contexts where teachers will be key stakeholders in the research of statistics teaching. As he noted, such contexts can strengthen the links between research and practice and provide a deep insight not only on teachers' professional learning but also on how this learning can have a strong effect on the formation of the actual teaching practice (Shaughnessy, 2014).

The next section of the theoretical part aims to illustrate how I view statistical teaching and learning as a social practice. Moreover, I aim to unfold our theoretical perspective for the development of this practice as well as for the professional development of the teachers who are engaged in this practice, and in particular the secondary mathematics teachers.

2.4. Conceptualizing DSTR Practice

2.4.1. Epistemological Approaches To Teaching Statistics

Is statistics a subfield of mathematics or not? How does statistical activity compare to mathematical activity? What is the role of probability in statistics? What is the role of probability in teaching and learning statistics at the school level? What statistics should be taught at school? Who is the most appropriate person to teach statistics, a statistician or a mathematician?

Questions regarding the nature and the specificities of statistical activity and statistical thinking have been many times the focus of debate in the statistics education literature from the infancy of the field's development (Moore, 1988; Steinbring, 1990; delMas, 2004; Scheaffer, 2006). In the statistical investigation cycle of Wild and Pfannkuch (1999), as discussed earlier in section 2.3.1, the specificities and differences between statistics and mathematics have been illustrated in much detail. One of the leading scholars in the statistics education community, David Moore, has given particular emphasis on the differences between statistics and mathematics and he strongly highlighted that "statistics do not originate within mathematics" (Moore, 1988; p. 3). According to this point of view, statistics has its own territory and needs to be taught separately from mathematics, since the focus on theoretical probability and abstract mathematical constructs constitutes obstacles rather than asset for the learning and understanding of the statistical context (Moore, 1988; Moore, 1997; Moore & Cobb, 2000;). The recognition of statistics as a separate discipline raises also concerns whether statistics should be taught by mathematicians or by statisticians (Moore, 1988).

However, despite the different origins and characteristics in the nature of the general discourse between the two fields, statistics are placed within mathematics curriculum worldwide (NCTM, 2000; MoE, 2007; Ontario, 2008; ACARA, 2008; New School, 2011) raising new challenges and new professional requirements for mathematics teachers. The harmonious development of both mathematical and statistical thinking in mathematics curriculum has been a point of focus in many studies and many have been written on how the fruitful cooperation between the two subjects can facilitate students’ understandings of both (Gattuso, 2002; Scheaffer, 2006; Gattuso & Ottaviani, 2011). More specifically, notions of uncertainty and random variability underpin all statistical problems and thinking processes, linking the learning of statistics to concepts of probability theory and chance models. On the other hand, probability is the branch of mathematics that study randomness, hence the probabilistic problems often constitute a paradoxical and counterintuitive area of inquiry that needs to link to real contexts, everyday situations, and empirical data in order to enrich conceptual understanding as well as to build correct intuitions. The different epistemological perspectives for statistics as a field of inquiry, as well as the exploration of the relationship between statistics and mathematics, and more particularly between statistics and probability, resulted in a number of different approaches for the teaching and learning of statistics.

Burril and Biehler (2011) discerned four perspectives for the teaching of statistics with regard to the relationship between probability and statistics. The first perspective is the framework of statistical thinking provided by Wild and Pfannkuch (1999). This framework concentrates on thought processes entailed in statistical investigations and sets random variability in the core of statistical reasoning. The second perspective views statistics as a process different from mathematics and is provided by the Guidelines for Assessment and Instruction in Statistics (GAISE) K-12 Report (Franklin et al., 2005). According to this perspective random variability or variability in data as long as the context, constitute decisive dimensions that distinguish statistics from mathematics. The third perspective is the statistical literacy perspective (Watson, 1997; Schield ,1999; Gal, 2002). According to this perspective, learners are seen as users instead of producers of data or statistical results (p. 59) and the emphasis is put on skills and abilities that are required for adults to understand and critically evaluate the statistical information around them. The last perspective is that of stochastics, a sub-domain of mathematics comprising probability and statistics. According to this perspective, statistics and probability both serve the same role of facilitating students to acquire appropriate experiences, knowledge and intuitions and to develop appropriate understandings and strategies in order to cope with situations where uncertainty and chance are inherent (Batanero, Green, & Serrano, 1998;. Batanero, Henry, & Parzysz, 2005; Borovcnik, 2006; Engel, & Sedlmeier, 2005; Saldanha, & Liu, 2014). Each perspective provides a ground for the curriculum guidelines with respect to the teaching of statistics. For example, in a study that investigating links between statistics and science, Watson (2014) discussed some core differences between the Australian curriculum (ACARA, 2013) and the US Common Core State Standards: Mathematics (CCSSM, 2010) based on the GAISE report (Franklin et.al., 2007) for the teaching of statistics. Particularly, in the case of the Australian curriculum, statistical content covers all grade levels developing gradually various concepts of chance and data representations and interpretations but without mentioning the word inference nowhere. On the other hand, in US CCSSM statistics and probability notions are first introduced in Grade 6 and there is a great emphasis on the context and the concept of variation. Moreover, the aim of drawing inferences from data populations become explicit from Grade 7 in the US CCSSM.

In this study, I view the teaching and learning of statistics through the lens of statistical thinking perspective provided by Wild and Pfannkuch (1999). Particularly, I concentrate on thought processes entailed in statistical investigations and I acknowledge random variability as a core concept in the development of statistical reasoning. Since in Greece, statistics is part of the mathematics curriculum taught by mathematics teachers, I am especially interested in how statistics and mathematics can be intertwined harmoniously within the mathematics curriculum. I view statistical learning in an environment where the statistical activity will reveal the specificities of the stochastic context and also the strong connection and the complementarity between statistics and probability. Groth (2015) describes the evolution of the statistics education community and mathematics education community, as two distinct communities of practice, who share common concerns and problems. In his study, he emphasizes the importance to encourage boundary interactions between these two communities, which will contribute to the vitality of both, on the one hand, and foster the research on teaching and learning in both statistics and mathematics. Sharing the communities of practice perspective for mathematics education and statistics education, we consider our work to be situated on the boundaries of these two communities of practice.

2.4.2. Fundamental Statistical Ideas In DSTR

The various epistemological approaches on the teaching and learning of statistics result in the complexity with regard to the identification of core concepts that are shared and foundational among the different perspectives. In an attempt to combine the different perspectives in a way that acknowledges the criteria for fundamental ideas in mathematics (Heymann, 2003) and statistics (Heitele, 1975), Burril and Biehler (2011) identify seven core statistical ideas which forms a common ground for what can be considered as fundamental in teaching and learning statistics. Moreover, these seven ideas allow for conceptual connections with other disciplines inside and outside mathematics, illustrate the specificities of statistical context and provide a view of the larger statistical landscape. However, the seven fundamental statistical ideas (Burril & Biehler, 2011) incorporate concepts that although can be found in both statistical and mathematical content, the meaning and the thinking that is related to them differs in the two disciplines. Below, we present the seven fundamental statistical ideas as well as tensions related to each idea with respect to the required thinking and meaning in the context of school statistics and school mathematics, as identified and discussed in Burril and Biehler (2011) (see pp. 62-65).

Data

In statistics data constitute numbers in context, the concept contains all the relevant notions and processes, namely types of data including categorical variables, methods of data collection and data measurement with a focus on biases and errors, and the interpretation of data displays. On the contrary, in mathematics data is context-free, measuring is a standard process with no error concern, and there is no concentration on categorical attributes. Moreover, in statistics data contribute to the development of probability notion while in mathematics data constitute numbers used to apply the probability formula.

Variation

Variation in statistics reflects the total effect of change in a stochastic situation. Since real data contains error and uncertainty, variation is about identifying and measuring variability to predict, explain and control. On the other hand, mathematical situations are always been addressed in an exact and precise manner and data are typically assumed to perfectly fit a mathematical model.

Distribution

Distribution is a concept developed in the context of the statistics to describe patterns of tendency and spread. This concept is crucial in reasoning about statistical variables (described or summarized in theoretical distributions/ empirical distributions/ sampling distributions). This concept has no particular tension to any mathematical concept.

Representation

Representation refers to graphical or other representation that reveals stories in the data. The idea of representation is essential in statistics exploration, since most statisticians begin with a graph and often change to different graphs or representations to see a different story in the data, this alternations from one representation to other is a meaningful and important process in statistical activity that let for engendering understanding from data, the term transnumeration has been coined to describe this process (Wild and Pfannkuch, 1999; p. 227). Contrarily, in mathematics visual representations of data have not a prominent role in mathematical activity and graphs are often used to show the same relationship in different representations (tables, graphs, and symbols).

Association And Modelling Relations Between Two Variables

In statistics, association and modelling relations between two variables refer to the study of the nature of the relationship among statistical variables. This study includes regression for modelling statistical associations, the creation, and study of scatterplots, the check of residuals and the consideration of how the context might relate to the choice of a model. On the contrary, mathematics usually uses Cartesian coordinates to draw graphs of functions with no attention to statistical aspects. Moreover, although mathematical modelling has been considered as a potential bridge between statistics and mathematics, this potentiality is seldom exploited. For example, the mathematics educators who do research and development in mathematical modelling (Blum, Galbraith, Henn, & Niss, 2007) data collection plays no systematic role in going from a real situation to the mathematical model nor does compare mathematical results to empirical data.

Probability Models For Data-Generating Processes

In statistics, variability in data can be quantified and modeled with probability models. On the other hand, the understanding of probability concepts can be facilitated by simulations, real data-sets and randomly drawn samples from a population. However, the traditional teaching of probability in mathematics relies mainly on a formal approach which neglects randomness and often takes independence and equiprobability as given and not to be considered or checked.

Sampling And Inference

Sampling and inference refer to the process of drawing samples from a population and drawing conclusions from these samples with some degree of certainty. These concepts are related to the sample size heuristic (Tversky & Kahnemann, 1971) by which people tend to rely on samples of small size to make generalizations for the population. The mathematical approach to proportional reasoning ignores variability and uncertainty and pays no attention to sample-to-sample variation and how this variation decreases as the sample size increases. Thereafter, a perfect proportional relationship is assumed by the students, which eventually acts as an obstacle for the development of statistical thinking.

The formation of a learning environment where all the above ideas will be central and illustrated is essential for allowing students to develop a deeper understanding of statistical content. Moreover, these ideas are not merely related to the acquisition of a specific piece of knowledge but rather to the development of a different type of thinking and reasoning with data information, namely statistical thinking and reasoning. The development of statistical thinking and reasoning constitutes a foundation stone for the present study, and I will refer hereafter to the practice that supports this development as *DSTR practice* (development-of-statistical-thinking-and-reasoning- practice). I next concentrate on elements of this teaching practice that seem to be crucial in supporting teachers to develop statistical reasoning and thinking in the mathematics classroom. A scheme for the conceptualization of what constitutes the DSTR practice is then suggested and illustrated further in section 2.4.6.

2.4.3. Resources For DSTR

As I discussed in section 2.2, the study of teaching resources has gained increased attention in mathematics education community (Adler & Reed, 2002; Pepin, Trouche, & Gueudet, 2012; Remillard, Herbel-Eisenmann, & Lloyd, 2009). In our conceptualization of teaching resources for DSTR practice, I use the term resources in a broad sense that goes beyond textbooks, tools, and material resources. In this sense, a resource can be anything that can be in use by teachers in order to support and facilitate the teaching and learning process in a DSTR learning environment. For example, except from digital tools, web-based sources and written documents, teaching can be supported by what teachers know, their previous experiences (everyday, teaching, academic or other), their collaboration with others (inside or outside mathematics teaching community), but also, the DTSR practice can be also supported by social or institutional resources, e.g. existing open sources of data in official websites, accessibility to databases of national organizations, the time spent to statistics teaching or the school assessment system. This broad view of teaching resources emphasizes their prominent role in the construction of meaning with regard to the practice of teaching and this why I acknowledge resources as one of the core elements of DSTR practice. Next, I will discuss some central resources that seem to play a crucial role in the teaching and learning of statistics and have been a point of focus in the statistics education literature. For the presentation of this discussion, I will use Adler's (2000) main categories for resources namely, cultural, material and human resources.

Cultural

In Adler's work, the time and the language constitute central cultural resources for the teaching of mathematics. However, due to the fact that statistical concepts gain meaning from the context they refer to, cultural resources play a more remarkable role in DSTR practice. For example, other societies have more data-oriented culture and other less data-oriented or other societies provide data availability and accessibility to a greater extent and others to a limited extent. The general cultural stance towards statistics has also an effect on the general policy and institutional means regarding the teaching of statistics. For instance, we consider as principal cultural resources the space that statistics subject occupies in the national curriculum, the required qualifications for the statistics teachers (e.g. degree in mathematics or in statistics, any special education), as well as the prevailing teaching approach towards statistics (e.g. theoretical or data-driven). Moreover, the strong links between statistics and society are also reflected in the use of everyday life problems and real data, where people's preferences, habits, and attitudes form the context of the problem situations.

The strong links between statistics and socio-cultural context unveil the importance of cultural resources in the formation of statistics teaching, since on the one hand, cultural resources provide a context for designing for learning statistics, and on the other hand, affect the conditions under which the teaching resources, suggested by statistics education community, integrate smoothly in particular local societies.

Although cultural resources are not a focus of research neither in mathematics education nor in statistics education communities, I consider them important for the formation of DSTR practice.

Material

As I discussed in section 2.2, material resources are the most common studied resources in the field of research for mathematics education teaching resources. Adler (2000), categorizes material resources into four categories, namely mathematical objects, everyday objects, school mathematics material, and technologies (see section 2.2). I use these four categories in order to assort material resources emerged in statistics education literature.

Many statistical objects have been a focus of research in the statistics education literature and some of them have been especially highlighted as essential for statistics and for statistical learning. For example, as we saw earlier (see section 2.4.2) data constitute one of the fundamental statistical ideas for the teaching and learning of statistics (Burril & Biehler, 2011). Since data constitute a physical representation of particular information, the notion of data is context-related and many times in research, the study of data is closely related to their context. The importance of using real data and motivated contexts has been also related to the motivation of students to engage with statistical activity, and their support to develop conceptual understanding of statistical content (Heaton & Mickelson, 2002; Lehrer, & Schauble, 2002); Franklin et.al., 2007; Garfield & Ben-Zvi, 2008; Sheaffer, 2011; Newman, Hood & Neumann, 2013;). Apart from the use of real data, many researchers have also studied how the use of randomization tests and the production of simulated data can also have a positive impact on students learning. Such studies reveal very interesting positive results for the development of students' inferential reasoning even in the case of very young students (e.g. Ben-Zvi, Gil, & Apel, 2007; Paparistodemou-Meletiou, 2008; Saplamidou, 2019). But the use of contextual data has not only been studied and emphasized as a remarkable resource for students learning but also as a notable resource in teachers' professional development (Heaton & Mickelson, 2006; Leavy, 2006; Gould, Bargagliotti, & Johnson, 2017; Frischemeier & Biehler, 2018).

The data has been many times studied in accordance with another fundamental statistical idea, namely the notion of variability. Since variability is inherent to the statistical activity, it underlies many statistical objects. Some of the most frequently studied in the statistics education literature with respect to the notion of variability are, except from the data and data production, the samples and sampling distributions (e.g. Bakogianni, Potari & Paparistodemou, 2013; Meletiou-Mavrotheris & Paparistodemou, 2014; Noll and Hancock, 2015), the statistical graphs with an emphasis on boxplots as well as on the transumeration process(e.g. Lee, Kersiant et.al., 2014; Edwards, Özgün-Koca & Barr, 2017), or data sets comparisons (Watson & Moritz, 1999; Ben-Zvi, 2004; Leavy, 2006; Frischemeier, 2014), the simulations of random experiments and the role of randomness (Prodromou, 2014; Lee, Starling et.al. 2014; Saldanha, 2015) and more. The existence of variability and uncertainty in statistical situations has also brought to the fore a vital discussion for the use of probability objects and ideas in the statistics learning process (e.g. Biehler & Pratt, 2012; Chernoff et.al., 2016; Batanero & Borovcnik, 2016). Objects related to probability theory (e.g. the formula for classical/conditional probability, the probability rules and properties, the law of large numbers, etc) constitute the mathematical core of statistics and have been many times in the focus of statistics education research (e.g. Watson, 1995; Tarr & Jones, 1997; Tarr J.E., Lannin, 2005). Other statistical objects that have been of a broad interest in a learning environment where the focus is on statistical reasoning and thinking are the measures of center and spread with a special focus on students' interpretations and understandings around them. For example, mean value has been other times conceptualized as a balanced point in the learning process (e.g. Flores, 2008; O'Dell, 2012; Bakogianni et.al., 2015) and other times as a signal in noisy processes (Konold, 2002).

Everyday objects have been highlighted by Adler (2000) as an important subcategory for material resources in mathematics education, and she acknowledged mainly resources such as money, newspapers, stories, calculators and rules (see section 2.2.). However, this category seems to play an important role in the teaching and learning of statistics since statistical problems rely a lot on real situations and thus on everyday objects. A particular emphasis has been given to the use of media extracts and web resources in the teaching of statistics which have been associated a lot to the support of students' statistical literacy as well as to the development of critical thinking towards statistical information (Watson & Moirtz, 1997; Monteiro & Ainley , 2007; Budgett & Pfannkuch, 2010). The connection between statistical content and real-life has also brought to the fore a number of everyday objects that are related other times with students interests (e.g. social media, online games, movies etc.), other times with social news (e.g. specific news, subjects of discussion on web blogs or forums, predictions for national elections) and other times with resources related to wide social or consuming interest (e.g. weather forecasts, annual sales of cars/ mobiles). Such resources have also been many times studied, either for their motivated role in the development of students' statistical literacy (e.g. Watson, 2006; Sturm & Eichler, 2014).), or for potential challenges that are related to their use in classroom (e.g. Monteiro & Ainley, 2004; Bakogianni, Paparistodemou, Potari, 2014). Moreover, the strong connection between statistics and probability, brings to the fore a number of everyday objects that are related to randomness and random experiments such as playing cards, dices, coloured marbles and others (Kazak. Fujita & Wegerif, 2016; Doerr, DelMas & Makar, 2017; Case & Jacobbe, 2018). Moreover, the idea of using games to motivate students' statistical reasoning has been recently related to everyday objects such as mobile phones and tablets (e.g. Kyriakides, Meletiou-Mavrotheris, and Prodromou, 2016).

Another subcategory identified by Adler (2000) for material resources, is school mathematics material (see section 2.2). In this subcategory are considered resources beyond the mathematical content that are specifically related to the pedagogical approach of this content (e.g. textbooks, geoboards, specific software, etc). Such resources constitute also a central part of statistics education. There are available various reports of guidelines for statistical teaching and learning (e.g. GAISE Report, Franklin et.al., 2005; MoE, 2007; New School, 2011), a lot of applets and specially designed objects for statistical teaching, such as:

- *NCTM illuminations for data analysis and probability - https://illuminations.nctm.org/*
- *Explore learning GIZMOS for Mean, Median, and Mode - https://www.explorelearning.com/*
- *the coin-tossing applet - http://www.rossmanchance.com/applets/CoinTossing/CoinToss.html-).*

The availability of digital resources that are specially designed for statistical teaching and learning constitutes an essential part in statistics education literacy and digital resources are considered as a crucial part of statistics instruction (Rubin, 2007; Pratt, Davies, Connor, 2011; Biehler, et.al., 2013). For this reason, I will discuss this resource separately at the end of section 2.4 for the conceptualization of DSTR practice and I will consider this resource as an individual subcategory of material resources in our study.

General technologies such as chalkboards, calculators, computers or projectors are of great importance for statistics, as in any other mathematical subject. However, such technologies are considered with respect to other material resources (such as statistical objects, software tools, or particular lesson plans) and they are not a focus of research on their own. I also considered such resources as peripheral in the learning process and they are not as a point of focus in our study.

Human

Human resources refer to the humans themselves and all the resources that are related to how they act, as well as to how they interpret the situations around them. Such resources can be knowledge of content, professional or everyday experiences, beliefs, emotions as well as interactions with other people.

Such resources have been broadly studied in statistics education literature in terms of knowledge aspects, beliefs, affective issues or attitudes towards particular statistical ideas for both teachers and students. Moreover, issues of collaboration between students, between teachers or between teachers and researchers have been also acknowledged and discussed in the literature. I will discuss some central issues of this field of inquiry with respect to two main categories, namely studies that are related to students and studies that are related to teachers.

Resources Related To Students.

Human resources that are related to the study of students are mainly students' misconceptions or difficulties with regard to particular concepts. For example, there are various studies that research students' reasoning on variation either by exploring students' expectations through trials of spinners or six-sided die rolls (e.g. Canada 2006, Watson et.al. 2003). Other studies focus on students' understandings regarding distributions, and in more particular, in terms of students' conceptions about the center (Konold & Pollatsek, 2002; Watson & Moritz, 2000) and variability (Reading, 2004; Watson & Kelly, 2006). Other statistical concepts and processes that have been widely studied with respect to students' conceptions are: comparisons between data sets (e.g. Watson & Moritz, 1999; Lee, Angotti, & Tarr, 2010) or the study of association and covariation between variables (e.g. Batanero, Estepa, & Godino, 1997; Casey, & Nagle, 2016). Moreover, students' use of heuristics in statistical inquiry, as well as the role of intuition in making decisions under uncertainty have been also been widely studied and discussed (Tversky, & Kahneman, 1974; Batanero, Serrano, & Garfield, 1996; Fischbein, & Schnarch, 1997; Gauvrit, & Morsanyi, 2014; Papadatos, Bakogianni & Zachariades, 2019). Such studies helped in providing some insight on students' thinking strategies and intuitive conceptions that seem to guide them in statistical problem-solving situations. Among the studies that informed literature with respect to human resources related to students, are also those which concentrate to students' beliefs and attitudes towards statistics (e.g. Gal & Ginsburg, 1994; Williams, 2015). There is also an important number of studies concentrating on the conceptualization of students' reasoning and understandings with respect to particular statistical concepts and processes. Some of these studies have formed theoretical frameworks that unpacked aspects of students' knowledge and argumentation. For example, Jones et.al. (2000), based on the SOLO (Structure of Observed Learning Outcomes) taxonomy (Biggs & Collis, 1991), developed a 4 constructs framework to describe students' thinking in data handling. For each of the four constructs in the data handling process, namely describing data, organizing and reducing data, representing data, and analyzing and interpreting data, students' reasoning is developed in a four-level path based on the Biggs & Collis (1991) development model, that is starting from idiosyncratic to reach analytic reasoning. In another study focusing on students' reasoning, Shaughnessy et.al. (2004), investigated students' understandings of variability in repeated sampling tasks. In their study, they suggest three types that seem to characterize students reasoning in a repeated sampling environment, namely,(a) additive, students' explanations are driven by frequencies, (b) proportional, students' explanations are driven by relative frequencies and (c) distributional, students' explanations are driven by both expected proportions and spreads. The extensive use and essential role of graphs in statistics motivated the work of Friel, Curcio & Bright (2001). In their work, they highlight the complexities inherent in graphing comprehension and they identify three levels to when students make sense of quantitative information displayed in graphs, namely, (a) reading data, which refer to information directly presented on the graph (e.g. values or scale), (b) reading between the data, which refer to defining relationships in the data (e.g. which column is highest or which is the range of the data), and (c) reading beyond the data, which refer to making, predictions, or inferences based on the data that is beyond the data presented on the graph (e.g. determine if the relationship between two variables presented on a scatterplot is linear or not). The complexity of statistical graphs and the different levels of graph comprehension have been further studied by many researchers not only regarding students' comprehension but also with regard to teachers' comprehension (e.g. Batanero et.al.,2010). Recently, there is a growing interest in informal inferential reasoning, and many researchers work on identifying key aspects that influence students when they drawing statistical inferences. Building on this field of inquiry, Makar and Rubin (2009) identify three key principles that seemed to be crucial to the development of informal statistical inference: (a) generalization, including predictions, parameter estimates, and conclusions, that extend beyond describing the given data; (b) the use of data as evidence for those generalizations; and (c) employment of probabilistic language in describing the generalization, including informal reference to levels of certainty about the conclusions drawn (Makar and Rubin, 2009; p. 85). In a further approach, Leavy (2010) acknowledges implication for teacher education and she suggest some key features that characterize the design and the selection of tasks that aim to support students develop informal inferential reasoning. Particularly, Leavy suggests that teachers design can be informed by the degree to which tasks require students to: utilize prior knowledge to the extent that the knowledge is available (Zieffler et al., 2008), provide evidence-based justifications for generalizations (Makar & Rubin, 2007; Zieffler et al., 2008), and use probabilistic language in describing the generalizations while making reference to levels of certainty about the conclusions drawn (Makar & Rubin) (Leavy, 2010; p. 48).

Last, except the human resources especially related to the teaching and learning of mathematics, Adler (2000) refers also to some basic human resources that are especially important for the maintenance of schooling generally, such as teacher qualifications agreed as basic, the teacher-pupils ratio in the classroom, etc. Such resources are being studied only in contexts of educational studies in underdeveloped countries. However, the particular resource of teacher-pupils ratio, although not being a particular point of focus on itself, it is an important resource for statistical lesson design since it constitutes an important factor for the progress of an experiment or study that is based on classroom data collection.

Resources Related To Teachers.

A large part of the literature related to knowledge aspects of statistics teachers, as well as of teachers attitudes and affective issues related to them have been discussed earlier in section 2.3.3.

However, we still know little about the interaction between teachers and statistics education researchers and how this interaction can affect the actual statistical teaching. Shaughnessy (2014) emphasizes that "the co-production of knowledge by teachers and researchers provides the opportunity for a dynamic duality where teachers become key stakeholders in the research process, and on the flip side, researchers become key stakeholders in the teaching process". This duality, although constitutes a promising resource for both professional and scientific development (e.g. Noll & Shaughnessy, 2012) is still in its infancy in statistics education research.

In this study, I consider and acknowledge researchers as a human resource that influences the formation of DSTR practice. Particularly, researchers provide material tools, support conceptual understanding and pedagogical management of the statistical concept. Moreover, they encourage teachers to take initiatives, communicate thoughts and considerations and interact with each other. In this manner, researchers constitute a core human resource for the practice developed in our study's community.

Summing up all the above as well as the specificities that are inherent to the stochastic nature of the statistical activity, I believe that the study of teaching resources is especially important in the case of statistics. This importance stems mainly from the: (a) multiplicity of existing resources that can support the teaching and learning of statistics as described above; (b) fact that statistical resources constitute a great challenge for mathematics teachers (Bakogianni, Potari & Paparistodemou, 2013; Bakogianni, 2015; Burgess, 2011; Groth, 2007; Lee & Hollebrands, 2011; Skott & Østergaard, 2016); and (c) epistemological differences between statistics and mathematics that differentiate the teaching and learning in each area (delMas, 2004; Meletiou-Mavrotheris, 2007; Moore & Cobb, 2000; Scheaffer, 2006). Our focus with regard to the resources goes beyond to which resources are chosen or used by the teachers in an individual manner when they teach statistics. I am especially interested to study resources in a collective context of teachers' collaboration and follow the process from the moment a resource becomes available to the teachers to the moment there is a consensus for its role and potentiality in the DSTR practice. I will use the term re-source, as a verb, to describe this process by which a resource integrates into the teaching or being utilized alternatively by the teachers (Adler, 2000). I also consider this process not as an individual process, but rather, as a collective process that gives shape and reflects the locality of the teachers' collaboration to develop DSTR practice.

2.4.4. Learning Potentials

As I discussed in section 2.3.2 the current guidelines to statistical teaching (NCTM, 2000; Franklin et.al. 2005; MoE, 2007; New School, 2011; Arnold, Confrey, Jones, Lee, Pfannkuch, 2018) go beyond learning about formulas and mastering procedural skills to include the conceptual understanding of the fundamental statistical concepts and ideas and the understanding and appreciation of statistical methods and practices. In a learning environment where statistical reasoning and thinking are central, students are expected to understand the purpose and logic as well as the process of statistical investigations and develop the ability to identify and pose questions that can be answered by data (e.g. Garfield & Chance, 2000). Students are also expected to understand and deepen in the fundamental statistical ideas as for example the existence and the role of variability in the problem solving (e.g. Burril & Biehler, 2011) or the use of probability models and data simulations for sampling from or investigating the characteristics of a population (e.g. Borovcnik, 2011; Lee, Starling & Gonzales, 2014). They also need to use and evaluate appropriate statistical tools and methods in order to analyze data and to measure or control variability (e.g. Franklin et.al., 2005), to compare or explore data distributions (e.g. Biehler, 2001; Ben-Zvi, 2004;), to make interpretations and provide arguments to communicate statistical results (e.g. Gal & Garfield, 1997). Moreover, students are expected to appreciate contextual aspects of a statistical problem and to adopt a critical stance when they are faced with an argument based on data (e.g. Gal, 2002) as well as to make sense of graphs or media extracts (Friel, Cursio & Bright, 2001; Watson 1997).

Learning potentials reflect what it counts as important in the learning process and why. In this manner, learning potentials link particular resources with the goals and the rationale that guides teachers to use them, and this is the reason why I consider learning potentials as the second central element that shapes the DSTR practice. However, the links, created by the teachers, between resources and learning potentials are not always the same as those created and suggested by the researchers and curriculum designers when they develop or study teaching resources. These links constitute an internal process that has been widely studied in the literature and is described as *transformation* (Rowland et. al., 2009), as *instrumentalization* (Gueudet &Truche, 2009), as *participation with resources* (Remillard, 2005) or as *re-source* process (Adler, 2000) (see also section 2.2). In my work, I study such links as a collective result in teachers' interaction with resources while they collaborate to develop DSTR teaching. I am particularly interested to see such links as the consensus of the community of the teachers of our study, after negotiating what is important for their statistical teaching and why.

Since the two researchers of this study act as facilitators and brokers (Wenger, 1998) in the teachers' community we have many times a role to provide resources in this community. These resources are other times explicitly linked to particular learning potentials and other times is left to the teachers to inquiry such potential links. In the first year of the study, we put a particular emphasis on the conceptual connections between probability and statistics. Particularly, we emphasized the importance of supporting the stochastic context in the DSTR learning environment by focusing on concepts of randomness and variability. This special focus to the use of probabilistic tools in the statistics teaching aimed to move the content of the learning potentials from computational abilities to the development of interpretive and critical skills that can serve to control and manage the stochastic context of the problem situations. Viewing the teaching and learning of statistics through the lens of statistical thinking perspective provided by Wild and Pfannkuch (1999), as I discussed in section 2.4.1., we encourage teachers to acknowledge the specificities of the stochastic context and also to highlight in their teaching design the strong connection and the complementarity between statistics and probability.

2.4.5. Features

The features constitute particular teaching approaches, strategies, and means that shape the conditions within which the students are provided with the resources chosen by the teachers to achieve the defined learning potentials. For example, many times have been highlighted the importance of engaging students in statistical investigations and explorations of real data (e.g. MacGillivray & Pereira-Mendoza, 2011; Makar, 2008). Moreover, many researchers suggest students' involvement with either real or simulated data in order to generate sampling distributions (e.g. Burrill, 2002; Gourgey, 2000; Stohl Lee, Angotti, & Tarr, 2010), and they also emphasized students' engagement with dynamic software tools that support data explorations (e.g. Ben-Zvi, 2006; Biehler, et.al., 2013; Fitzalen & Watson, 2014).

Pfannkuch (2018) combine various curriculum approaches that are discussed in the current literature and summarize them into three central features that foster students' enculturation into statistical thinking. The first key feature according to Pfannkuch is the promotion of essential statistical experiences. Such experiences involve data-rich environments which set authentic data in the cornerstone of the curricula and leave space for uncertainties and powerful visualizations to reveal. An additional experience that is emphasized strongly by Pfannkuch (2018) is probability modelling and the students' involvement with constructing models that aim to capture and explain random behaviors. Such experiences connect strongly probability and statistics in a way that supports the development of both constructive reasoning and intuition. Other essential experiences are related to fostering students' conceptual development through appropriate visualizations such as comparing and interpreting boxplots or visualizing conditional probability situations through concrete or digital materials. Such experiences, although seemed to have a positive impact in assisting students to develop a statistical perspective, require further research in order to clarify how and what visualizations are appropriate to foster conceptual understanding in the various educational levels (e.g. Bakker, Biehler and Konold, 2005). Further, designing investigations is also considered as a key learning experience since it gives students the opportunity to link the design of a study with the analysis and the statistical outcomes. However, such an experience is often unmanageable due to time and other resources limitations, so Pfannkuch presents research examples that suggest the use of virtual environments to assist statistical investigations in the school context (e.g., Baglin, Bedford, & Bulmer, 2013; Bulmer & Haladyn, 2011; Darius, Portier, & Schrevens, 2007; Steiner & MacKay, 2009). Such environments seem promising because, on the one hand, allow for entire statistical investigations to take place in the classroom, and on the other hand, they support the development of statistical reasoning and thinking. However, research with regard to these environments is still in its infancy. Over and above the essential statistical experiences that are related to the actual learning process, Pfannkuch refers also to the importance of evaluating arguments and statistical literacy, namely to interpret, evaluate critically and challenge arguments in media, professional or other everyday contexts (e.g. Watson, 1997; Gal, 2002; Merriman, 2006; Budgett & Pfannkuch, 2010). She strongly emphasized that "the evaluation of data-based arguments must be in the spotlight when reimagining curriculum approaches" (Pfannkuch, 2018; p. 399).

The second feature that is defined in Pfannkuch's re-imagination of statistics curriculum is fostering statistical reasoning and argumentation. In particular, she refers to the importance of supporting students to make accurate verbalizations when they interpret and reason from data. Last but not least, statistics teaching can no longer be considered without technology. Software tools specially designed for statistics instruction (hereafter I will use the abbreviation STSI tools to refer to Software Tools for Statistics Instruction) are assumed as an integral part of statistics instruction (Pfannkuch, 2018; p. 390), but due to our special focus on STSI tools in this study I will discuss on this feature further in section 2.4.7.

As we saw, features define the environment in which DSTR practice can be developed and the learning potentials and the purpose inherent in this practice can be facilitated and encouraged. In this sense, a feature links resources and learning potentials to the particular conditions that these can be accessible and transparent to the students. For example, we often talk about the importance of students' engagement with data explorations in TinkerPlots, in order to develop an understanding of data distributions. Through the students' engagement with data explorations in TinkerPlots (*Feature*), we link resources such as data, software tools, data distribution and data graph with the learning potential of developing students' understanding for the concept of data distribution. Thus, I consider features as the third element that shapes the DSTR practice. Given the fact that features reflect the means and conditions that define how the learning potentials can be supported and how the resources can be utilized in the actual teaching, this element is partly a component of the particular means and conditions related to the school context of this particular community that I study. In a particular school context, the resources and the associated learning potentials will be further linked to a teaching approach depending on which teaching approach is both feasible and familiar. For example, in order for teachers to engage students with data explorations on TinkerPlots they need to have access to a computer lab with this specific tool available and in the same time to be technologically and pedagogically educated to support such explorations. In this sense, I view features as the feasible and familiar characteristics of the practice developed inside the community of our study that aim to support the communally negotiated learning potentials. Thus, the features reflect the locality and the content of the community's joint enterprise.

2.4.6. A Theoretical Scheme For DSTR Teaching

In my conceptualization, I view DSTR teaching in a dynamic triangle that captures the three dominant aspects that interact in the formation of this practice, namely the features, the learning potentials, and the resources (see Figure 2-11).

In the community of the eleven teachers of our study, DSTR teaching constitutes a new practice that is introduced by the researchers and developed by the members of this community. The researchers, as brokers in this newly established CoP, bring new resources to this community and also present features and learning potentials that are linked to them (either through particular teaching examples or through the discussion on pieces of research in statistics education). Once these features and learning potentials become transparent in the community, they constitute reified objects that come to be added in the set of resources negotiated in this CoP. The resources constitute the primary impetus for mobilizing negotiation of meaning and reflecting the history of mutual engagement of the community. In the triangle of the DSTR teaching practice, the CoP of our study negotiates the meaning with respect to what, why and how, producing a repertoire of resources, which reflect the shared history of learning and consequently the development of this practice. In this sense, the learning potentials and the features constitute core parts of the community's joint enterprise. Particularly, the learning potentials reflect a collective consensus of what is important to be achieved in the learning process and why, so they form the goals of the community's enterprise. Similarly, the features are linked to the characteristics of the teaching that support the communally negotiated learning potentials. Thus, the learning potentials reflect the purpose, and the features reflect the means and conditions of the joint enterprise.

Building on the idea of negotiation of meaning (Wenger, 1998) I see the process of participation in the members attempt to link the three elements of the dynamic triangle and the process of reification in the transparency of the emerging links. For example, when teachers negotiate meaning with regard to real data, then through the process of participation they debate how they can use this resource and for what particular learning goals, when this negotiation results in particular links that become transparent and shared among the members of the community, then these links are considered as reified objects, namely the outcome of the reification process. However, reified objects are not only links among the three elements, but generally resources that are related to these links (e.g. documents, digital tools, acquired knowledge/experiences, etc).

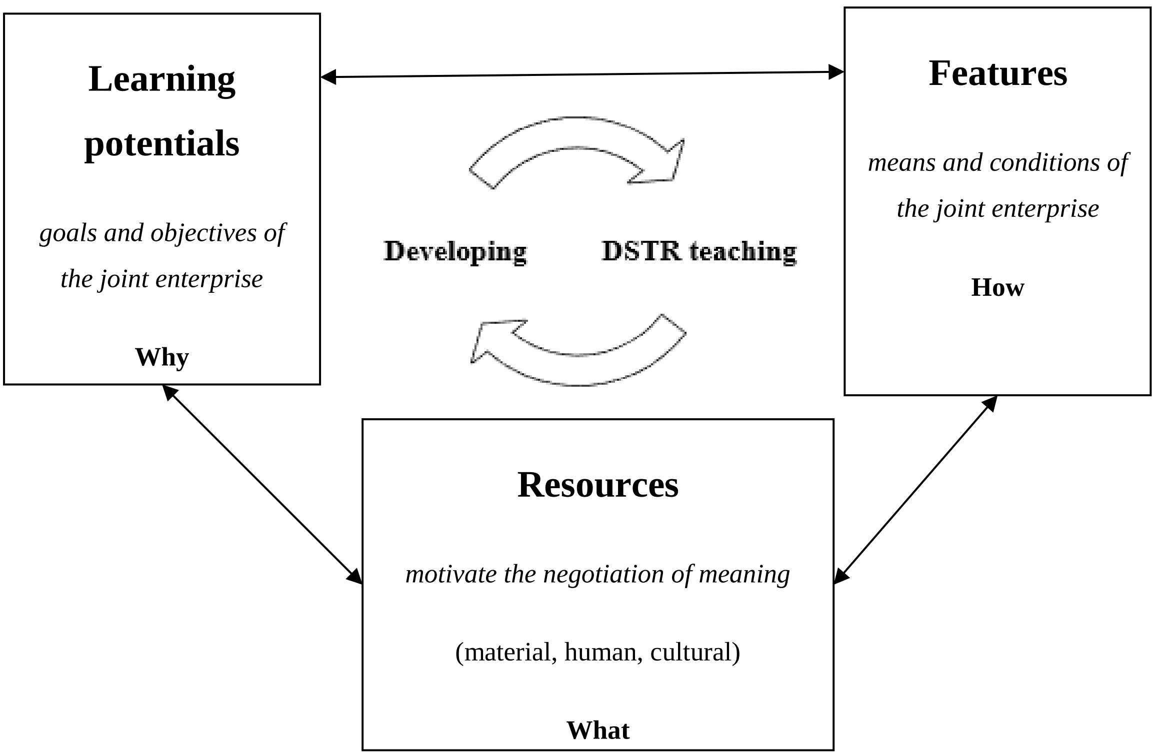


Figure 2-11 - The conceptualization of DSTR teaching scheme

In my study, I concentrate on a particular resource, namely software tools specially designed for statistics instruction (STSI tools). The rationale for this special focus will be explained in section 2.4.7 that follows. I am especially interested to see how the three elements of the dynamic triangle will interact while the members of the CoP negotiate the meaning of STSI tools, and what links will contribute to the formation of the CoP's DSTR practice.

2.4.7. A Particular Focus On STSI Tools

The development of statistical reasoning and the DSTR teaching practice set statistical inquiry and data explorations at the center of statistical teaching and learning. Biehler, Ben-Zvi, Bakker, and Makar (2013) compared statistical reasoning with "traveling" from statements to conclusions based on data and acknowledged environmental aspects, such as uncertainty, variation, or lurking variables. In this metaphor "the role of a computer tool is to make travelling (whichever way) easier and faster, inevitably with some "black box" effect: when traveling by plane or train we see fewer details along the road than when walking or cycling" (Biehler, Ben-Zvi, Bakker & Makar, 2013; p. 678). Many researchers provide empirical data to show how technology can facilitate the development of students' statistical reasoning even at very young ages (e.g. Konold & Kazak, 2008; Makar, Bakker and Ben-Zvi, 2011; Paparistodemou and Meletiou; 2008). These studies indicate a strong impact of such tools in assisting students to gain fluency in data explorations, develop connections between data and chance, and for inferential reasoning.

Biehler et.al. (2013) identified three main areas where software tools can support the development of statistical reasoning, namely (a) data exploration, (b) connecting data and chance, (c) preparing for statistical inference and assigning particular standards of digital tools to each area. For example, data exploration can be supported by characteristics such as dynamic dragging, ability to organize and represent data as well as the ability for multiple and linking representations of data. The area of connecting data and chance is mainly associated with the ability to simulate random experiments and modelling probabilistic processes by randomization tools, samplers, and prediction models. Moreover, the area of preparing for statistical inference can be facilitated by abilities such as random number generators and tools for creating sampling distributions and confidence intervals. Such abilities can build valuable learning experiences for students in understanding statistical activity and develop statistical reasoning and thinking, and consequently are essential characteristics in the case of what I consider as STSI tools. Some of the most frequently used STSI in the research and curriculum design are Fathom[2] and TinkerPlots which met the above requirements in an outstanding and efficient way (Biehler et.al., 2013). Both tools are developed to facilitate learning and doing statistics and provide opportunities for dynamic graphs, link multiple representations, data explorations, and simulations of random experiments. On the one hand, Fathom enables data analysis both visually and computationally, and it fits better in secondary and tertiary education. For example, Meletiou (2000) used Fathom in assisting students, in an introductory statistics course in college-level, to build intuitions about variation and she concluded that a learning environment where technology plays a constructive role can support students' understandings of statistical concepts and the development of students' statistical reasoning (see also Meletiou, Lee & Myers, 1999; Meletiou & Lee, 2002). In later research Fathom was also used with future teachers, to explore their intuitions and understandings with respect to simulations of random experiments and inferential statistics (Maxara & Biehler, 2007; Maxara & Biehler, 2010). On the other hand, TinkerPlots is designed for creating simulation models without necessarily the use of symbolic input. Moreover, TinkerPlots give the opportunity to young students to invent their own elementary graphs through a specially designed graph construction tool. The special tools provided by TinkerPlots have been designed to facilitate the development of statistical reasoning to young students, and it is suggested and explored by researchers for students from the age of 9 onwards (Biehler, 2013). However, in some cases, it has been used to study the development of students' statistical reasoning with even younger students. For

2 Fathom is a dynamic software for teaching data analysis and statistics (Finzer, 2001).

example, Saplamidou (2019) explored the development of informal inferential reasoning in grade 3 students through specially designed tasks and the use of TinkerPlots.

The significance of STSI stems from their potentiality to support statistical reasoning and argumentation, as well as from the potentiality to foster conceptual understanding of fundamental statistical concepts and ideas. STSI's potentiality to support conceptual understanding is addressed through the opportunity it gives to link multiple representations in a dynamic way, to simulate random experiments by producing random samples and supporting authentic statistical practices, and to make feasible the use of a large volume of authentic and realistic data that gives space for statistical inquiry and exploratory activity in school classrooms (Bakker & Derry, 2011; Olive & Makar, 2010). This particular power of technology gives students' access to statistical concepts and ideas that it was impossible until now and also supports students to gain a deeper understanding of these concepts and restructure their thought towards statistics (Pfannkuch, 2018). This is why Pfannkuch (2018) acknowledges STSI tools as an integral part of statistics curricula, and the approaches that integrate such tools, as one of the three core features of statistics teaching.

But above from their potentiality as a powerful tool for statistics instruction, STSI tools, and especially Tinkerplots, have been widely used in teachers' professional development programs. For example, in the context of a European Project aiming to support the professional development of mathematics teachers, Meletiou-Mavrotheris, Mavrotheris and their colleagues (2007) urged the formation of a virtual community of practice in which the members shared experiences that are similar to those expected to provide to their students. In this context, teachers had the opportunity to model and investigate statistical problems with the aid of TinkerPlots and Fathom. This experience seemed to assist the development of both teachers' statistical understanding and their statistical teaching. For example in

Germany, there are especially designed courses (Modeling, Magnitudes, Data and Chance (MMDC), 2017; Developing Statistical Reasoning with TinkerPlots; 2017) where teachers have the opportunity to engage with the whole PPDAC cycle (Wild & Pfannkuch, 1999) and use Tinkerplots to organize, analyze and explore data as well as to compare data distributions and assist inferences based on gathered data. The particular design of the course as well as the use of TinkerPlots as a primary tool in teachers' statistical investigations, both contributed in the development of teachers' statistical reasoning and conceptual understanding (Frischemeier & Biehler, 2018). In another study, TinkerPlots have been also used in a graduate course aiming to support teachers develop understandings of distribution, samples and sampling distributions, as well as inferential statistics, with a special focus on randomization approaches (Jacob, Lee, Tran & Doerr, 2015). The evaluation of this course showed that teachers' experiences on statistical investigation and reasoning, as well as the opportunity to engage with a simulation approach to inference assisted by TinkerPlots, helped teachers to improve their understandings of sampling variability.

For all the above reasons, STSI tools are strongly connected to the DSTR teaching practice and they constitute a key resource for instruction that targets the development of statistical reasoning and thinking. Thus, such tools have a prominent role for DSTR teaching practice and they also constitute a new and challenging resource for statistics teaching that is expected to affect mathematics teachers' practices in the near future (Biehler, Ben-Zvi, Bakker & Makar, 2013; Eichler & Zapata-Cardona, 2016). However, although it is strongly suggested that STSI tools need to be integrated into the teaching of statistics we still have no empirical evidence of how this integration can be achieved and encouraged. Particularly, in order for STSI tools to become part of DSTR practice, we need to move the lens of the research to the actual teaching and to study how teachers re-source the teaching of statistics with STSI tools and how they actually use them in teaching. My study aims to contribute to this latter and unexplored field of inquiry.

2.5. Summarizing The Theoretical Propositions Of The Study

This study builds on concepts of social practice theory to study the re-sourcing process in a subject which poses new challenges in secondary mathematics teaching, namely the DSTR teaching. I view the DSTR teaching in the boundaries of three well-established communities, namely the statistics education community, the mathematics education community and the mathematics teaching the community of practice. In my research, I acknowledge innovations and new resources that aim to cross the boundaries of statistics education to enter the secondary mathematics teachers' repertoire. Focusing on a CoP of eleven mathematics teachers, I investigate the path from the moment a boundary object enters in the CoP to the moment this object consolidates to the practice of this community. I particularly study the teachers' interaction with resources and the development of the DSTR practice through a dynamic triangle which acknowledges *what*, *how* and *why* to take part in the negotiation of meaning that results in the development of the community's shared repertoire. The three vertices of the triangle represent the *resources* that mobilize the negotiation of meaning in the CoP, the *features* that describe means and conditions that characterize the use of resources-in-practice-in-context and last the *learning potentials* which refer to the goals and the rationale behind the use of resources. Since I view the re-sourcing process as a social process which concludes to the development of the community's repertoire, I focus not at each vertex of the triangle separately, but rather on emerging links among them. I see such links as being communally negotiated and shared, and thus as constitute indications for the knowledge developed inside the CoP and the development of their practice. Moreover, I believe that the study of such links will provide insight into both global and local aspects that influence the consolidation of a resource in the practice of secondary mathematics teachers. In this research journey, I concentrate on the particular case of STSI tools because this resource summarizes all the characteristics of a boundary object (see p. 5), namely they are new, innovative, closely connected to

DSTR practice and also challenge the secondary mathematics teachers' usual practices. Acknowledging the prominent role of STSI tools in the DSTR practice, I believe that the research of the re-sourcing process of such tools in the CoP of the secondary mathematics teachers of our study, will reveal specificities of both these tools and of the DSTR teaching and its development.

3. Methodology

3.1. The Context Of The Study

In Greece, the content of statistics constitutes a very small part in the mathematics curriculum, and the teachers often leave this subject at the end of the school year or neglect teaching it. Particularly, according to the current curriculum of secondary education, the students are first introduced to some fundamental concepts of descriptive statistics in Grade 8. The emphasis in this Grade is on statistical formulas, calculations, and graph construction, while the main concepts included in this level are population and sample, statistical graphs, frequency, and relative frequency distribution, a grouping of observations as well as measures of location and dispersion in data sets. In Grade 9, students are introduced to some core probabilistic ideas, such as sets, sample space, and events, as well as the classical definition of probability. The emphasis is mainly based on applications of the classical definition of probability and the idea of randomness is introduced through experiments of equiprobable events in dies, coins, etc. After Grade 9, statistics appear in the curriculum in Grade 12. In the school textbook for that grade, there is one chapter on descriptive statistics which also includes some elements from linear regression and linear correlation and one chapter on probabilistic ideas that extends the content of Grade 9 to include some elements of conditional probabilities and combinatorics. The two chapters constitute separate parts of the curriculum and there are no conceptual connections between them.

The content of statistics and probability in secondary education, as presented above, is part of the official mathematics curriculum and is taught by mathematics teachers. In Greece, all mathematics teachers have a degree in mathematics but without a compulsory specialization in mathematics or statistics education. However, there are several postgraduate programs in mathematics education and there are also some regular professional development programs run by the Ministry of Education which aim at educating teachers in new teaching approaches and digital technologies with respect to mathematics teaching and learning. On the contrary, there is no particular education with respect to statistics teaching and learning. In particular, mathematics teachers receive a formalist approach to statistical content from their graduate studies, and there is also some contact with stochastic mathematics in their post-graduate studies related to mathematics education but only in terms of familiarizing with quantitative methods of data analysis. Until the period of this study in teacher education and professional development programs, there were no courses that focused on statistics teaching and learning. Consequently, mathematics teachers in Greece have no particular education with respect to DSTR, they have no particular experience with real data and data analysis as well, and the statistics teaching tradition relies on procedural approaches.

However, my aim to explore the development of DSTR teaching coincide with a reformed curriculum (New School, 2011) which was in pilot implementation the time this study began. This curriculum reinforces the role of statistics, puts statistics and probability in all grade levels, and puts an emphasis on data exploration and statistical reasoning. An example that illustrates the turn on the teaching and learning approach of statistics in the reformed curriculum is in the case of Grade 8 where the main learning potentials in the existing curriculum were for students to familiarize with measures of tendency, calculate them and construct bar-charts and histograms of data. In contrast, in the reformed curriculum students in Grade 8 are expected to be able "to collect, manage, interpret and represent statistical data, to make conclusions based on data, as well as, to interpret statistical graphs (New School, 2011; p. 18).

This movement in the statistical content in school curriculum opened the discussion for inquiring approaches to statistics teaching and brought to the fore teaching resources and learning activities that were far from the tradition inside mathematics classroom in terms of both content and epistemology. In this changing landscape, we decided, as a pilot attempt, to add a two-hour session for an introduction to DSTR teaching in a postgraduate course run by the supervisor of the study. Particularly, the postgraduate program was in Mathematics Education, and the course run by the supervisor was on «Connecting Research and Practice in mathematics teaching». This course had usually four main modules, namely teaching algebra, teaching calculus, teaching geometry, and mathematical proof and argumentation. In the spring semester of 2012, and while the reformed curriculum was on pilot implementation, I organized and run an additional module aiming to introduce teachers with some ideas and approaches related to DSTR teaching and included in the reformed curriculum.

This session was attended by 15 mathematics teachers (4 practicing and 11 prospective) and it included the presentation of the main guidelines with regard to DTSR practice (e.g. the fundamental statistical ideas - Burril & Biehler, 2011; definitions of statistical literature, reasoning, and thinking - Ben-Zvi & Garfield, 2004) as well as some popular problems that were analyzed and discussed among the postgraduate students (e.g. *the hospital problem* – Fischbein, & Schnarch, 1997; *the gummy bears question* – Rubin, Bruce, Tenney, 1990).

On July 2012 I addressed a call to the 15 post-graduate students who attended the session on DSTR teaching, to participate in a teacher group with the aim to inquire DSTR practice and as well as to develop and explore teaching materials for the secondary mathematics classroom. This call was also addressed to the other three teachers who had recently been graduated from the postgraduate program and who had a special interest in innovative approaches to teaching. Eleven teachers responded positively in our call (5 practicing and 6 prospective mathematics teachers) and agreed to work voluntarily in our project. Ethical issues were taken into consideration, as all participants knew that they were participating in a research study. Each had given their informed consent and knew they could leave the project if they desired to.

3.2. Research Aims And Research Questions

In my work, I utilized the communities of practice (CoP) framework (Wenger, 1998) to investigate a group of secondary mathematics teachers who had been working systematically and collectively for two years to form statistics teaching that promotes statistical reasoning and thinking. In this collaborative context, I aim to get insight on the re-sourcing process of this community, to explore how resources are being transparent and negotiated and by which path this process results on the resource integration to the community's shared repertoire. Our main focus in this investigation is on STSI tools due to their prominent role in DSTR teaching.

My study is guided by three research questions:

What elements of the DSTR teaching were revealed and negotiated in the formation of the community's practice, with respect to the various professional development tasks in which the mathematics teachers were engaged?

Focusing on the resource of STSI tools, what phases seem to characterize the re-sourcing process inside the community?

How does the secondary mathematics teachers' CoP develop the DSTR practice through negotiating meaning with respect to STSI tools? How is the process of negotiation of meaning formed within the various phases of the re-sourcing process?

3.3. The Participants

The group of teachers consisted of eleven mathematics teachers who responded positively in our call (see section 3.1). I present them in brief on Table 3-1 below. The names used here are pseudonyms. As we can see five of them were practicing teachers (Akis, Dinos, Lidea, Kimon, Marcos) most of them with more than 10 years of teaching experience, while six were prospective teachers (Athina, Chloe, Eva, Lia, Ria, and Sofi) with no particular experience in secondary mathematics classrooms. All participants were about to finish their postgraduate studies in Mathematics Education, except from Kimon, Lidea, and Marcos who had been already graduated from the same program 1-2 years before the beginning of this study. As regards the content in relation to statistics in the program, there were two related courses that the postgraduate students had the opportunity to attend. The one was in statistics and probability, where issues of descriptive statistics, confidence intervals, and hypothesis testing were included, as well as, some core issues of probability theory, such as the law of large numbers, independence, conditional probability, expectation, and some common distributions. The other course was on quantitative research methodologies, where students had the opportunity to engage with real data, with statistical software (particularly the SPSS) and to familiarize with statistical investigations and inference. None of these courses were mandatory for the postgraduate students but all of the participants of our study had attended at least one of them. No courses were offered with respect to statistics education or more particularly to the statistics teaching and learning in secondary education. Thus, the content and the guidelines regarding DSTR practice was an unfamiliar and unexplored landscape for the mathematics teachers in our study.

Table 3-1 - Community of Practice members

Participants	Professional background	Teaching experience
Akis	Practicing	<10 years
Dinos, Kimon, Lidea, Marcos	Practicing	>10 years
Athina, Chloe, Eva, Lia, Ria, Sofi	Prospective	no particular classroom experience

As it emerged from the initial interviews, teachers' motives for volunteering were mainly their personal interest in working with colleagues and mathematics education researchers, their desire to learn more about statistics and teaching statistics meaningfully, and the general context of statistics teaching reform.

Most participants admitted a lack of confidence towards the content of statistics and viewed this experience as an opportunity to explore not only alternative approaches to statistical teaching but also deepen their own understanding of the statistical concepts and methods. However, there were two participants who showed high confidence in statistics, namely Marcos and Dinos. Marcos had a personal interest in statistics education and in his master's dissertation, he worked on the reformed curriculum guidelines for teaching and learning statistics worldwide. Moreover, due to his strong interest and background in statistics education, he had been a member of the committee that had designed the Greek reformed curriculum (New School, 2011) and he was one of the members who designed and suggested the materials for the subject of probability and statistics. On the other hand, Dinos had professional experience as a statistical analyst and was very familiar with statistical content and statistical methodologies. Marcos and Dinos' comfort and confidence with statistical content was determinant in them having a prominent and often leading role inside the community.

As regards STSI and their potential use in teaching statistics, none of the participants, except Marcos, had had any previous experience with such tools and almost half of participants had no previous experience with any digital tools related to statistics (e.g. the commercial statistical analysis software package SPSS or Microsoft Excel spreadsheets). Thus, the dominant view towards software tools in statistics teaching focused on quick and accurate calculations and chart design. However, as Marcos had a strong personal interest in STSI and had explored many of them by himself (e.g. Fathom, TinkerPlots, illuminations applets by the NCTM), he was aware of their potentiality to support the development of statistical reasoning. He hadn't, though, used any of them with his students at that time.

From the beginning, I worked with this group as an emerging community of practice (Wenger, 1998) by encouraging the development of mutual engagement, joint enterprise, and shared repertoire. However, these characteristics were not only due to our impact. They also stemmed from the teachers' willingness and interest to establish norms that promoted participation and the development of meaningful products that reflected the history of participants' active engagement and sharing.

3.4. The Role Of The Researchers

The researchers of this study were the author of this dissertation (Researcher 1) and the supervisor (Researcher 2). As I mentioned in the previous section the researchers of the study acted as one of the main motives for the participants to work voluntarily in this study. This stems from the fact that the researchers and the participants had already worked together in other academic contexts. The previous experience from collaborating in other contexts helped in developing quickly a relationship of trust and cooperation that supported further the establishment of a community among the teachers, although most of the teachers first met with each other in the context of this study.

Both researchers participated in the meetings and had a role of facilitator during them. However, as regards the design of the meetings, Researcher 1 was mainly in charge of finalizing meeting agendas, while Researcher 2, acted as discussant during the design of the meetings. Both researchers encouraged teachers' active participation and their collaboration by addressing less active members, encouraging members to take the initiative to present materials or share ideas, and creating small working groups with a common goal. Moreover, they also challenged teachers to reflect on emerging issues and discuss their views and personal experiences

In the emerging community of practice, the researchers had the role of brokers (Wenger, 1998), introducing resources and practices from the statistics education community. Particularly, they provided: research papers for members to read and then discuss in the groups' meetings; examples of statistical tasks and teaching materials which members discussed in a view of defining the learning requirements and potentials. They also presented research findings with regard to students' difficulties and they also introduced tools, suggested in the literature, that could support students' understandings of statistical ideas. Moreover, they tried to support teachers to develop awareness for both the specificities and the complementarity between statistics and mathematics. As necessary, they made suggestions or asked critical questions regarding statistical teaching and wrote reports summarizing the main themes and emerging issues from each meeting for further thought and reflection.

However, the researchers tried to have minimum impact on teachers' work with the provided resources and strongly supported teachers to engage with resources and use their imagination to link them to their practice. Particularly, they encouraged and let them contribute on the formation of the following meetings' agenda, and also they paid great attention to teachers' needs and difficulties as well as on teachers' desires and dynamics throughout their collaboration.

3.5. The Main Agenda Of The Meetings

The group of volunteer teachers worked collectively for two academic years (2012-2013 and 2013-2014) on a regular basis (about two meetings per month lasting approximately two and a half hours each). The main agenda of the meetings was formed around a cyclic route of five core Professional Development Tasks (PDT). This PDT is described in brief below.

a) *exploration of statistical tasks/situations (SI): in this PDT the teachers collaborated in analyzing statistical tasks with regard to the statistical concepts and ideas included in them, or addressed these statistical tasks themselves*
b) *discussion on instructional materials (IM): in this PDT the teachers collaborated in analyzing statistical tasks with regard to their didactical characteristics (features/learning potentials) and they discussed the possibility to use them in the statistics teaching as well as potential transformations*
c) *discussion of research papers (RP): in this PDT the teachers were provided with research papers by the researchers. They read these papers and then discuss during the community's meeting research findings regarding the teaching and learning of statistics*
d) *design of tasks for the classroom (DES): in this PDT the teachers were asked to collaborate in designing a task for the students. In this task, they were encouraged and expected to use their experience from the previous PDT. They were also encouraged and asked to implement the tasks they designed in the classroom. During the implementation, the practicing teachers were those who implemented the designed task and the prospective teachers observed the teaching process keeping field notes from their observation*
e) *reflection on the classroom implementation (REF): in this PDT the teachers were asked to reconsider their design and reflect on teaching and learning issues.*

This agenda constituted the basic structure of the meetings, although the number of meetings dedicated to each aforementioned dimension, and the specific content of the group's work, was finalized in accordance with teachers' needs and requests.

The two years of the collaboration in this community resulted in 26 meetings and five tasks implementations in mathematics classrooms. However, in order to address our research questions, I restrict our study on the first 10 of the 12 meetings held in the first academic year (2012-2013). These 10 meetings constituted a complete cycle of inquiry (content inquiry, design, implementation, reflection) showing a full picture of teachers' actions and interactions, while the last two meetings of this year introduced a new inquiry cycle. Table 3-2 presents the duration of each meeting and the number of teachers who participated in relation to the main PDT.

Table 3-2 - Main tasks on the agenda of teachers' collaboration.

		CoP's meetings / Tasks discussed in each meeting									
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Main tasks discussed in CoP's meetings	Introductory discussion (**INTR**)	√									
	Discussion of research papers (**RP**)		√	√	√	√					
	Exploration of statistical tasks/situations (**SI**)	√						√			
	Discussion on instructional materials (**IM**)			√		√		√			
	Design of tasks for the classroom (**DES**)				√	√	√	√	√	√	
	Reflection (**REF**)										√
Meetings' duration		138min	120min	135min	135min	140min	110min	180min	150min	95min	120min
#Participants in each meeting		10	9	9	11	10	9	8	7	7	9

During the design phase, the two researchers asked teachers to design a task for the students with a general aim to engage them with statistical inquiry and make conceptual connections with probabilistic notions. The teachers were free to design one or more tasks and to work in one or more group. The only prerequisite was to collaborate during the design and also to guarantee that the task would be implemented in a classroom. As discussed earlier, Dinos and Marcos both had a leading role inside the community. These two teachers made two different suggestions for the design and the teachers were separated into subgroups around Marcos and Dinos on their own initiative. In this process, the researchers didn't intervene, as long as there was at least one practicing teacher in each subgroup. In Table 3-3, I present a brief description of the tasks that the two subgroups focused on.

Table 3-3 - Brief description of the two tasks designed by the teachers

	Subgroup A	**Subgroup B**
Participants contributed to the design	Marcos, Kimon, Lidea, Lia, Ria	Dinos, Athina, Chloe, Sofi, Eva
The main idea of the task	Marcos suggested engaging students with the simulation process in the context of a random experiment with a further goal to engage students with informal statistical inference. Kimon and Lia showed interest to explore the idea of data simulations while Lidea and Ria showed a preference to work with Marcos who was quite familiar to statistics education resources.	Dinos suggested giving students a simple and easily manageable question in order to engage them with all the steps of the statistical inquiry cycle with a further goal to understand the role and the meaning of the standard error of a statistic. Athina, Chloe, Sofi, and Eva had collaborated again during their postgraduate studies and after Dinos' suggestion, they immediately offered to collaborate with him.
Statistical question aimed for the students	Does the meaning in words have an impact on our ability to recall them or not?	How could we estimate how much money on average the students in our school spend in the school canteen every week?
Students' Grade level	12 Grade students	8 and 9 Grade students
Time spent on the task	5 teaching hours (45 minutes each)	4 teaching hours (45 minutes each)

Except for the five PD tasks, in my analysis, I also encounter the introductory discussion which took place in the first meeting (INTR). This discussion aimed at presenting to the teachers the aims and main agenda of the community's enterprise, as well as, to encourage the participants to get to know each other. Although this discussion was not part of the researchers' PD agenda, it brought to the fore various information about participants' background, interests, and motives (human resources), or about subject matter and educational resources (material resources) that aimed to facilitate participants' work in this community.

3.6. The Data Of The Study

The data of the study were mainly produced inside the communities' meetings. Particularly, I made audio and video recordings of all meetings and I transcribed fully the 10 meetings (see Table 3-2). These data constituted the main source that guided our research.

However, I also conducted semi-structured individual interviews at study begin and end. The beginning interviews aimed at gaining insight into the participants' personal histories (academic background, teaching experience, views towards statistics and towards the relationship between statistics and mathematics, etc.). The final interviews were conducted about a year or more after the last meeting with all the participants, aiming mainly at exploring the impact of the groups' work on the individuals who had participated in it. I made audio recordings of these interviews and used them as an additional data source.

Other data were teachers' written reflections on issues they considered as central in each meeting. Although these reports were not a primary source of data, they often constituted a useful source of triangulation for corroborating the study's findings. Please note that the original discussions were in Greek and I have translated some extracts to English for the purpose of this manuscript.

3.7. The Method Of The Study

To achieve my research goals, I followed an exploratory case study methodology (Yin, 2014). According to Yin, the scope of a case study is *"to investigate a contemporary phenomenon (the "case") in depth and within its real*-world context, *especially when the boundaries between phenomenon and context may not be clearly evident"* (Yin, 2014; pg. 16). Our aim is to gain a deep insight into the development of DSTR practice while teachers collaborate to inquire teaching and learning of statistics. I see this development through the process of re-sourcing, namely the integration and use of resources-in-practice-in-context (Adler, 1998). The endeavors of our investigation are context-dependent in their nature, and the case study methodology provides a fruitful and valuable ground to develop access and knowledge for the social phenomenon of re-sourcing the teachers' practice.

Our case was the community of 11 secondary school mathematics teachers (see Table 3-1) who responded positively to our call to participate voluntarily in this study. The newly-established community cannot be considered as a typical group of mathematics teachers, however, the diversity of the backgrounds, the internal individual motives, and the familiarity with collective work indicated that this group would:(a) remain together for a notable period of time despite the voluntary character of the work; (b) develop significant interactions among group members; and (c) engage actively with the design of teaching materials.

Situating our analysis within this particular community and especially for such an extended period of time is acknowledge both as limitation and benefit. The limitation is the existence of similar communities of practice and the possibilities for generalizations of this study's findings. On the other hand, the very existence of such a community gives us the ground to investigate in depth the development of a new practice in a real context, with real conditions, so I believe that the results of my investigation will reveal aspects of the re-sourcing process with regard to DSTR teaching that can be transferable and valuable to other systems and communities. Flyvbjerg (2011) argues in favor of the force of a case study that "*knowledge may be transferable even where it is not formally generalizable. A purely descriptive, phenomenological case study without any attempt to generalize often helped cut a path toward scientific innovation*" (pg. 305), and he adds that especially the development of social science is based on concrete, context-dependent knowledge.

3.8. The Data Analysis Process

Our data analysis process consisted of three successive levels which are described briefly in Table 3-4 and discussed in more details next. Particularly, the first level was a descriptive level aiming at giving a picture of the elements emerged in the formation of the DSTR practice. In this level, I used grounded approaches with a focus on the three core elements of DSTR teaching (see Figure 2-11) regarding the five PDT. The other two levels concentrated on a specific resource, namely the STSI tools. More specifically, I tried to get a deep insight into the process of negotiation of meaning regarding this resource within the CoP. I was especially interested to identify phases that characterize the re-sourcing process of STSI tools, as well as particularities and complexities that seem to designate the process of negotiation of meaning in each phase.

Table 3-4 - Description of the data analysis levels

Levels	Process	Data	Aim
Level 1	Open coding with respect to: Features Learning Potentials Resources	Video and audio recordings from the 10 meetings in the academic year 2012-2013	Identify key elements of the DSTR teaching as they emerged in the group's negotiation of meaning
Level 2	Tracing STSI tools with the use of CoP (**what** is the content of the negotiation and **who** participated)	*Thirty-one episodes* related to the code "software tools" which emerged in Level 1analysis of the data	Identify phases that the resource passes during the process of the negotiation of meaning
Level 3	Use CoP tools to look for links among features, learning potentials and resources for each of the identified phases (**what**, **who,** and **how** in terms of emerging links and reified objects**)**	Same as level 2 and *Teachers' written reflections*	Study the development of DSTR teaching in the re-sourcing process

3.8.1. The First Level Of Data Analysis

The first level of our data analysis aimed at identifying key characteristics of teaching and learning statistics that emerged while the teachers were collaborating to develop their practice of DSTR teaching. Particularly, at this level, I split the data in terms of the PDT (see Table 3-2). Then, for each PDT I worked on a grounded-theory base (Glaser 1998, Charmaz, 2006) using an open coding process. Particularly, starting from the utterances of each participants' contribution in relation to each task, as a unit of analysis, I worked line-by-line assigning codes with respect to our three core theoretical constructs, namely the features, the learning potentials, and the resources. In this process there was a continuous comparison between specific features, learning potentials and resources that reveal in the data, and those that have been emphasized and discussed by other researchers (Charmaz, 2006). Particularly, for the identification of the resources I was based on Adler's (2000) categories (see section 2.2) while for the identification of features and learning potentials I was guided by the literature's guidelines as described in section 2.4. In Table 3-5 I present some examples of how I assigned codes in moments of the discussion where there was explicit reference to elements of DSTR teaching or where the codes described an implicit reference to these elements.

The open coded process was assisted by the qualitative data analysis software ATLAS.ti (https://atlasti.com/), which constituted a valuable tool in the organization and coding of our data. Moreover, it helped us to define code families as well as to summarize the main findings of the open coded process with respect to the appearance and frequency of codes.

Table 3-5 - Examples from the open coding process

		Explicit reference to codes	Implicit reference to codes
Resources	Cultural	"such tasks require a lot of time" (Lidea, 4th meeting) –**time**; "the students usually overestimate samples because they are used of their homogeneity in the way we use them in life, for example when we say blood sample, or a product sample" (Dinos, 2nd meeting) – **everyday use of statistical terms**	"but you see, the situation here is very different than in the USA or New Zealand" (Ria, 4th meeting)- implying the general **educational policy** **(1)** **in terms** of the emphasis given on statistical literacy
	Material	"Athina had the idea to discuss with students real polls from media" (Researcher 1, 10th meeting) – **media extracts**	"See here the difference on the two distributions" (Marcos, 7th meeting) – **screen, STSI tools**(2)
	Human	"You need to see it in order to understand, if you were here in the last meeting where Marcos present as to how simulations work you would have understood" (Lia, 8th meeting) – **group interactions**	"It can be random but unlucky, I mean that the fact that it is random doesn't mean that it is representative" (Chloe, 1st meeting) – **everyday experience**(3) "Marcos, can you present something on Fathom in the next meeting?"(Researcher 2 -4th meeting) – **technological tools**(4)
Features		"to engage students with real data" (Marcos, 4th meeting) "to encourage students negotiations in the classroom" (Chloe, 1st meeting) – **engagement with real data**	"now I'll put the data in a Fathom file to see what happens" (Marcos, 7th meeting) – **engagement with STSI tools**(5)
Learning Potentials		"it is important that students be able to interpret the results of a statistical investigation" (Dinos, 5th meeting) - **interpret statistical information**	"intuitively I still have difficulty to deal with compound simple events"(Akis, 2nd meeting) – **overcome probabilistic misconceptions**(6)

Notes:

(1) Here Ria implies the general **educational policy** of the USA and New Zealand in terms of the emphasis given on the development of statistical literacy

(2)Although they don't refer to **STSI tools**, they use them to analyze their data, so we consider the resource as being transparent to the community

(3)In this extract, Chloe doesn't seem to base her argument on statistical tools but rather on her **everyday experience**

(4)Here, although not explicitly, Researcher 2 refers to Marcos' experience with **technological tools** and make this experience transparent to the community

(5)In this case, the community use Fathom and explore its potentialities for their own learning, so the feature of **engaging learners with STSI tools** is transparent here

(6)Here Akis discuss himself as learner sharing his own difficulty to overcome the simple and compound events misconception and by this, he brings to the fore a learning potential as to how to **overcome this probabilistic misconception**

A more combined example, from this line-by-line coding process, is given below (see Table 3-6). The presented extract refers to the PDT Exploration of statistical tasks/situations (SI) where the teachers conducted a pilot study trying to recall two lists of words: one list while listening to music and the other without doing so. This experiment's intent was for students to explore whether listening to music can affect the capability to recall words or not, but here teachers conducted the experiment on their own and selected pilot data from inside the community. Marcos put the two sets of the selected data in a Fathom file and used a simulation tool to produce many differences of means in order to test if the initial difference between the means of the two data sets was significant or not.

Table 3-6-. Example from the first level of analysis. Extract from 7th meeting - Episode 2

Note: Inside the text, I **bolded features**, put *learning potentials* in *italics* and underlined resources. If, for example, a particular phrase is assigned to both features and resources, it appears in ***bolded italics***.

Moreover, the numbers (e.g. [1,2,..]) indicate the correspondence between code and quotation.

Line	Participant	Extract	**Feature**	*Learning potential*	Resources
	Marcos:	**The simulation showed**[1] that the difference we got is a difference that can appear frequently, consequently *we can't deduce that the factor 'music' had an impact on the one group or the other*[2]. Actually, we need to be very careful in our interpretations. I mean this difference may have to do with the small sample size[3], I mean we can't be sure that in the experiments' design we *acknowledge all the factors that affect the results. As we show,* ***some factors that we didn't anticipate emerged***[4]. I mean that it was not only if the persons heard music or not but also if they had to recall the words in the list A or B[5]. Moreover, as I observed another factor was if someone worked first with list A or with list B, I mean after the first time someone may already have created strategies to recall words so the second time they would be more prepared in a way and more easily recall the words in the second list.	[1]Engagement with randomization processes [4]Stochastic nature of an activity	[2]Interpret the difference of means between the two data sets [4]Interpret the impact of a particular factor in the study [4]Acknowledge possible factors that affect the results of the study	[1]Simulated data [2]Music [3]Sample size [4]Factors [5]Lists of words Statistical tools

This level of our data analysis gave us an insight into how the DSTR develops throughout the various PDTs with respect to the three core elements as set in our conceptualization for this practice. In particular, it helped us to see how the various PDTs brought to the fore different elements of the DSTR teaching scheme. Moreover, it also testified empirically to the prominent role of STSI tools in DSTR teaching. The important role of STSI tools, as discussed in the theoretical section, combined with their prominent role in all elements of the DSTR practice as they appeared to various categories of both features and learning potentials (see also the results section), led us to focus on this material resource and trace the related codes throughout the dataset.

3.8.2. The Second Level Of Data Analysis

In the second level of the data analysis, I used again the utterances of the contribution from each participant, but I now divided the data in relation to the part of the discussion where STSI tools were discussed. By this process, I distinguished 31 episodes. I considered an episode as part of a discussion related implicitly or explicitly to software tools in statistics teaching. Examples are when the teachers (or the researchers): experimented directly with STSI; illustrated a particular teaching example or idea using STSI; made a point regarding their view to STSI tools; or even when a participant presented a specific example making use of software tools – even if no further discussion about that example ensued. An episode spans from the beginning of the relative topic of discussion until when the discussion topic changes. Particularly, I focused on the negotiation content and the participation process in terms of actions and interactions among participants. Table 3-7 below illustrates the process I followed with the 31 episodes by displaying two examples.

Table 3-7 - Examples from the second level of analysis.

Meeting (episode) Participants	Negotiation of integrating STSI tools in the teaching	
	Participation	**Negotiated meaning**
1st (1): R1	No interaction among participants	STSI tools were not visibly negotiated
10th (5): Dinos, R1, R2	Dinos interacted with R1, R2 and shared his experience from using Fathom in his teaching	Reconsidering the role of technology in the process of learning statistics Reflecting on modifications and alternative uses of Fathom

This process helped us to identify patterns that characterized how the community participants negotiated the meaning regarding the integration of STSI tools in the teaching chronologically. By this process, I identified three phases that STSI tools passed through as DSTR teaching developed through the teachers' collaboration. I briefly describe these three phases in Table 3-8 and discuss further details in the results section.

Table 3-8 - Summary of the three phases with respect to the results of the second level of analysis

Phases	**Core theme related to the process of negotiation of meaning**	**Corresponding data**
Emergence	No interaction between teachers and STSI tools	Initial interviews, 1st – 4th meeting (1 episode)
Exploration	Discussing STSI potentialities as a teaching tool	4th-9th meeting(25 episodes)
Immersion	Sharing experiences from classroom reality	10th Meeting (5 episodes)

3.8.3. The Third Level Of Data Analysis

In this final level of my analysis, I deepened the analysis of the three phases identified in the previous level. Here, I aimed at investigating more systematically the re-sourcing process in terms of the teaching scheme of Figure 2-11. Particularly, for the episodes of each phase, I tried to record links that emerged among the three core elements throughout each episode. Table 3-9 presents schematically the sequence of links as they emerged in the 5th episode of 10th meeting (see also Table 3-5).

Table 3-9 - Example from the process of analysis of Level 3 (extract from 5th episode-10th meeting)

Participants' contributions	**Feature**	***Learning potential***	**Resource**	**emerging links and other**
Dinos: I am very positive that *students need to do their own data explorations*. When you *engage them with sampling, they need to take samples on their own*. We could omit the paper and pencil task of *finding all possible samples of size 3 from a population of 5* and instead of this, we could engage them somehow with technology, to *allow them to make data explorations*. These explorations cannot be made without technology, technology is prerequisite, students to work in groups of 2-3 with a computer.	(F1) Encourage students to make their own data explorations (F2) Encourage students to use physical tools to explore data (F3) Encourage students to use digital tools to explore data (F4) Encourage students to work in small groups	*(L1)*Make data explorations *(L2)*Create a sampling distribution *(L3)*Find all possible samples from a given population	(R1)Students (R2)Data (R3)Samples (R4)Paper and pencil (R5)Population (R6)Digital tools (R7)Statistical tools (R8)Teaching tools	He refers explicitly to students as a resource for the point he makes. He reflects on their choice to engage students with small populations (F2), which he had strongly insisted on during the design of the task, and he now connects explicitly (*L1*) with the use of STSI (F3). He also refers explicitly to the importance of (F4) in supporting (*L1*).

The information included in Table 3-9 allowed as to keep track of the following processes throughout the three phases (emergence, exploration, immersion):

- Participation: who contributed to the negotiation of meaning regarding STSI tools in each episode; what interactions were recorded among participants; and how the negotiation of meaning with respect to STSI tools was formed and developed in each phase.

- Reification: how the negotiation of meaning with respect to STSI tools linked to teaching; what specific features or learning potentials became transparent in the community; what other resources were connected with STSI tools; and what links emerged among them (were the links direct/indirect, explicit/implicit)

- Re-sourcing development: shifts in the elements or the links among them as identified in the various phases that can indicate a development in the re-sourcing process (e.g. in Table 3-9 we have the feature "Engage students directly in data explorations using Fathom," which also appeared in the exploration phase. The difference in the immersion phase is that this feature links directly to the learning potential "Connect statistical investigations with probability," and this link was not visible in the discussions of the exploration phase.)

In this level of analysis, I use the theoretical idea of links between the three core elements of DSTR practice, as an analytical tool to explore both the interactions among these elements but mainly how these interaction shape and develop the practice of this community.

4. Results

4.1. Results Of The First Level Of Data Analysis

The results from the first level of analysis are presented with respect to the five PD tasks aimed at the teachers' professional development, namely (a) the exploration of statistical tasks/situations (SI), (b) the discussion on instructional materials (IM), (c) the discussion of research papers (RP), (d) the design of tasks for the classroom (DES) and (e) the reflection on the classroom implementation (REF). Except for the five PD tasks, in the analysis, I encounter the introductory discussion which took place in the first meeting (INTR) as well. This discussion aimed at presenting to the teachers the aims and main agenda of the community's enterprise, as well as, to encourage the participants to get to know each other. It brought to the fore various information about participants' academic background, interests, and motives (human resources), or about statistical objects, such as graphs or formulas, and educational resources (material resources) that aimed to facilitate participants' work in this community.

The open coding process resulted in a total number of 1474 of codes, and in Figure 4-1 I present the percentage of codes that appeared in the various PD tasks.

As can be seen in the figure below, most codes appeared while the participants were designing for their classroom. The time spent on each PD task was a significant parameter in the number of codes that appeared in each one. However, a large number of codes in some specific PD tasks may indicate a more focused discussion in relation to resources, features and learning potentials. In the next part of this section, I will try to get insight in such qualitative aspects (e.g. nature of the task) by focusing on patterns in the emerging codes as well as possible differences among the various PD tasks.

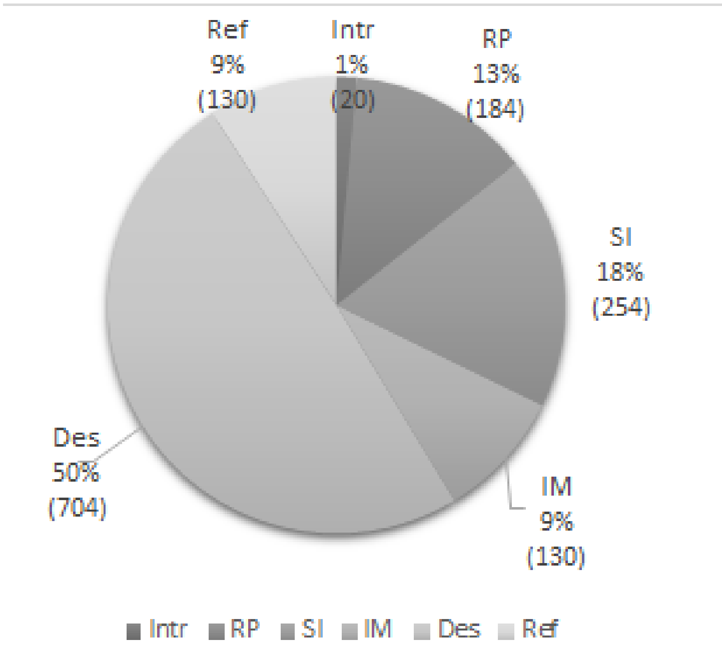


Figure 4-1 - Frequency of codes attributed in the various PD tasks

In Figure 4-2 I summarize the relative frequency of codes in each PD task, with respect to the three directions which guided our coded process, i.e. the features, the learning potentials and the resources as emerged throughout CoP's discussions.

As we can see in Figure 42 the codes related to resources were the most dominant in all PD tasks. In the case of the introduction, there was also a significant reference to various resources, which constituted 90% of the total codes assigned in this discussion. Moreover, as we can see, the category of features is the least represented among the various PD tasks. Particularly, in Introduction, there is an absence of codes related to features, while in the other PD tasks the category of features covers the smallest part among the three studied dimensions. However, the reflection task seems to constitute an exception, since, in this case, Features and Learning Potentials share almost the same part of the pie. In this task, features are also the most frequently represented among all PD tasks (20% of the total codes).

In the next part, I continue the presentation of the results with respect to the three core dimensions of our conceptualization of DSTR practice, namely the resources, the learning potentials, and the features. For each dimension, I discuss the results in each PD task. A summary and final comments from this level of analysis follow and close this section.

N: Total number of codes attributed in each PD task

Figure 4-2 - Frequency of the three dimensions' relative codes in each PD task

4.1.1. The Resources

The codes that were attributed to resources were 846 in total which was further summarized into 9 categories regarding the nature of the resource. In Table 4-1 below I display the 9 categories and some indicative examples of codes as appeared in each category.

Table 4-1 - Categories appeared in Resources dimension, illustrated by characteristic examples of codes

	Main Categories		Examples of codes
Resources	Cultural	Language	e.g. wording, everyday use of statistical terminology
		Social/Institutional	e.g. school context, educational policy, time issues, accessibility to data sources, classroom norms
	Material	Everyday objects	e.g. media extracts, computer games, playing cards, phone books, dies, political polls, social media
		Statistical objects	e.g. mean value, median, probabilities, boxplots, data summary, statistical estimation
		Digital tools	STSI tools (e.g. Fathom, TinkerPlots) and other digital tools (e.g. Excel, SPSS)
		Research Literature	e.g. research articles, web educational resources, web PD resources
		Curriculum material and Infrastructures	e.g. Greek curriculum materials, software license school computer laboratory, school projectors
	Human	Human resources related to teachers/Researchers	e.g. teachers' knowledge of statistical tools, teachers' previous experience with data explorations, experience from teaching, everyday experience, experience with digital tools, group-based interactions, teachers' difficulties/misconceptions
		Human resources related to students	e.g. students' background in statistics, students' attitudes towards statistics, students' interests, students' misconceptions/difficulties, students' everyday habits

The emerged categories

As we can see in Table 4-1 a plethora of resources emerged in teachers' discussions. The nine categories were grouped into three families with respect to Adler's (2000) categorization, namely cultural, material and human resources.

Cultural Resources.

Cultural resources are related to the Greek context and more particular to language as well as to social and institutional issues. For example, when the teachers discussed misconceptions related to the representativeness of a sample, Dinos said:

> *"it is not only about students in statistics, but it is also generally on how we use the term sample in our everyday language. I mean, what Researcher 1 told before, we speak about blood sample, perfume sample, shampoo sample, .., all these samples are homogeneous, so it is easy for someone to generalize about the representativeness of a sample. We need to build on counter-examples." (2nd meeting)*

The everyday language emerged also in various discussions and it was a code with a 25% frequency among the cultural resources. Another example comes from the teachers' talk about statistical simulations. In this discussion, Lia said "I have only met this word in computer games. In this case, you pretend to be a particular character, to drive a car or an airplane, this is simulation" (5th meeting) bringing to the fore meanings from the word's everyday use. Similarly, when they talked about random sampling and randomness Chloe said: "Ok it can be random and unfortunate, I mean if it is random you can get the extreme case, you need to control it somehow" (1st meeting). In this case, Chloe seems to use the word random in everyday language and not in a probabilistic manner. Words like simulation, randomness, probability, samples brought to the fore the strong connection between statistics and real-life not only in terms of the nature of the statistical problems but also, in terms of how people construct meaning in the statistical context.

Except for language codes, there were various codes that are related to cultural resources but linked to the school reality. The next table aims to illustrate the category of social/institutional codes.

Table 4-2 - Examples aim to illustrate the Social /Institutional category of cultural resources.

School context	"I need an official authorization in order to permit observers in my classroom, my school principal is too bureaucrat" (Lidea, 4th meeting), "there is only one computer lab and it is not easy for mathematics teachers to use it" (Dinos, 6th meeting)
Educational policy	"Statistics is part of mathematics curriculum, so you always need to have a mathematical goal in what you do" (Researcher 1, 4th meeting), "I will give some examples from USA and New Zealand curriculums which pay great attention to statistics education"(Marcos, 5th meeting)
Time issues	"I don't think that the time spent normally in the teaching of statistics is enough for open discussions" (Akis, 2nd meeting), "in order to do the experiment we will need many hours, probably five"(Marcos, 6th meeting)
Accessibility to data sources	"and how will you get the students' scores to study them, do you have access to them?" (Lidea, 4th meeting)
Classroom norms	"You don't need to do something but to give your students an appropriate problem and let them free to solve it" (Dinos, 2nd meeting), "the students seemed very familiar to work in groups"(Athina, 10th meeting)

As we can see from the examples presented in Table 4-2, social/institutional codes are strongly related to the particular context of the Greek educational system. These codes brought to the fore limitations and issues related to the implementation of statistical activity inside Greek mathematics classrooms.

Material Resources.

Material resources constituted about 60% of the total resources emerged in teachers' discussions. These resources were related to five categories namely i) statistical objects, which includes objects related to statistical content, ii) everyday objects, which include objects related to everyday life, iii) digital tools, which include two forms of digital tools, namely *STSI tools* which refer to statistical software especially designed for statistics instruction (e.g. Fathom or TinkerPlots), and *other digital tools* which refer to statistical software used in statistics generally (e.g. SPSS or excel), iv) research literature related to statistics education, and v) curriculum materials and infrastructures, which include the Greek curriculum guidelines and school infrastructures related to the teaching of the curriculum. In Table 4-3 below, I present some characteristic examples that aim to illustrate the various codes into the five categories of material resources.

Table 4-3 - Examples aim to illustrate the categories emerged related to material resources

Statistical objects	"I would prefer to let students formulate the **statistical question** on their own"(Athina, 6th meeting), "a question is if we will ask for the difference between **means** or between **medians**"(Marcos, 7th meeting), "when we gave them the **dot plot** they immediately understood"(Dinos, 10th meeting)
Everyday objects	"we could use social media as a source, the **Facebook** friends for example" (Kimon, 4th meeting), "I think a question related to **students'pocket money** would be of value for them" (Athina, 5th meeting), "I realized that they were very familiar with many probabilistic concepts. Of course, this is also due to the **computer games** they play."(Marcos, 10th meeting) "Athina's idea to use **real polls** from internet worked very nice"(Dinos, 10th meeting)
Digital tools	"It is so important to have an experience with simulations in **Fathom**" (Lia, 8th meeting), "students entered their data in an **excel sheet**"(Dinos, 10th meeting)
Research literature	"The example I am presenting here is the same with the one presented in the **video** I sent you (referring to a video retrieved from a web professional program for teaching statistics)" (Marcos, 5th meeting), "how other researchers have used a statistical investigative cycle? Are there any **papers** we can read about it" (Athina, 5th meeting)
Curriculum materials and Infrastructures	"and how you will connect it to the **curriculum**" (Ria, 4th meeting), "confidence intervals in **8th grade**?" (Chloe, 7th meeting), "we couldn't **use the computer lab,** so I used the school projector inside the classroom" (Dinos, 10th

meeting),

As we can see in Table 4-3, the negotiation of meaning regarding DSTR teaching brought to the fore a plethora of material resources that illuminate both the rich statistical content and the high material requirements related to the statistical teaching.

Human Resources.

The variety of human resources emerged throughout teachers' discussion. The codes attributed to this category were mainly implicit in the discussions, however, human resources seemed to be determinant in teachers' negotiations of meaning with regard to DSTR. The emerged codes were summarized into main categories. The first category refers to human resources related to the teachers and the researchers of the CoP, namely those who develop the DSTR teaching, while the second category refers to human resources related to the students, namely those who are taught statistics with an emphasis on statistical reasoning and thinking. The category Human resources related to the teachers/researchers includes a plethora of codes that correspond to resources especially linked to the participants of the CoP, namely their professional experience, their knowledge of statistics and mathematics, their previous experiences with data or with STSI tools, their familiarity with digital technologies or their experiences form mathematics classrooms. Except the codes directly related to the content and the teaching of statistics, the coding process revealed various codes that were closely linked either to the participants' everyday life or to their experiences from their work inside the community. For example, teachers' everyday experiences, heuristics that they employ in their everyday judgments, their interests, as well as, experiences stemming from their interactions inside the community, affective issues or attitudes towards statistics were also apparent and influential in teachers' discussions.

In Table 4-4 below, I provide some illustrative examples from some of the most frequently appeared codes of this category.

Table 4-4 - Illustrative examples of the category of human resources related to teachers/researchers.

Teachers' knowledge of statistical tools	"When you take 2 out of 10 it is like you have 10 cells and you choose 2 to put something inside. When you take 8 it is like the opposite, like you choose 2 to not to put something inside", (Dinos, 2nd meeting)
Teachers' experience from teaching	"But here you need to develop the opposite reasoning, this is difficult for students, it doesn't come easily." (Marcos, 2nd meeting)
Teachers' previous experience with data explorations	"if it is random, it is random, it is not more or less random, random samples have power on their own", (Dinos, 1st meeting)
Everyday experiences	"Ok the question regarding pocket money is nice, but as I told you before some students maybe they don't have pocket money. In my case, my parents used to prepare my snacks themselves and never gave me pocket money at that age" (Sofi, 6th meeting)
Affective issues	"Hospitals again! Oh please, no more examples with Hospitals, I have so bad experiences that I can't even hear about them" (Lidea, 4th meeting)

Group-based interactions	"Vasilis example helped us to understand and if it was clear for us, this example may also be very helpful for students too" (Sofi, 6th meeting)
Teachers' difficulties/misconceptions	"We do have difficulty with representativeness, and we have a tendency to make generalizations without any accuracy in them. It is not only a students' problem it is a difficulty that we also share as well" (Kimon, 1st meeting)
Teachers' knowledge of STSI tools	"Now I will put the data in Fathom. This tool is powerful, you can make easily summary tables, graphical representations, simulation and many more. I will see you some of these as we explore our data" (Marcos, 7th meeting)

The second category emerged in Human Resources was human resources related to students. Although I didn't study directly students and their learning in my work, this resource was rather frequent in teachers' discussions providing meaning in the development of DSTR teaching. Especially, as the PD task was closer to the actual; teaching, issues regarding students' background knowledge, interests or attitudes towards statistics were revealed. In Table 4-5 I present some of the most frequent codes emerged in our data, illustrated by some characteristic examples

Table 4-5 - Illustrative examples of the category of human resources related to students

Students' background in statistics	"We can't allow the formulating of the research question open to students. Students will be lost, it is not easy to

	formulate a question that can be addressed by quantitative data, they don't have such experiences" (Dinos, 6th meeting)
Student's attitudes towards statistics	"students have many times a bad picture for statistics, this is why I believe is more important to give them good instead of bad examples. In particular how statistics help doctors or biologists to make decisions from data and not how companies may publish misleading polls" (Marcos, 1st meeting)
Students' interests	"they seemed to be very interested in studying about their pocket money, more than studying on facebook friends" (Athina, 6th meeting)
Students' everyday habits	"they play a lot with web games and in such games, they became very familiar with frequencies and estimations based on probabilities" (Marcos, 10th meeting)
Students' misconceptions/difficulties	"We know that students have a tendency to think proportionally about samples and we want to challenge proportionality" (Sofi, 7th meeting)

Human resources had a prominent role in our data in both volume and variety of codes. In the next section, I will discuss how the various resources, including human resources, appeared in several PD tasks and what seems to be their role in the formation of DSTR teaching.

Resources In Relation To The PD Tasks.

The nature and the content of the PD tasks constituted important parameters for the codes emerged in each task. This became quite obvious in Table 4-6 below, where we can see the frequency of the emerged categories in the various PD tasks. A clearer picture of the same information is given by the histograms of Figure 4-3 as well.

Table 4-6 - Frequency of categories related to Resources in the various PD tasks

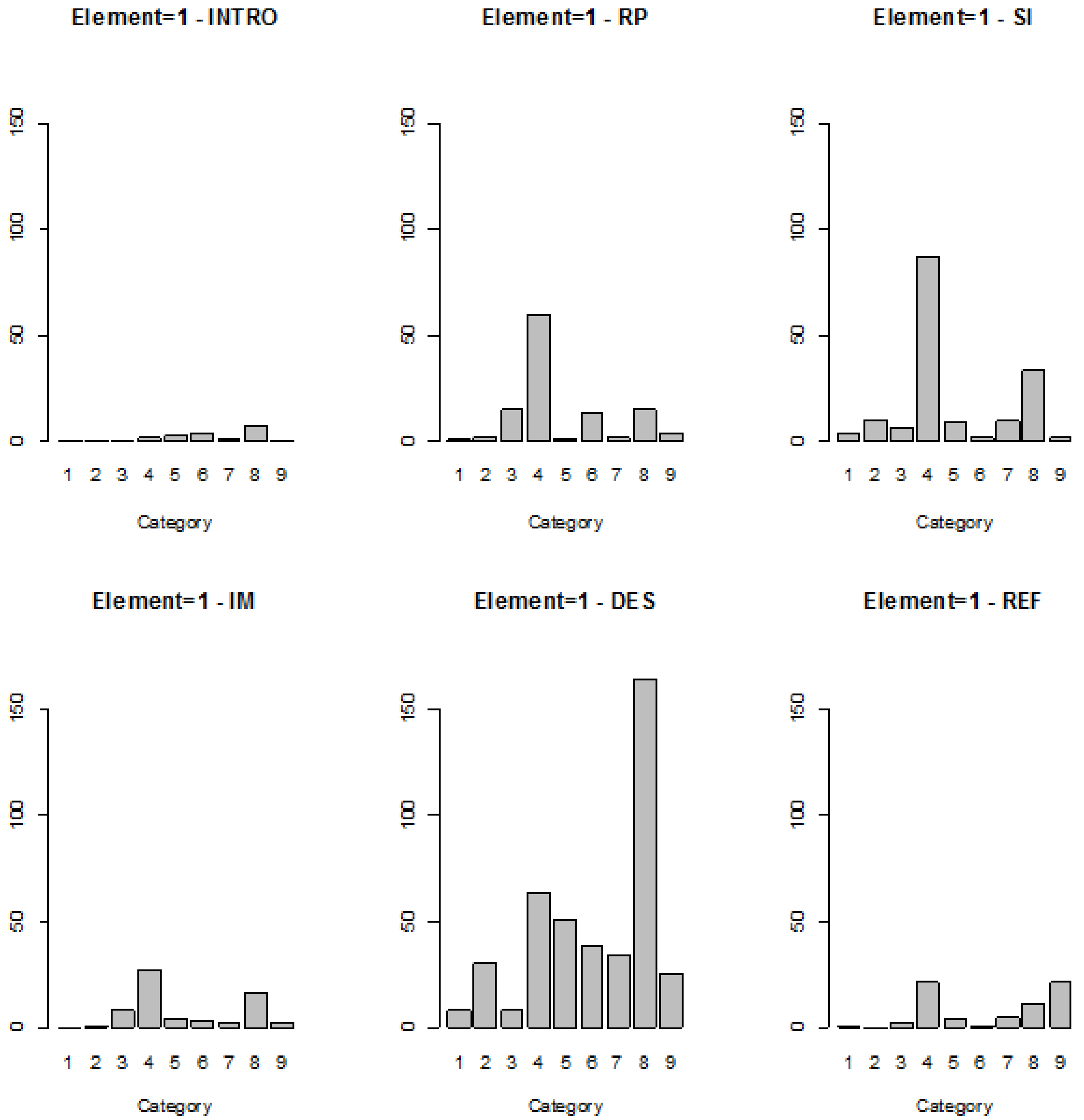
			Intr	RP	SI	IM	Des	Ref	Total
Resources in the various PD Tasks	Cultural	Language		1	4		8	1	14
		Social/Institutional		2	10	1	30		43
	Material	Everyday objects		15	7	8	8	2	40
		Statistical objects	2	60	87	27	63	21	260
		Digital tools	3	1	9	4	51	4	72
		Research Literature	4	13	2	3	38	1	61
		Curriculum material and Infrastructures	1	2	10	2	34	5	54
	Human	Human resources related to teachers/Researchers	8	15	34	16	164	11	248
		Human resources related to students		4	2	2	25	21	54
Total			18	113	165	63	421	66	846

As we can see in the table above, the resources appeared in most PD tasks were content-oriented. Particularly, we can see that the majority of the emerging codes are around two categories, namely the statistical objects and the human resources related to teachers/researchers. However, as we can see, during the Design task the spread of the emerged resources in the various categories was notable. The codes revealed in this PD task were related to all the categories of resources, and also especially during this task, the resources related to the Greek curriculum, the school infrastructures, and institutional issues like available time were central in group's discussions. Moreover, the resources that are related to students came to the fore mainly during the design and the reflection task. In particular, the codes related to students constituted a 31% of the total number of codes emerged in the Reflection task, while in the Design task constituted a 5% and less than 3% in the other tasks. This was mainly due to teachers' unfamiliarity with many of statistical tools that, except for the case of the design and the reflection, they discussed more on a direction to inquire their own learning than students' learning. So, despite researchers' attempts to bring students on the fore of teachers' inquiry, the teachers moved the discussion to their own understandings. The next extract is characteristic of teachers' need to inquire into their own learning:

1st meeting –Exploration of statistical tasks: Explore different sampling methodologies to address a statistical question

> *Researcher 2: Do you think that you could do with your students this particular discussion we do here now? What would be the teaching goal of such a discussion?*
>
> *Kimon: I would suggest doing this discussion with our mathematics teachers colleagues first (laughs). I mean we need to make clear that all these show a tendency. This needs to be clear first among our colleagues because I think we are those who have also the habit to make not documented generalizations.*

The histograms in Figure 43 shows clearly that in the majority of PD tasks the dominant resources in teachers' discussions were related to statistical objects as well as to teachers' knowledge of statistical tools which was expected since the inquiry of statistics was in the core of teachers' enterprise in this community. The other resources emerged in each PD task stemmed from the particular content of the task as designed and provided by the researchers, as well as from contextual aspects regarding the Greek context or judgments interventions made by the teachers. However, the PD tasks related to the design of instructional materials and the reflection on the implementation of them were mainly guided by the teachers themselves, since it was the teachers who had the responsibility for the design. So, throughout these tasks, the emerged resources stemmed mainly from teachers' needs, decisions, ideas and less from the researchers' agenda. Specifically, in the design of the task, as we can see in Figure 4-3, we have a launch into the appearance of human resources related to teachers/researchers and a notable spread of the categories of resources emerged, while in the reflection task, we can see a considerable appearance of human resources related to students.



Note: Where 1-9 on the x-axis refer to the 9 categories of resources presented in Table 4-1, namely 1. Language, 2. Social/Institutional, 3. Everyday objects, 4. Statistical objects, 5. Digital tools, 6. International Literature, 7. Curriculum Material and Infrastructures, 8. Human-Related to teachers/researchers and 9. Human-Related to students

Figure 4-3 - Histograms of the frequency of Resources in the various PD tasks

To sum up I can say that the landscape of the resources, as revealed while the teachers and the researchers were working to develop DSTR practice, is rather rich and complex. Many of them are closely related to the subject of statistics (material or human resources related to the teaching and learning of statistics) while others are related more to contextual and thinking issues related to statistical activity (use of everyday experiences/objects, social issues, intuitions/difficulties/ misconceptions) which seemed to be quite central in the case of DSTR practice. As I discussed, the frequencies displayed in Table 4-6 are helpful in giving us an overall picture of the central categories of resources emerged in each PD task, and they are also illuminative of the PD tasks that seemed to motivate the richness and interaction among resources. The categorization of the resources, as resulted by the open-coding process, was also helpful in highlighting the complexity of the underlying resources when working on DSTR practice. Particularly, as we saw except the material resources related to the statistical objects and the human resources related to statistical knowledge, a significant amount of other categories of resources (cultural, from everyday life, digital, etc) seemed to play an important role while teachers negotiated meanings regarding the teaching and learning of statistics. Many of the material resources are quite rare or non-existing in the teachers' work in the other disciplines of mathematics such as in algebra or calculus. For example, real data, simulated data, STSI tools, randomization process, media extracts, or web data sources are quite new and innovative material resources that are very frequently appeared in the development of DTSR teaching. Similarly, a range of human resources that are also rare apparent in other disciplines of mathematics (e.g. everyday knowledge, knowledge of students' habits/ interest) seemed to have an important role for the teaching and learning of statistics.

4.1.2. The Learning Potentials

The open coding process revealed 374 codes with respect to learning potentials which were further summarized into 7 main categories. In Table 4-7 I present these categories and I give some examples of the emerged codes, which were quite frequent among all codes, in order to illustrate the content of each category.

Table 4-7 - Categories appeared in the dimension of Learning Potentials, illustrated by characteristic examples of codes

	Categories	Most frequent codes appeared in each category
Learning potentials	Develop particular types of reasoning	e.g. students develop distributional reasoning, students contrast to their intuition, students contrast to their proportional reasoning, make an inference based on data simulation
	Performing procedural tasks	e.g. calculate the mean value, calculate the theoretical probability of an event, calculate the medians of data sets
	Deepen on statistical inquiry and Inference	e.g. compare two data sets in terms of means, make a hypothesis for the comparison of two data sets, compare different sampling methods, make an inference based on data, make a decision based on data, interpret the results of a statistical investigation, explore different samples, understand the impact of the sample size
	Familiarizing with digital tools designed for statistics instruction	e.g. make data explorations in Fathom, use technological tools to produce samples, create a sampling distribution on Fathom, familiarizing with Fathom
	Develop graphical competencies	e.g. compare data graphs, interpret data graphs, create dot-plots, create sampling distributions, create boxplots
	Connect or deepen to probability notions	e.g. define the sample space in a random experiment, understand the meaning of a random sample, calculate the theoretical probability, estimate probabilities using relative frequency, find all possible combinations of samples in a given population, understand the mathematical process behind computer simulations, estimate the probability of an event
	Acknowledge sources of variability and uncertainty	e.g. appreciate stochastic parameters of the problem situation, explore influential factors in a random experiment, develop strategies to control variability, acknowledge the distance between a sample mean and the population mean

The Emerged Categories.

The first category relates to particular types of reasoning and thinking that was emphasized within teachers' discussions. These particular types of reasoning and thinking were either directly or implicitly linked to the teaching expectations and goals developed in the CoP. For example, when they discussed a particular research paper Ria said: "These examples illustrate how students use proportionality to compare data distributions". In this extract, Ria through the presentation of a particular study (Watson & Shaughnessy, 2004), brought to the fore issues related to how students use proportional reasoning when they address statistical tasks. This presentation motivated a discussion on the role and the potentiality of proportional reasoning for the learning of statistics. In a further example, in the 7th meeting during the design task, Dinos emphasized: "it is important for students to see all possible combinations, to find all samples by hand on their own. Otherwise how? I mean how they will reach probabilities … it would be a black box". In this extract, Dinos make explicitly his learning potential to facilitate students' combinatorial reasoning in order to help students understand the role of probabilities and the idea of the standard error. Similarly, in various instances teachers linked either implicitly or explicitly types of students' reasoning and thinking, such as make an inference based on data ("it is not only to engage students with the PPDAC cycle but more to engage them with statistical inference and making judgments based on data"; Marcos, 4th meeting), contrast with intuition ("it is important to do both sampling and census study so that students can contrast the findings and address misconceptions related to samples"; Dinos, 5th meeting) or students' develop distributional reasoning ("what seem rather challenging is to facilitate students' thinking in a statistical manner"; Dinos, 10th meeting).

The second category summarizes codes that describe the skills and abilities used in performing procedural tasks inherent in the statistical activity. For example, when Marcos suggested students to make comparisons between the means of the two groups of the suggested experiment (see Table 3-3, Subgroup A), he acknowledged students' ability to calculate the mean value. The following extract from the 7th meeting presents a small debate on whether students' should calculate the mean value or the median of their data:

> *Researcher: Marcos why do you prefer means instead of medians?*
>
> *Marcos: It fits better to what I want to see here.*
>
> *Chloe: And what do you want to see?*

Marcos: Look the point is that I need to be able in the end to give a reasonable explanation; you know what happened here, what is the point of this process in the end. So, although medians are more manageable and easier for the students, in this case, means will help to make the points clear. I mean, you see here what we have (he means in the results of the pilot study), with so small differences in the data of the groups it won't be easy with medians to make a point, to find sets of data with the same difference or bigger. I mean that for this particular problem, the mean value will support the activity better.

In this extract, Marcos describes his rationale for asking students to calculate the means instead of medians in the task he suggested. As we can see his choice is very dependent on the results he expects to come out from the experiment, and, although he doesn't refer directly, he considers students ability to calculate the mean and the median of a data set.

Similarly, in Subgroup B, an important learning potential, in which Dinos insisted in various instances, was students to be able to calculate the theoretical probability. Particularly, in the following extract Dinos discussed the main learning potentials of the task designed by subgroup B.

Researcher 1: But why Dinos you choose just 5 particular values? Why it must be so specific?

Athina: So that they can take all possible combinations of 3, how else?

Dinos: If they take all possible sets of three, they would then calculate the means for each triad, and they will find the probability to be within a particular range. If they don't find this probability themselves then what, it will be a black box for them all this thing with samples and standard error.

In Dinos' words, we can see that this procedural task, namely, to calculate the theoretical probability of a sample to be within a particular range, is considered as essential to help students understand the notion of the standard error and it became a central learning potential during the design phase.

The third category, namely *Deepen on statistical inquiry and inference*, was the category with the highest frequency among learning potentials. The codes attributed in this category are purely linked to the statistical activity suggested for the students. For example, students to be able to compare two data sets by comparing their mean values, or to be able to make a hypothesis based on data (see task of Subgroup A), or students to understand the impact of the sample size in the reliability of a sample or to make an inference based on data (see task of Subgroup B). However, except those learning potentials that were directly connected to the teachers' design, there were also some learning potentials that became a point of focus for the teachers although not explicitly connected to final goals regarding the classroom implementations. Similarly to the choice between the median and mean, as discussed in the second category above, the next extract aims to illustrate a similar debate regarding the third category.

> *Athina: The point is that we will then use the software to generalize from the small population to the big population of their study, the school population. I mean to tell them that this software does what they did with the five values.*
>
> *Researcher 1: Yes but I still have some doubts if it is a good idea to work out of your problem with this small population. I mean you could work inside your problem, with the 350 values of the school population and choose three different sample sizes, let's say samples of 10, of 80 and of 180 to explore various simulations on Fathom. I mean so that students will see how possible is for each sample size to be within a particular range.*

Chloe: I like this suggestion. I think it is better than getting out and in from the initial statistical problem.

Dinos: Ok but they are 8th Grade, what does it mean for them take from 350 values a sample of 10? It means nothing. I mean if they don't take on their own all possible combinations then it is a black box.

In the extract above we can see Researcher 1 bringing to the discussion learning potentials such as *understand the impact of the sample size* or *explore different samples*. These learning potentials became a point of focus and debate among teachers but Dinos emphasized students to be able to explore all possible combinations and insisted on a peripheral role of STSI tools in students' activity.

The degree of students' engagement with software tools, appeared in various instances in teachers' discussions. Particularly, if students would engage directly with the software or if the teacher will use the software in a plenary presentation, if students will explore data on their own or produce samples, or how they would familiarize them with the software (e.g. by making printed handouts or by some particular training) were some of the students' skills and abilities linked to the use of STSI tools. Such codes, although not very frequent, could not be included in the other categories, which are more content-oriented, so I created a category which summarized codes related to *Familiarizing with digital tools designed for statistics instruction*.

The other three categories that are presented in Table 4-7, summarize codes that are related to the particular content of statistics that seemed to have a prominent role within teachers' discussions. Particularly, learning potentials related to graphical competencies, to conceptual connections with probability notions and to the acknowledgment of variability and uncertainty were rather apparent in almost all PD tasks. The examples presented in Table 4-8 below aim to illustrate some characteristic codes appeared in this category.

Table 4-8 - Illustrative examples for some of the learning potentials' categories

Category	**Examples of implicit reference**	**Examples of explicit reference**
Develop graphical competencies	"Look the distributions of the results, this difference is what we want the students to see" (Marcos, 7th meeting); implying students to be able to interpret the data distributions and to compare data graphs as well"	"Students will find the means of all samples of size 3 and then they will put the values in a dot plot" (Dinos, 6th meeting); refers directly to students ability to create dot plots
Connect or deepen to probability notions	"Randomness has power, there is mathematics behind"(Dinos, 1st meeting); implying the importance of students to make conceptual connections between statistics and probability in order to understand the meaning of a random sample	"They will first do the process by hand and then take samples randomly in Fathom, it is important in order to understand how computer simulation tool works" (Marcos, 8th meeting); refers directly to students understanding of the mathematical process behind computer simulations
Acknowledge sources of variability and uncertainty	"The fact that the difference is not significant doesn't mean necessarily that music doesn't influence our ability to recall words, there may be other factors in the experiment that we should acknowledge in the design of the study" (Marcos, 7th meeting); implying the importance of being able to explore influential factors in a random experiment	"They need to make two groups do the experiment but we won't give separate the classroom into two groups. We will let the students do this separation by giving them some options so that they will explore which option is better by acknowledging the stochastic parameters" (Marcos, 8th meeting); refers directly to students' ability to appreciate

stochastics parameters of the problem situation in order to make a choice

Learning potentials are determinant in the formation of DSTR practice. As we show in the categories emerged in our data, there are various learning potentials appeared in teachers' discussions, others related to the specific knowledge of statistical tools and procedures, others related to the actual practice of statistical investigations (thinking, reasoning, inference) and others related to the technology that assists statistical activity. In the next section, I will discuss how these learning potentials, appeared in the several PD tasks of our study.

Learning Potentials In Relation To The PD Tasks.

In Table 4-9, we can see how codes of the various categories appeared throughout the different PD tasks as well as the total frequency of each category within the 10 meetings. The frequencies of the emerged categories of the learning potentials are also displayed for each PD task separately in the histograms presented in Figure 4-4.

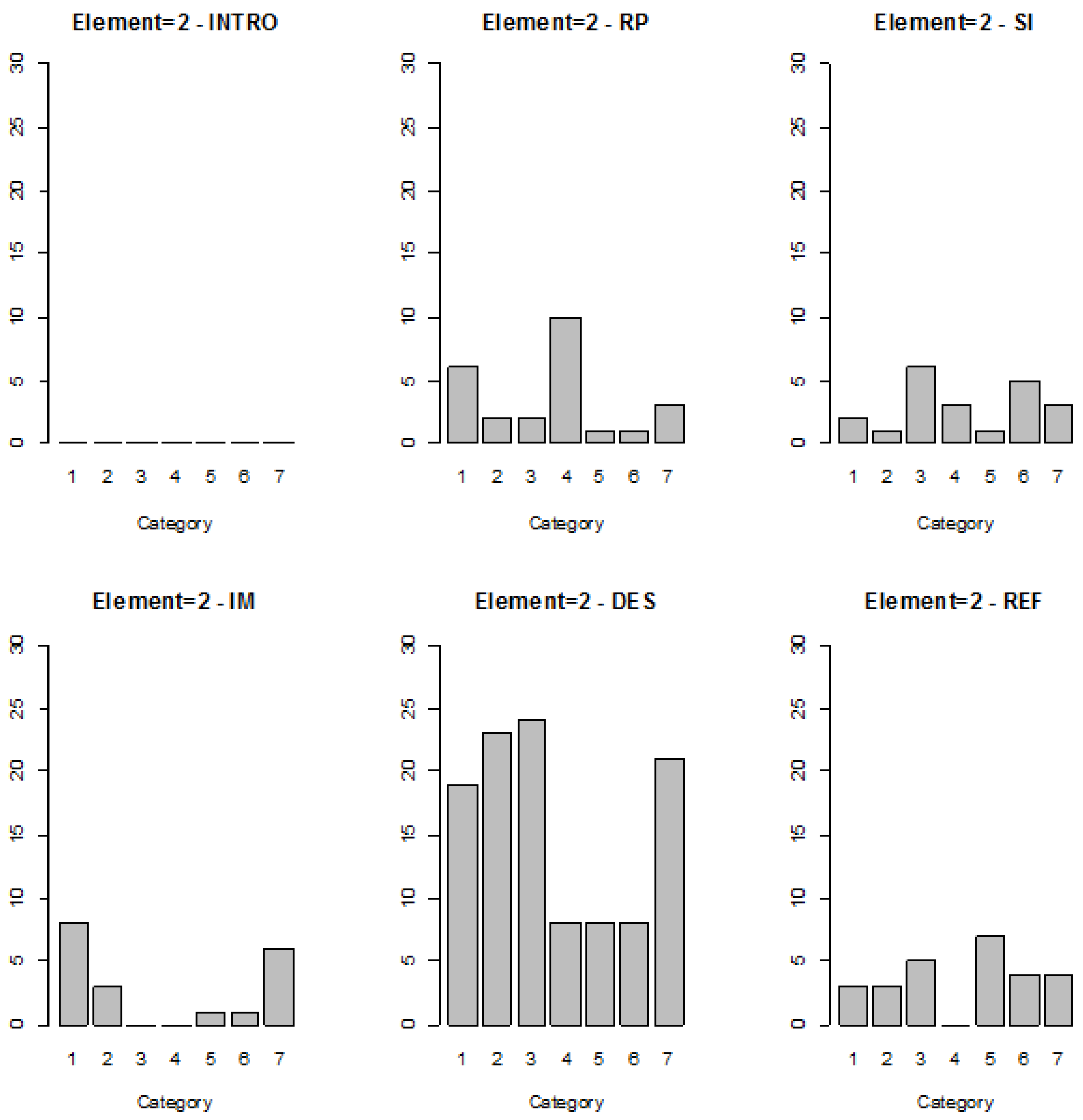
Table 4-9 - Frequency of categories related to learning potentials in the various PD tasks

		Intr	RP	SI	IM	Des	Ref	Total without Design	Total
Learning potentials in the various PD Tasks	Develop particular types of reasoning		15		1	41	5	21	62
	Performing procedural tasks			1		9	1	2	11
	Deepen on statistical inquiry and Inference	1	8	42	29	29	25	105	134
	Familiarize with digital tools designed for statistics instruction			1		3	1	2	5
	Develop graphical competencies		4	1		5	1	6	11
	Connect or deepen to probability notions	1	12	10	5	64	4	32	96
	Acknowledge sources of variability and uncertainty		7	13	13	21	1	34	55
Total		2	46	68	48	172	38	202	374

As can be seen in the table above, in the majority of the PD tasks, there are one or two categories that seem to be dominant in teachers' discussions while the appearance of others is very limited. Since the design task revealed a sound amount of learning potentials in both terms of quantity and quality, in the table I also present a column with the total frequencies after I removed the column refers to the design.

The case of the design differs a lot from the other PD tasks because, first, we can see that teachers refer explicitly to learning potentials in this case, and second, we can see learning potentials from various categories to appear in the group's discussions. The explicit reference to learning potentials during the design phase may stemmed either from the insistence of the researchers for making their goals clear, either from leading teachers' attempts to illustrate the rationale of their suggested tasks, or from the process of negotiating the goals and potentiality of the task they designed within the context of the community

As we can see in the Des column, the category "deepen of statistical inquiry and inference" constitutes a central learning potential discussed in all PD tasks while "Connect or deepen to probability notions" and "Acknowledge sources of variability and uncertainty" seem to be dominant too. The description of particular codes and goals that are related to these three categories of learning potentials presented in Table 4-7 testify to the strong relationship between the three of them and the DSTR practice. However, the strong appearance of these categories during the teachers' design and in particular in an explicit manner indicates that the goals of DSTR became shared in the practice of the teachers' community. Especially the category "deepen of statistical inquiry and inference" and the "Connect or deepen to probability notions" were also dominant when teachers reflected on their implementation. Moreover, we can see that the category "Develop particular types of reasoning", except for the design and reflection, has also a frequent appearance in the discussion regarding particular research papers. This is also related to the content of the research papers, selected by the researchers which aimed to reveal particular aspects of students reasoning when they are confronted with statistical tasks (e.g. proportional reasoning, reasoning based on intuition, distributional reasoning, or inferential reasoning).



Note: Where 1-7 on the x-axis refer to the 7 categories of learning potentials presented in Table 4-7, namely 1. Develop particular types of reasoning, 2. Performing procedural tasks, 3. Deepen on statistical inquiry and inference, 4. Familiarizing with digital tools designed for statistics instruction, 5. Develop graphical competencies, 6. Connect or deepen to probability notions, 7. Acknowledge sources of variability and uncertainty

Figure 4-4 - Histograms of the frequency of Learning Potentials in the various PD tasks

Aspects of students' potential reasoning were often and more explicitly discussed during the design and the reflection. In these PD tasks, we can see not only particular types of students' reasoning, which were already discussed in other PD tasks and became a point of focus in teachers' design but also types of reasoning that came up while teachers negotiated the rationale and potentiality of the teaching actions intended for their students. Examples of such types of reasoning were "the development of students' inferential reasoning based on simulated data" or "the development of combinatorial reasoning" when students work on sampling processes (see for instance the extract from the 7th meeting below). Last, we can see that "Familiarizing with digital tools designed for statistics instruction", although not being directly related to the statistics subject matter, became a learning potential on its own. Particularly, we can see that in three instances during the design, the teachers referred explicitly on the importance of familiarizing students with the Fathom software in order to be aware of the statistical tools behind the software functions and be in a position to take initiatives and make meaningful explorations. This category was also apparent during the reflection phase.

To sum up, the teachers' discussions revealed two learning potentials which seem to be central in all the PD tasks, namely a deeper understanding of the statistical inquiry and inference processes as well as the deeper understanding of the connections between statistics and probability. Both of these learning potentials were supported by the researcher's design in RP, IM and SI PD tasks, but as I saw they were rather frequent during the design and reflection as well, where it was the teachers those who led the discussions. This may indicate that these learning potentials were also important for the teachers and maybe became shared within their community in the formation of DTSR practice. Especially, during the design, the teachers referred in many instances explicitly to the aim of supporting students in understanding probabilistic notions and proceeding to statistical inference, even in an informal manner. It also seems that the emphasis the teachers gave to facilitate statistical investigations in their classroom went along with an aim to support connections to probability theory as well as to help students develop an awareness of sources of variability (e.g. appreciate stochastic parameters of the problem situation or explore influential factors in a random experiment). Moreover, we saw that the discussions during the design task were linked to the development of particular types of reasoning (e.g. students develop distributional reasoning, or contrast to their proportional reasoning), while in the other PD tasks there is very limited reference to types of students' reasoning (except for the discussion on research papers that concentrated on students' reasoning). Last, the appearance of the category related to students' familiarization with digital tools during the design and the reflection phase, highlights that the use of STSI tools presupposes a level of familiarity in order to be used in a meaningful way for DTSR teaching.

4.1.3. The Features

Features constitute the third dimension in the open coding process I followed. The codes referred to this dimension were the less frequent and the less negotiated among the three dimensions in almost all the PD tasks. Particularly, I identified 202 codes in teachers' discussions which have were summarized into 7 categories. The description of the emerged categories, as well as examples of codes that were frequently appeared in each category, are presented in Table 4-10 below.

Table 4-10 - Categories appeared in Features dimension, illustrated by characteristic examples of codes

Categories		Most frequent codes appeared in each category
Features	Encouraging the development of particular types of reasoning	e.g. engage students with informal statistical inference, use small populations to illustrate a data simulation process, create situations to contrast intuition, procedural approach to a problem situation
	Using/Producing authentic data	e.g. use real data from school population, engage students with authentic data production, engage students with data exploration, use authentic media extracts, use
	Engagement with data simulations	e.g. use physical tools to do data simulations, use digital tools to do data simulations, use a combination of physical and digital tools to do data simulations, use data
	Engagement with random experiments	e.g. engage students with a randomization process, engage students with random repetitions, engage students with random experiments, use of unusual objects in random experiments
	Creating appropriate classroom conditions	e.g. encourage classroom discussion, encourage group work, engage students with a digital tool, build on students experiences
	Coping with uncertainty and chance	e.g. management of unexpected statistical results, utilization of stochastic context, the anticipation of chance parameters
	Engagement with statistical inquiry cycle	e.g. engage students with sampling methods, engage students in a full PPDAC cycle, engage students with the rationale of statistical investigation

The Emerged Categories.

As in the case of the learning potentials, the codes I assigned in the case of features were other times explicitly expressed by the teachers, and other times the appearance of feature stemmed from the nature of the particular PD task that teachers were engaged in.

In the following table, I aim to illustrate the various categories of features as appeared among the teachers' discussions and also clarify the qualitative difference between the explicit and implicit appearance of the features.

Table 4-11 - Illustrative examples for the Features categories

Category	**Examples of implicit reference**	**Examples of explicit reference**
Encouraging the development of particular types of reasoning	Marcos: The equiprobability bias. It is not easy to understand that 6,5 is different from 5,6. It is not simple and students use to consider them as equiprobable. Akis: No it is not easy at all. You need to need appropriate experiences to contrast intuition. (2nd meeting: implying the importance of creating situations to contrast intuitive misconceptions)	"But the thing is to help them understand why we use this sample size, why this sample is good for our study " (Dinos, 6th meeting: refers directly to facilitating the development of students' statistical reasoning)
Using/Producing authentic data	Teachers produced their own authentic data in a pilot study in the context of Subgroup A design. Since they negotiated various limitations and factors that influence the data production process we assigned the whole extract to the code: "engage	"There are many sources that you can use authentic data even if you cannot produce them in the classroom. For example, there is the CensusAtSchool database where you can find authentic data from students worldwide" (Marcos, 5th

Category	Examples of implicit reference	Examples of explicit reference
	students with authentic data production” (7th meeting)	meeting); refers to the use of online databases and educational resources where teachers can derive real data
Engagement with data simulations	Teachers made data simulations in Fathom to check a hypothesis resulting from the pilot study in the context of Subgroup A design. Since they negotiated various limitations and factors that lie behind the data simulation process we assign the whole extract to the code: “use digital tools to do data simulations” (7th meeting)	“At first the students would use paper and pencil to understand the process and then they would use the computer simulation tool to repeat the situation many times to see how frequent the particular result is. To use simulations in order to make an inference.” (Marcos, 4th meeting); Marcos refers to the use of a combination of physical and digital tools to do data simulations, and to encourage the Informal Statistical Inference.
Engagement with random experiments	In various PD tasks teachers had the opportunity to engage with random experiments (e.g. 1st meeting, they were divided into subgroups through a randomization process; 2nd meeting discussed about experiments with dices; 7th meeting, produced random samples in Fathom) and negotiated various features such	Dinos: But how you can validate students’ inference? How can you convince that the inference is reliable? Marcos: The only means that you have is to engage them with random repetitions. How else?” (4th meeting) Marcos refers to students’ engagement with random

Category	Examples of implicit reference	Examples of explicit reference
	as the use of unusual objects in random experiments see for example: Marcos (2nd meeting) "But you can imagine that you throw the dice twice, or better you can imagine using dices of different colors, for example, a red and a green dice"	repetitions of an experiment
Creating appropriate classroom conditions	"What we have here, namely to build on each other's ideas and share experiences, is very important especially when you are doing statistics" (Chloe, 10th meeting); she implies that group work and building on each other's experiences is important for the learning of statistics	"We will make some handouts regarding the software environment and the will explore the data on their own, maybe in pairs" (Marcos, 8th meeting) refers to the creations of handouts in order to engage students with Fathom.
Coping with uncertainty and chance	Marcos: But we need to see how it works before we do it in the classroom. Lia: Yes, this is a very good idea, we can do it here in the next meeting, what do you think? (6th meeting) They imply the management of unexpected statistical results and the anticipation of chance parameters	Marcos: I tried so hard to control bias in the list of words we gave to students but what actually happened is that my list was full of bias since I was based on the English example and in English language, things work differently than in Greek. Researcher 1: How does this bias as you say, affect your design? Marcos: Dramatically. I mean this

Category	Examples of implicit reference	Examples of explicit reference
		task aimed to introduce somehow informally, the idea of hypothesis testing, but, what hypothesis to test when the difference is too big. (Marcos' reflection) refers directly to the management of unexpected statistical results
Engagement with statistical enquiry cycle	In various PD tasks, teachers had the opportunity to engage with statistical enquiry cycle (e.g. 1st meeting, exploring various sampling methods; 7th meeting, conducting a pilot study) and to negotiate features such as to engage students with sampling methods, in a full PPDAC cycle, with the rationale of statistical investigation	Akis: In my opinion, it is important for students to understand the idea of sampling. What is the role of the sample in a study and to evaluate a sampling method? Dinos: Engage students with the whole statistical cycle, to produce their own data, to explore different parameters and to see that there is a rationale behind sampling, it is not magic. (4th meeting)

As we can see in Table 4-10, and illuminate further in Table 4-11, the features were identified to respond to how of developing DSTR practice. Particularly, the codes attributed to the promotion of particular types of reasoning, to how this promotion can be facilitated, namely by using small populations to illustrate a data simulation process or to engage students with informal statistical inference by asking them to make a conclusion based on data. The how of the DSTR practice, that is the particular means and conditions related to the teaching and learning for developing statistical reasoning and thinking, brought to the fore characteristics that are related to the intended reasoning and thinking; the statistical context including the stochastic nature of statistical situations (uncertainty and chance) and the particularities of statistical activity (dimensions of statistical enquiry cycle); the classroom conditions; the fundamental statistical idea of data, including the idea of authentic data, simulated data or randomized procedures. The particular appearance of each category in the various PD tasks and differences in the way they have been discussed in each case are presented in the following section.

Features In Relation To The PD Tasks.

In Table 4-12 below, I present how the seven categories appeared in the various PD tasks. Additionally, the histograms of Figure 4-5 help in the visual representation of the observed frequencies as well as in the discussion that follows. As can be seen in Table 4-12, we have no features identified during the introductory discussion. While teachers discussing research papers, the features that seemed to dominate their discussion were features that relate to students' engagement with random experiments as well as to the encouragement of particular types of reasoning. The features related to particular types of reasoning were also dominant during the discussion about particular instructional materials. In this PD task, the features that are related to students' engagement with a statistical enquiry cycle were also very frequent (e.g. engage students with a comparison of different sampling methods, engage students with the rationale of statistical investigation, etc). In the three other PD tasks, there was a strong spread among all seven categories. This spread is also apparent in the total frequencies, presented in the last column in Table 4-12. In this column, we can see that there is no particular category among Features that we could say stand up among the others. On the contrary, it seems that all the revealed categories of features had a similar intensity in their negotiation. The picture of the histograms shows clearer this uniformity.

Table 4-12 - Frequency of categories related to Features in the various PD tasks

		Intr	RP	SI	IM	Des	Ref	Total
Features in the various PD Tasks	Encouraging the development of particular types of reasoning		6	2	8	19	3	38
	Using/Producing authentic data		2	1	3	23	3	32
	Engagement with data simulations		2	6		24	5	37
	Engagement with random experiments		10	3		8		21
	Creating appropriate classroom conditions		1	1	1	8	7	18
	Coping with uncertainty and chance		1	5	1	8	4	19
	Engagement with statistical enquiry cycle		3	3	6	21	4	37
Total		0	25	21	19	111	26	202

As have been already discussed the Design and the Reflection were the PD tasks with the highest level of teachers' freedom in forming the agenda of the discussion in the group meetings. As we can see during the design the most frequent category was the one related to the development of particular types of reasoning. This result stems from the fact that during the design the features appeared in order to address a particular learning potential negotiated by the teachers. For example, in the 4th meeting, where Marcos said:

> *Marcos: But can I say something else? I think that the main idea behind the problem with the hospitals (Watkins, Scheaffer, & Cobb, 2008; p. 220) is the idea of the simulation of a situation. I really*

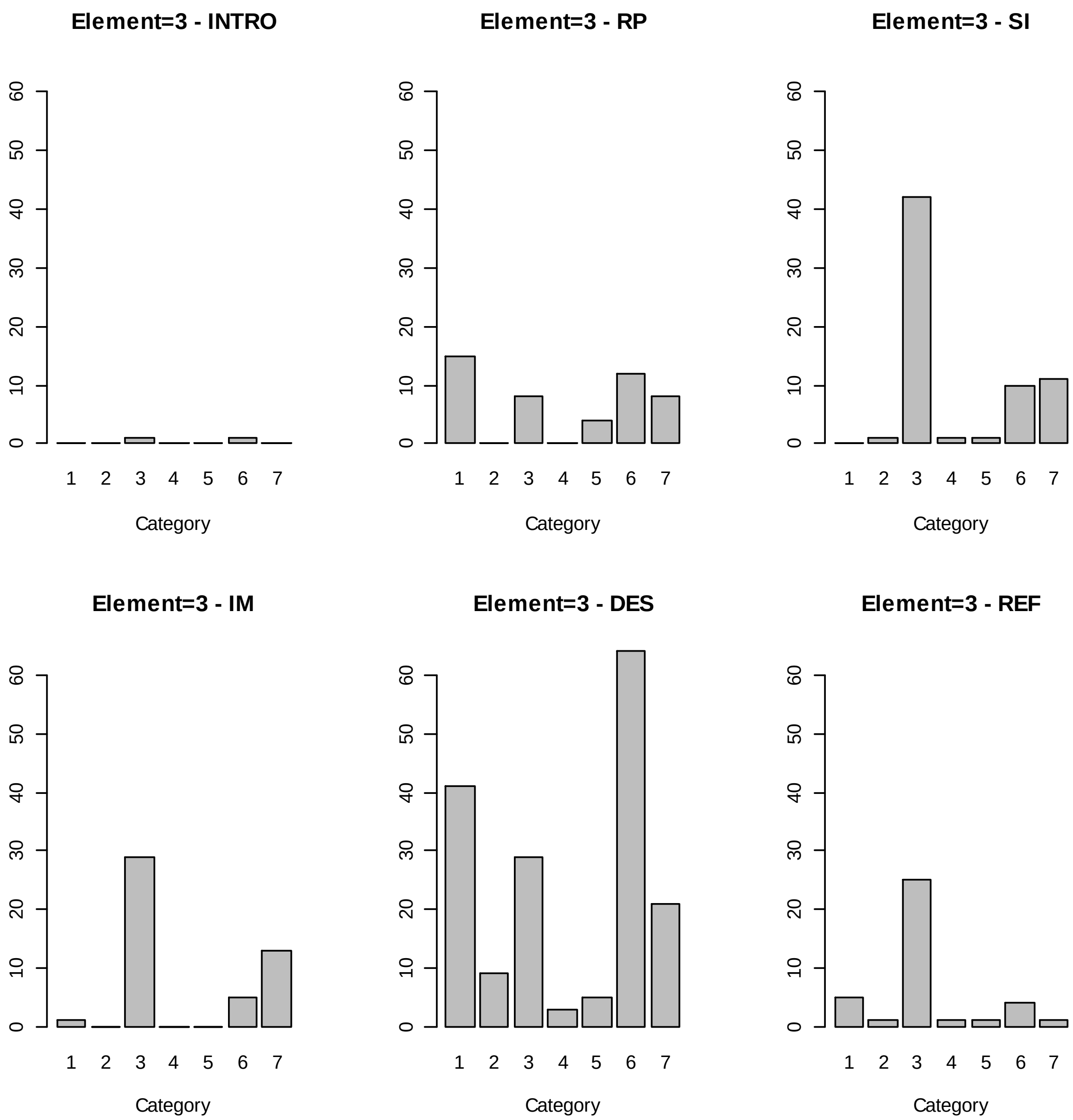
like the idea of data simulations and I think it is something that we can do in both Gymnasium (Grades 7-9) and Lyceum (Grades 10-12). I mean not to do data simulation just in order to produce data, but rather in order to make an inference based on this data.

Dinos: But how you can validate students' inference? How can you convince that the inference is reliable?

Marcos: The only means that you have is to engage them with random repetitions. How else? I mean you can use simple means that are available even for young students, namely the frequencies, and the estimation of probabilities through the frequencies, and make an inference on judging how frequent is a result to appear and what can I conclude from this.

Here Marcos' idea to engage students with informal statistical inference was first related to how this particular reasoning can be supported and then more specific features came into play by Marcos (e.g. data simulations, randomization processes, etc). This strong interplay between features and learning potentials was also apparent during the reflection. As we can see in Figure 4-5, features and learning potentials during reflection were almost equally apparent in teachers' negotiations (20% and 29% respectively) and this was the only task with a so high representation of features. However, we can see that, although one of the tasks implemented in the classroom was based on a random experiment, the related feature (namely Engagement with random experiments) did not appear in the reflection.

This result was due to several management issues in the implementation of the random experiment which linked the random experiments more to means for coping with uncertainty and chance (e.g. management of unexpected statistical results or anticipation of change parameters). In particular, the random experiment the teachers designed approved to be difficult to connect with the learning potentials that teachers defined due to several parameters (design failures, unexpected outcomes, high variability in the classroom context – small sample, etc). For this reason, the discussion regarding the designed experiment led to a more general discussion of how issues of uncertainty and chance can be acknowledged during the design and how unexpected outcomes can be managed in order to support the initial learning goals.



Note: Where 1-7 on the x-axis refer to the 7 categories of features presented in Table 4-10, namely 1. Encouraging the development of particular types of reasoning, 2. Using/Producing authentic data, 3. Engagement with data simulations, 4. Engagement with random experiments, 5. Creating appropriate classroom conditions, 6. Coping with uncertainty and chance, 7. Engagement with statistical enquiry cycle

Figure 4-5 - Histograms of the frequency of Features in the various PD tasks

Last, we can see that the use of STSI tools and of digital technology, in general, does not constitute a category in our data analysis. This is not because digital technology was absent but instead because it was almost omnipresent in the data. In this way, students' engagement with digital technology is not a category on its own but is inherent in almost all categories. In Table 4-13 I present some of the most frequent codes with respect to the use of digital technology, as related to each category.

Table 4-13 - Emerging codes related to the use of digital technology as emerged in each category of Features

Features Category	Examples of Codes related to the use of technology
Encouraging the development of particular types of reasoning	use Fathom to support the development of distributional reasoning, use STSI tools to facilitate students' develop informal inferential reasoning
Using/Producing authentic data	Use data sets available on the internet or STSI tools' webpages
Engagement with data simulations	use Fathom to produce simulated data
Engagement with random experiments	use Fathom to design a random situation, use STSI tools to design a randomization process
Creating appropriate classroom conditions	students work directly with Fathom in pairs, teacher use Fathom to facilitate students' visualizations of data
Coping with uncertainty and chance	to use STSI tools in conducting a pilot study
Engagement with statistical enquiry cycle	use Fathom to engage students with data exploration, USE STSI tools for transnumeration activities, use STSI tools to analyze quantitative data

As we can see in Table 4-13, in all categories there is at least one code related to digital technology. The central role of STSI tools in the learning of statistics has been widely discussed in the literature review and this role is also reflected in the results of our study. As we can see all the teaching means and strategies revealed in the teachers' discussions are somehow connected with the use of STSI tools. Additionally, we can see that codes related to STSI tools are not only linked features that aim to support the learning potentials as defined by the teachers, but are also linked with features that aim to support the actual implementation of a statistical task, namely the appropriate classroom conditions and the management of uncertainty and chance throughout the implementation. Particularly, the actual use of STSI tools in the classroom varies considerably from direct interaction between students and STSI tool to a mere presentation on a classroom projector. Moreover, STSI tools seemed to be essential not only for the development of students' learning but on the development of teaching as well. Specifically, we saw that in order for teachers to deal with uncertainty in their design, they conducted a pilot study before the classroom implementation so that they could acknowledge as many stochastic parameters as possible. In this study, they used Fathom in two ways. The one was to analyze the data they collected from the pilot study and discuss the didactic management of them. The other was to do data simulations in .order to investigate potential scenarios that could take place in the classroom. In this way, STSI tools, except for facilitating the learning process, also facilitated the design process.

To sum up, I can say that the open coding with regard to features of the DSTR teaching highlighted their concrete role in the formation of the teaching practice. Particularly, as we saw the highest negotiation of features took place in the last two PD tasks, namely the design and the reflection, where these features were strongly linked to particular learning potentials. During the other PD tasks, we saw that the negotiation of the features was mainly related to the content of the task and consequently, I can say that these negotiations were guided to some degree by the researchers' design. On the contrary, during the design and the reflection, the features mainly appeared to respond to particular goals defined by the teachers or were connected with the general context of the curriculum and school. Moreover, when a statistical task is connected to the classroom reality, a variety of stochastic and management issues appears that brings to the fore a number of features that stem mainly from the teachers and the particular norms of their classroom and less from the researchers' support. Last, the significant role of digital technology was apparent in the study of the features and it also shows the importance of this resource not only in terms of supporting learning potentials but also in terms of supporting the teaching design process.

4.1.4. Conclusion Of The First Level Of Analysis

The open coding process I followed helped me to get a deeper insight into the development of the teaching practice with respect to the three core dimensions as set in our conceptualization for DSTR. Particularly, we saw how different PD tasks motivated different sources of negotiation with respect to resources, learning potentials and features. Moreover, we saw that in the case of teachers' design the picture in all three dimensions changed considerably. This was mainly due to two aspects. First, because during the design, the researchers tried to give teachers initiatives and also to have a minimum impact on the preformed agenda of the meetings. This fact left space for a stronger interaction among the teachers, brought to the fore various human resources, revealed learning potentials closer to the teachers' interests and closer to the curriculum, as well as supporting a more explicit negotiation of particular features that appear in order to facilitate the defined learning potentials. Second, because the task of design is linked both to the classroom reality generally and to the particularities of the Greek context specifically. In this sense, all the resources, learning potentials and features that are reported in statistics education literature and introduced by the researchers, needed to be negotiated, modified and adjusted in order to fit into the teachers' classrooms with its specific norms and conditions. Thus, in this PD task the available or suggested resources interacted with the classroom related resources, either material (infrastructures, student-teacher ratio, classroom norms, etc) or human (teachers' knowledge and experience on statistics, students' background knowledge in statistics, everyday experiences, colleagues' interactions, group-based interactions etc). On the other hand, during the reflection task, the discussion was quite focused on particular resources, learning potentials and features. In this case, we saw that the features were more explicitly connected to specific learning potentials and the resources related to students were more apparent. As regards the other PD tasks, the negotiation was closely related to the content of the task. The resources revealed in these PD tasks were mainly related to the resources included in the researchers' agenda, while the features and the learning potentials appeared in the discussions often implicitly and rarely explicitly. The results of this phase gave us an insight into the differences occurring in the main dimensions of the DTSR practice scheme, as the PD tasks are more closely connected to the actual teaching. Additionally, STSI tools seemed to have a prominent role in all dimensions of the statistics teaching and linked to various categories of both features and learning potentials.

The open coding process was helpful in giving us an overall picture of how the practice of the community is developed throughout the various PD tasks in terms of resources, learning potentials and features. However, we still know little of how these three separate dimensions act and interact in the formation of DSTR practice. For example, how strong was the negotiation of a particular resource, namely did the majority of the teachers contribute to the discussion or only a few of them? How can we interpret that during the reflection task the frequency of codes related to learning potentials and the frequency of codes related to features were so close (see Figure 4-2)? Were particular resources connected to particular learning potentials or features? How such connections may influence the developing of the shared repertoire of the community? What seems to affect the integration of new resources in this shared repertoire? Questions as to the above guided us to proceed in the next level of analysis of our data. In this level, I aim to go deeper and address such questions.

In order to go deeper on the DSTR practice and focus on potential interactions among the three core dimensions of this practice, I decided to concentrate on a particular resource and follow its path from its introduction to the community until its integration to the community's shared repertoire. The resource that seemed to have strong interactions with features and learning potentials and it is also essential for the DSTR practice is the STSI tools. In the next section, I present the findings from tracing the STSI tools in the data discussing the path of their integration in the community's practice along with the phases, as well as aspects that seem to frame the passage from one phase to another.

4.2. Results Of The Second Level Of Data Analysis

4.2.1. The Phases Emerged On The Re-Sourcing Process

The second level of the data analysis focused on a particular code emerged on the first level, namely STSI tools. The trace of this code in the discussions within the CoP, restrict the data into 31 episodes as described in the methodology section (see section 3.8.2). As shown in the results of the first level (see section 4.1), STSI tools appeared in almost all PDT of the CoP's work, from the introductory discussion to the design and the reflection of the teaching, so this is a resource that seemed acted as a boundary object that entered in the community and influenced the practice that was formed within it. The analysis in this level concentrated on what was negotiated with respect to this resource and by whom. This analysis revealed three distinct phases which cover the route from the moment the resource became available to the community to when it became part of the community's shared repertoire.

Table 4-14 - Summary of the three phases emerged in the second level of analysis

Phases	**Core theme related to the process of negotiation of meaning**	**Corresponding data**
Emergence	No interaction between teachers and STSI tools	Initial interviews, 1st – 4th meeting (1 episode)
Exploration	Discussing STSI potentialities as a teaching tool	4th-9th meeting(25 episodes)
Immersion	Sharing experiences from classroom reality	10th Meeting (5 episodes)

The three phases were identified with respect to the characteristics of the negotiation of meaning of STSI tools in the 31 episodes. Particularly, as we can see in Table 4-14., what characterized each phase were the actions and interactions among participants with respect to STSI tools. The actions and the interactions motivate the process of negotiation of meaning in relation to STSI tools.

Next, I will discuss in detail the three phases and I will illustrate the characteristics of negotiation of meaning in each one.

4.2.2. The Emergence Phase

This phase extends from the moment the STSI enters in the community until this resource becomes a point of focus in the community's discussion and the participants start to explore its characteristics and potential uses. This phase is not characterized by negotiation of meaning but rather by an explicit reference to the resource. In the case of STSI tools, this phase included only one episode from the introductory discussion. The next time the code appeared (4th meeting) it was when Marcos suggested using Fathom to engage students with data simulations. Thus, the second time STSI tools were revealed in our data, they motivated a negotiation of meaning with regard to their use in the teaching of statistics.

Particularly, in the introductory meeting with the teachers where they presented the motives for forming this group of teachers who would work together for studying the teaching of statistics, presented some PowerPoint slides to talk about "supportive material." In this part of the discussion, we referred to software tools specially designed to support the learning of statistics in secondary school that would be available to the CoP and gave the examples of Fathom (Finzer, 2001) and TinkerPlots (Konold & Miller, 2005). We linked these tools with some related resources, such as the official web pages (https://www.tinkerplots.com/, http://fathom.concord.org/), and available license keys for access. More specifically, Researcher 1 said:

> *"In this slide, we bring some web sources regarding these tools (she means STSI tools), we also have available some license keys to*

help you explore their potentialities and we are sure in the next meetings we will have the opportunity to discuss more these tools."

Overall, I only referenced the use of these tools to support statistics learning and teaching in a limited way, for example by saying,

"These are research products aimed at supporting the learning process at secondary schools. They are not similar to SPSS, most of you probably know, but they are more directed towards supporting students with data explorations."

Here, I made explicit the potentiality of these tools to support students' explorations with data, as well as their adequacy to be used even at lower levels of education.

By saying that in this phase there was no negotiation of meaning with respect to STSI tools, I refer to a communal negotiation that becomes observed and transparent within communities' members. Since STSI tools became available to the members of the community, they simultaneously became a potential teaching tool that each member of the community can integrate to their teaching repertoire. However, here I focus on the communal negotiation of meaning as well as on the development of the shared repertoire of the community. In this manner, I count the emergence phase as the phase of the resource being emerged but without being further negotiated within the community.

Moreover, in the case of STSI tools, there was only one episode assigned to the emergence phase. However, in other cases, there would be more episodes assigned to this phase and not guaranteed passage to the next phase. For example, in various instances throughout the 10 meetings, the researchers, as well as Marcos, referred to the CesusAtScool project (https://censusatschool.ie/) as a potential database that they could use for their classroom task design. However, although this resource became available and referred in more than one instances of teachers' discussion, it never became a point of a communal negotiation among the teachers and they no further explore their potentiality as a teaching resource up to meeting 4.

4.2.3. The Exploration Phase

The second phase was the exploration phase. This was where the members started to interact with the resource in the context of the community and explored potential uses and limitations regarding statistics teaching. The exploration phase extended from the moment the resource became a point of focus in the community's discussion until it was incorporated into the actual teaching. The exploration phase contained 25 episodes (out of 31 in total) and in Table 4-15 I present a brief description of the content of the negotiation of meaning in the various meetings.

Table 4-15 - Content of the negotiation of meaning in the various meetings

Meeting	**Participation**	**#episodes**	**Negotiated meaning in each meeting**
4th	R1, R2, Marcos, Lidea, Lia, Akis, Ria (plenary discussion)	3	Marcos suggested using the idea of data simulation in their imminent design. Questions raised on the process of data simulation and how this could be assisted by technology.
5th	R1, Marcos, Lia, Ria (plenary discussion)	3	Marcos presented some particular teaching examples to illustrate the idea of data simulation. The question raised in the process of data simulation as well as an emerged need to engage in such a process and develop a deeper understanding of what it is and how it can be assisted by Fathom

Meeting	Participation	#episodes	Negotiated meaning in each meeting
6th	Marcos, Lidea, Ria, Kimon, Ria (Subgroup A)	6	Marcos discussed with his colleagues on a particular random experiment and he suggested the use of Fathom to make several repetitions of the experiment and proceed to an inference. Questions raised on the statistical process of this random experiment as well as on the possibilities provided by Fathom.
	Athina, Dinos, Sofi, Chloe (Subgroup B)	2	They discussed using a software tool in order to produce various samples form a data population and make quick calculations to find the averages of the samples. There was a reference to Fathom (that Marcos suggested in the previous meeting), and they discussed asking for Marcos' support.
	Marcos, Lidea, Dinos, R1 (plenary discussion)	1	A brief discussion on how each subgroup was planning to use STSI tools in their design
7th	R1, R2, Marcos, Lia, Ria, Dinos, Kimon, Athina (plenary discussion)	5	After the teachers did a pilot study based on the random experiment designed by Subgroup A, Marcos used Fathom to explore the gathered data and to show the other teachers how they could use the simulation tool of Fathom to make an inference based on these data. Further questions on other possibilities of Fathom and other useful tools were raised by the members of Subgroup B and discussed.
8th	R1, Michalis, Chara, Aggeliki, (Subgroup A)	3	Lia referred to her experience from exploring data in Fathom in the previous meeting

Meeting	Participation	#episodes	Negotiated meaning in each meeting
9th	R1, Marcos, Lidea, Lia, (Subgroup A) Athina, Sofi, Chloe (Subgroup B)	2	Small discussion in both Subgroups on the particular questions that could be addressed to the students with respect to the outcomes produced on Fathom.

As we can on the Table 4-15 above, Marcos was the one who makes STSI tools a point of focus for the community by suggested using such tools in their design. Particularly, on the 4th meeting, in which the participants were asked to discuss ideas for classroom implementation, Marcos said:

> *"Can I suggest something else? I mean I am thinking of the task we discussed (he refers to Watkins, Scheaffer, & Cobb, 2008; p.220), and I am very interested in the idea of simulating a random situation. I find such a process very interesting and fruitful for students in both Gymnasium (Grades 7-9) and Lyceum (Grades 10-12). I mean to proceed somehow to statistical inference with simple tools, not necessarily with formal tools, but rather with experimentation and empirical observations."*

Marcos' suggestion motivated a rich negotiation since the community's members were familiar with neither STSI tools nor the process of data simulation. Some indicative examples of questions raised in several instances of the exploration phase were:

> *"But what do you mean when you say simulation?" (Lia, 4th meeting and again in 5th meeting).*
>
> *"And how you will validate the inference to the learners" (Dinos, 4th meeting).*

"The idea of computer simulations seems very interesting, I mean it sounds like it can be easy and quick, but we need to see it. Could you present us with an example?" (Dinos, 4th meeting).

"But how students can engage in the simulation process" (Ria, 7th meeting).

"I still have difficulty on how does it work" (Lidea, 8th meeting; she refers to the data simulation on Fathom).

All the above questions motivated a discussion that through further members' interactions, many potentialities of STSI tools were illustrated and discussed. Particularly, teachers discuss on using STSI tools to produce simulated data based on a random experiment, to produce random samples given a particular data population, they also explored and discussed representational tools and data summary tools that they could use for the statistical investigations in the classroom. As we can see in Table 4-15 many of the possibilities regarding STSI tools were revealed and discussed while teachers explored the data they collected during the pilot study of Subgroups' A design in the 7th meeting.

However, the negotiation of meaning, in this phase of the re-sourcing process, was characterized by an absence of experiences and connections to the actual teaching. STSI tools constituted a new and innovative resource that none of the teachers had known about or had used before. Thus, their exploration regarding STSI tools more informed their own experience as learners and less by their teaching experience for their students' learning.

The exploration phase requires the emergence of the resource but is not necessarily the ground for the immersion phase. For example, another resource that was explored within the community was physical tools (e.g. playing cards or dices) to simulate random phenomena. Such resources were discussed and explored through particular examples that were provided by the researchers. However, this resource didn't immerse in the practice of the community despite the positive attitude of the members towards it. An important issue with STSI tools was that, although initially, it was only Marcos who was willing to explore them in the actual teaching, it became a strong point of focus in teachers' discussions (25 episodes in 6 meetings) and it influenced both subgroups' design and the subsequent implementations in the classroom.

4.2.4. The Immersion Phase

The third phase in the re-sourcing process, namely the immersion phase, occurred when the resource immersed in the actual learning environment. Particularly, the task designed by subgroup A was implemented in Marcos and Kimon's 12th-grade classrooms. Ria and Lia participated as observers of Marcos' implementation, Kimon video-recorded his lesson and shared it with the other members of subgroup A, while Lidea and Akis did not participate in the implementation. The task designed by subgroup B was implemented only in Dinos' 8th and 9th-grade classrooms and all the other members of this group were observers during the two implementations. This phase contains five episodes. Four episodes come from the tenth meeting where the participants reflected on what they had done in the classroom, while the last episode comes from the discussion of Marcos' implementation. During the implementation, the members had the opportunity to use in the classroom the tasks they had designed, and to observe students' reactions and difficulties regarding STSI tools. In this phase, the negotiation of meaning was mainly about limitations and potentialities that seemed to be crucial in the students' learning. In Table 4-16 I present a brief description of the content of the negotiation of meaning in the various episodes of this phase.

Table 4-16 - Content of the negotiation of meaning in the 5 episodes of the immersion phase

Episode	**Participation**	**Negotiated meaning in each meeting**

Episode	Participation	Negotiated meaning in each meeting
1st	Sofi, Dinos, Chloe	Discussed students' reactions in the view of sampling distribution created by Dinos in Fathom
2nd	Sofi, Dinos, Chloe, Researcher 1	Students acknowledging variability in sampling while exploring sampling distributions on Fathom
3rd	Chloe, Athina, Dinos, Marcos, Ria Researcher 1	Students' familiarity with the stochastic context and random phenomena
4th	Kimon, Marcos, Researcher 1, Researcher 2	Difficulties in using the school computer lab in order to engage students with Fathom
5th	Marcos, Dinos, Researcher 1	Issues regarding students' direct engagement with STSI tools

As we can see in the Table 4-16 the resource is mainly negotiated with respect to students and the school reality. The immersion of STSI tools in the actual practice of statistics teaching gave the teachers the opportunity to explore further the Fathom potentialities as a learning tool. They acknowledged the representational dynamics of Fathom and its role in supporting students' observations and to facilitate the development of students' statistical reasoning. For example, in episode 1 Sofi discussed that: "When we created the data dotplot they immediately linked to the shape of a mountain" and Dinos agreed that "it was easy to see that most values are spread around a particular value, to see variability". They moreover appreciated STSI tools for their potentiality to support students to develop statistical reasoning. It is characteristic what Dinos said in the second episode "In order to help them develop an alternative way of thinking we need to spend time on this, we need to encourage them to make their own data explorations, they need to use the software themselves" and they also acknowledged students' everyday experiences and sources of familiarity with stochastic context. The next extract from the 3rd episode is indicative:

> *Athina:* *The big surprise for me was how familiar students were with the stochastic context. At least much more than us.*
>
> *Chloe:* *This is also maybe due to the era of their age. I mean in the current days, children are very used to the technology, to the media, to… you know.*
>
> *Dinos:* *Yes, this was surprising for me too.*

Furthermore, very central to the negotiation of meaning at this phase seemed to be the school reality. For example, Kimon, although he was positive and intended to use Fathom in the implementation of the task of Subgroup A, he actually admitted school-based limitations that hinder him from using the schools' computer lab. Marcos also provided some limitations and difficulties in engaging students with STSI tools at school reality.

As we can see the negotiation of meaning during the immersion phase is characterized by teachers developing an awareness of the actual use of STSI tools and how this use can indeed facilitate the learning process. Moreover, they also become aware of what seems to frame this integration in terms of school-based conditions and norms. This developing awareness indicates a potential consolidation of the resource in the shared repertoire of the community.

The consolidation of the resource in the shared repertoire of the community refers to the final impact of the resource in the repertoire of the community, which in the case of STSI, indicates an emerged necessity of using such tools in statistics teaching. Although a consolidation would presuppose a longitudinal study of the community's practice, the final interviews with the participants conducted about 1.5 years after the last meeting, constitutes a good indicator for the resource's integration into the community's shared repertoire. Moreover, the teachers' voluntary participation in the CoP for a second year is also indicative of acknowledging this experience as a fruitful one.

In their final interviews, almost all the participants referred to the use of STSI in statistics teaching spontaneously and without being prompted. Moreover, most of the participants seemed willing to use such tools and considered them as essential.

Particularly, the participants with no strong background in statistics admitted that they were benefited from their collaboration with the other teachers and especially from Marcos and Dinos and they emphasized the importance of deep content knowledge as well as more teaching experiences in developing familiarity with STSI tools potentialities. The words of Athina and Lia are indicative of this shared view among the participants of the CoP:

> *Lia: I don't think that Fathom was easy to become familiar with. I watched Marcos do all these moves and construct these simulations. A strong mathematical and statistical background is needed aside from technological familiarity. I learned a lot from my experience in the group, but I feel like I need to learn more.*
>
> *and*
>
> *Athina: Ok I believe that such software is considerably important and theoretically, I would definitely use it. But, ...I mean practically, I think I need some time to feel…you know. I mean OK, we saw how we can use the software, we saw what we can do with it and how this can support learning, but if I was supposed to use it tomorrow morning, ...I feel I need some more experiences with that.*

On the other hand, Marcos and Dinos both admitted the value of their collaboration, but they also emphasized that the contribution of others helped them to develop a deeper understanding of what they discussed and linked it more explicit with particular learning potentials and features. Moreover, they both mentioned that the classroom implementation helped them to link more efficiently STSI tools with students' learning and also appreciate their colleagues' collaboration and contribution during the implementation of the statistical tasks. The next two extracts aim to illustrate how Marcos and Dinos experienced their participation in this study:

> *Marcos: I can say that I learned a lot from my interaction with the researchers and working with Dinos and Akis was also valuable. However, I can say that the other questions and comments helped me to go deeper to what I already knew. I mean I needed to justify what I had in mind, to make it clearer, to find appropriate examples, to acknowledge possible constraints and difficulties. This was a really fruitful and informative experience too.*

and

> *Dinos: I had no idea before about Fathom or anything similar, what I had in mind were tools that enable fast calculations. But this [he means Fathom], this is different, this is really useful ... What students can do with Fathom, is not something you can describe in a theoretical manner, the student can't reach the same point without such a tool, because without it, it is difficult to develop understanding.*

However, as in the previous phase, the immersion phase requires that the resource has been already explored by the community in terms of teaching and learning potentialities and alternative ways of using, but, is not necessarily a piece of evidence for the resource consolidation in the community's shared repertoire. Particularly, there were examples of resources, other than STSI tools, that although immersed in the community's practice we don't have any evidence of being shared and communally integrated into the actual teaching. An example of such a resource is the case of media extracts. Media extracts was a resource included in Subgroup's B design and immersed in their practice with a positive influence for students, as the members admitted. However, the re-sourcing process with respect to this resource showed that some of the aspects that seem to frame the integration of this resource in the teachers' practice are: (a) teachers' familiarity with the context in media reports, (b) difficulties in defining learning goals and (c) teachers' low self-confidence regarding classroom management (see also Bakogianni, Paparistodemou and Potari, 2014).

4.2.5. Summarizing The Results Of This Level

The second level of the data analysis revealed that the re-sourcing process constitutes a three-phase process in relation to the process of negotiation of meaning within the CoP. Particularly, as the resource moves deeper in the practice of the community the negotiation of meaning structures a different content and a different focus. This process is displayed schematically in Figure 4-6.

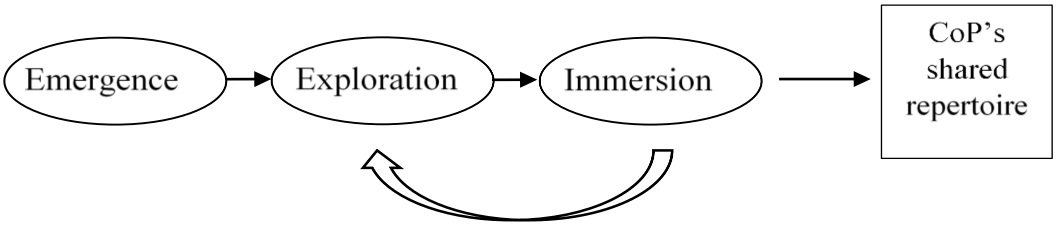


Figure 4-6 - The three phases in the re-sourcing process of STSI tools

The emergence phase is the initial phase where the resource is being visible and available to the community but not communally negotiated among its members. The exploration phase is characterized by a rich negotiation of meaning. The members start to interact with the resource in the context of the community and explore the potentialities and utility of the resource with respect to teaching and learning. The third phase is the immersion phase This phase occurs when the resource immerses in the actual learning environment and the participants have the opportunity to use it in the classroom and gain experience from its impact on the learning process. As we saw, the immersion phase may lead either to the consolidation of the resource in the shared repertoire of the community, or to a new ground for the exploration phase.

At this level, we saw that the three phases are characterized by differences in the content of the negotiation of meaning in relation to the resource. We also saw that the three-phase re-sourcing process is nor a linear process, but on the contrary, the passage from the one phase to the other is neither guaranteed nor inconvertible. Although the concentration of this level of analysis on the negotiation content and the participation process in terms of actions and interactions among participants was helpful in identifying the various phases in the re-sourcing process, it gives us a poor insight in the passage from the one phase to the other. The results from the third level of analysis aim to give light on this passage and go deeper in the process of the negotiation of meaning in relation to STSI tools, looking for emerging links among resources-features-learning potentials, that seem to frame or facilitate this passage.

4.3. Results Of The Third Level Of Data Analysis

The second level of the data analysis focused on a particular code emerged on the first level, namely STSI tools. STSI tools appeared in almost all PDT of the CoP's work, from the introductory discussion to the design and the reflection of the teaching, so this is a resource that seemed acted as a boundary object that entered in the community and influenced the practice that was formed within it. Our analysis in the second level concentrated on what was negotiated with respect to this resource and by whom. This analysis revealed three distinct phases which cover the route from the moment the resource became available to the community to when it became part of the community's shared repertoire. In the last level of the data analysis, I delved deeper into the three phases emerged in the second level, namely the emergence, the exploration, and the immersion phase, and investigated how the DSTR teaching scheme (see Figure 2-11.) was formed and developed in the various phases.

4.3.1. The Emergence Phase In The Re-Sourcing Process

As have been discussed in the results of the second level of the data analysis, the emergence phase is characterized by an absence of interaction among participants with respect to STSI tools. The researchers, in an informative presentation, referred to some popular software tools (e.g. Fathom and TinkerPlots) and discussed their potentialities in the statistics learning process.

In this phase, which lasted from the 1st to the 4th meetings of the CoP, there was no negotiation of meaning regarding the STSI tools use in the teaching of statistics among participants and I could not identify explicit links between the three elements of DSTR teaching.

4.3.2. The Exploration Phase In The Re-Sourcing Process

The exploration phase was extended from the moment the resource became a point of focus in the community's discussion until it was incorporated into the actual teaching. During this phase, the participants started to interact with STSI tools, with Fathom, in particular, they investigated technical possibilities and discussed potential uses. To make the presentation of the results more intelligible, I need to mention that given that the PD tasks in the first four meetings had not made use of STSI tools (for example the papers discussed or statistical tasks addressed), this phase practically covered the design steps of the tasks intended for their students. In the 4th meeting, the researchers asked teachers to start the design of a task that they would implement in their classroom. No direction was given by the researchers about the number of the tasks that they could be designed and on how the teachers would collaborate in the design.

As discussed in the methodology section, Dinos and Marcos both had a leading role in the design phase, but the difference between them was that Marcos had pedagogical experiences related to STSI tools, while Dinos was familiar only with specialized statistical software. The two teachers made two different suggestions for the design and the teachers were separated into subgroups around Marcos and Dinos on their own initiative (see Table 3-3 in Chapter 3). In this process, the researchers did not intervene, as long as there was at least one practicing teacher in each subgroup.

Since the process of negotiation of meaning was dominant in this phase, I will discuss its results with respect to the processes of participation and reification as observed in the 25 episodes. Although these were not distinct, they each revealed different aspects of the process of negotiating meaning in this phase.

Participation: Challenging And Justifying Potential Uses Of STSI.

The exploration phase was characterized by a strong interaction among the participants. This helped in making transparent to the community various features, resources, and learning potentials related to STSI in DSTR practice. Since only

Marcos (and the researchers) were familiar with STSI tools, the other participants had a more peripheral role in the 25 episodes. Particularly, in most cases, the participants challenged an idea that had come from Marcos or the researchers. At times, though, they asked for clarifications and justifications. Usually, Marcos and the researchers were those who brought to the fore particular elements related to learning potentials and features or made explicit links among them.

An average of five participants contributed to the exploration phase episodes. Generally, the episodes where many of the participants were actively engaged in the discussion were those where the teachers negotiated the statistical content, namely a particular statistical task, concept, or process. The interaction among those with a strong background in statistics and those without it motivated the process of negotiation of meaning and supported forming a shared view and joint activity intended for the students.

The 2nd episode of the 5th meeting, as presented below, was the episode with the strongest interaction among participants, particularly, almost all teachers present in the meeting interacted while Marcos presented a PowerPoint file with resources and suggestions for subgroup A's design. Moreover, before this meeting, Marcos had sent an e-mail to the other members of subgroup A with a written description of his suggestions and relevant resources (e.g. a link with a video describing the suggested experiment). Here I present an extract where Lia asked Marcos to justify his suggestion regarding data simulations. This extract illustrates a rather typical form of interaction in the exploration phase.

Extract 1: from the 2nd episode of the 5th meeting (5th episode in the total of the 25 episodes)

Lia: Marcos, what do you mean by "simulation?" I have only met this word in computer games. In this case, you pretend to be a particular character, to drive a car or an airplane, this is a simulation. But what is simulation here?

Marcos: But in order to explain this, I need to explain something else first. Let's say that **we have 30 people divided into two groups.** The example I am presenting here is the same with the one presented in the video I sent you with the words with meaning and the words without meaning [he is referring to http://www.learner.org/courses/learningmath/data/session6/video.html#]. So, **the one group tries to recall words with meaning and the other group tries to recall words without meaning**, and *we note down the number of the words in each case*. Then I *calculate the mean* value for the one group, then the mean value for the other group and I subtract the two values. The point now is *to decide whether this difference is usual or unusual*. Is it expected or unexpected? If it is usual, there may be many *reasons for being usual*, maybe it is the way we divide the people into two groups, whether we select them at random or not, etc? So, the only characteristic that could affect the results is that in the one case, the words had meaning and in the other, they didn't. In such a case, the fact that a particular person recalled a particular number of words could not be affected by the fact that the words had meaning or not. So, *my <u>hypothesis</u> is that the ability to recall a particular number of words is irrelevant to the fact of whether the words have meaning or not.* So, if I were in the first group, with the list of words with meaning, and I recalled 15 words, I could achieve the same score even I have the list with the words without meaning. Here is where the <u>simulation</u> comes in. I start **to blend the <u>data</u>** from the two groups and separate them again at random and take again the difference between the mean value of the two groups. If **I repeat this process many, many times**, *how many times will I make a <u>difference</u> equal to or bigger than the initial difference*? This is the simulation. I repeat it many times, many, many times, and *I count the frequency that the initial difference appears*. In that way, *I can judge whether the initial difference is usual or unusual*.

Ria: And in which grade do you intend to **implement this experiment**?

Marcos: We can discuss this.[3]

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3 *<u>Reminder:</u>* In all the presented extracts we refer to the three elements by the following system: I **bolded** a **feature**, put *learning potentials* in *italics*, and <u>underlined resources.</u> If for example, a particular phrase is assigned to both features and resources then I note this by using **<u>both</u>** underlining and bolding.

As can be seen in the extract, Marcos was aware of the statistical concepts and processes related to the suggested experiment. However, it was Lia's question that mobilized the idea of computer simulation to be negotiated and prompted Marcos to make transparent to the other participants the underlying statistical processes and concepts. In this way STSI tools were explicitly linked to the "Feature: blend data sets," and the "Feature: repeat many times a random process." Moreover, links among features, resources and learning potentials became visible and were negotiated within the community, such as the "Learning potential: decide whether a difference in means is usual or unusual," which was explicitly linked to the "Resource: computer" and to the "Feature: repeat many times a random process." At the end of the extract, Ria's contribution acted as a starting point for a negotiation related to possible connections with the official curriculum.

The next extract is from the 8th meeting. This meeting followed the one where participants collected their own data and explored them in Fathom. In this episode, the participants of Subgroup A discussed on the design of a worksheet that would support the implementation of their task

Extract 2: from the 2nd episode of the 8th meeting (22nd episode in the total of the 25 episodes)

Researcher 1: The issue now is that Lidea, who was not here in the last meeting, has no particular experience with Fathom. I think it would be important for her to see *how simulations work in Fathom*. What do you think Lia? How was your experience?

Lidea: I have heard a lot in this group about it but never seen live how it really works.

6	Lia:	It is so important <u>to have experience with simulations in Fathom</u>. At least for me, it was essential. In the last meeting, I really felt that <u>*I realized a lot about how simulations work*</u>. It is very important.
7	Researcher 1:	It is also important because this experience would help you to understand better what is going to happen inside the classroom.4

As we can see in this extract, the fact that, in the previous meeting, Marcos used Fathom and explained to his colleagues the steps and the rationale behind the actions he made, helped Lia develop an understanding of what is data simulation and how this can be done in Fathom. Similar understandings were also admitted to being developed by other participants like Athina, Eva, and Ria, some of them explicitly in groups' discussion while others during the final interviews.

The next extracts reveal issues that aim to illuminate both the development of links among resources, features and learning potentials, as well as, the interactions between researchers and teachers during the negotiation of meaning in the exploration phase.

The extract 3 presented below, occurred during the seventh meeting where the group discussed the learning potentials of the task they were designed. In this extract, the discussion is about Dinos' idea to engage students in a full PPDAC cycle where the main activity would be data explorations with different samples.

4 <u>*Reminder:*</u> In all the presented extracts we refer to the three elements by the following system: I **bolded** a **feature**, put *learning potentials* in *italics,* and <u>underlined resources.</u> If for example, a particular phrase is assigned to both features and resources then I note this by using <u>**both**</u> underlining and bolding.

Dinos insisted that students needed to experiment with a small population where they could take all possible samples of a particular size, for example, to take all sample of size 3 in a population of 5 elements.

<u>Extract 3:</u> from the 1st episode of the 7th meeting (the 16th episode of the 25 episodes)

1 Dinos: *My goal is that the students will be able to compare their*
2 *results with the actual values of the population*, so **this won't be something that will remain in the air** [he
3 means the deviation between a sample and the
4 population]. Now, since **<u>we have the opportunity to use this software</u>** [he means Fathom], we don't need to take
5 only one sample, we can take several samples so that *the*
6 *students can observe that in some of them we are very close and in others, we are further away*.

7 Researcher 1: And this is not a learning goal on its own [she means to assist students in understanding variability through exploring sampling distributions]?.

8 Dinos: No, because *it is not sufficient mathematically*. I mean
9 <u>how this number comes out</u> [he means the mean of a specific sample]? How? Otherwise, *this is a "black box."*

(some minutes later)

6 Athina: Have other researchers worked on similar problems? Can we read something relevant?

As we can see, Dinos appreciated the use of the software (F.) in helping students to develop distributional reasoning (L.P.) and a deeper understanding about samples (L.P.), but in contrast, he doubted that students' engagement with such a tool could adequately provide rich experience for the underlying mathematical and statistical learning potentials. Dinos used his statistical and mathematical knowledge (H.R), and his professional knowledge (H.R.) to focus on mathematical concepts. He also emphasized the importance of students' understanding of the underlying mathematical relationships (L.P.) and he recommended students' experimentation with physical tools (F.) and the limited role of STSI in the teaching.

As regards the contribution of the researchers, the researcher here tried to support Dinos in making his learning goals clearer and be more focused on the tasks' learning potentials. The episode which included this extract lasted about an hour and in it, the participants negotiated stochastic and mathematical aspects of their design and shared ideas of how they could engage students in inquiring statistical and mathematical concepts. The researchers contributed to this negotiation by encouraging participants to acknowledge different aspects of the task, make their views transparent to their colleagues, and become more focused on the task's potentiality. Lastly, as we can see at the end of the episode, Athina called for relevant material in the literature. Generally, the researchers contributed with various resources whenever participants invited us to, or we judged that it was important for the members. For example, we brought particular research papers, or prepared PowerPoint files of curricular material or research results (M.R.). An additional issue in Athinas' contribution was that she explicitly considered the group as a group of researchers rather than a group of teachers. Particularly, from her words seems that she considers the enterprise of the community to concentrate on exploring the teaching than the actual teaching.

In another instance of the same episode (see extract 3), Dinos challenged Researcher 1's suggestion to engage students with data exploration in Fathom and motivated a debate regarding a paper and pencil activity as opposed to one based on digital tools. Particularly, Dinos had a difficulty to link the use of Fathom with specific learning potentials and he seemed to trust a paper and pencil process in order to illustrate the notion of sampling to students. In subgroup B, unlike Subgroup A, the technological background of participants seemed to play a crucial role in their unwillingness to engage with Fathom. Actually, only Athina was open to engage with the simulation process and contribute to the design of engaging students with Fathom, while the others seemed to rely on Dinos to lead the activity on Fathom. The uncertainty of the participants in Subgroup B is quite apparent in the next extract:

<u>**Extract 4:**</u> from the 2nd episode of the 9th meeting (the last episode of the 25 episodes) –Dinos was absent in this meeting

Athina: Now we need to proceed in the design of how we will use Fathom, what we are going to present to students and what we will ask them. But how, I mean, we need to familiarize with Fathom first

Sofi: <u>Let’ s leave this part to Dinos</u>. He is already more familiar with this tool.

Cloe: I don’ t feel that I can work on this right now.

Sofi: Let’ s leave it for the next time that Dinos will be with us, we need his guidance anyway. [5]

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5 <u>*Reminder:*</u> In all the presented extracts I refer to the three elements by the following system: I **bolded** a **feature**, put *learning potentials* in *italics,* and <u>underlined resources.</u> If for example, a particular phrase is assigned to both features and resources then I note this by using <u>**both**</u> underlining and bolding.

As can be seen, when focusing on the participation process, it was mainly the participants with strong backgrounds in statistics who brought to the fore various teaching resources (e.g. specific suggestions, teaching examples, central concepts/processes). However, it was a component of the interaction among all, and especially those who were unfamiliar with the negotiated content, that helped achieve transparency and the sharing of these elements and links within the community. This process gradually led to a consensus for the joint enterprise.

Reification: A Continuous Interplay Among Potential Features, Learning Potentials, And Resources.

During the exploration phase, the participants had the opportunity to interact with STSI tools in many ways and be engaged with several activities, such as: negotiation of tasks aimed for their students; worksheet design; exploration of particular concepts/ideas; engagement in a pilot study where they collected their own data; and use of Fathom to analyze and simulate data they produced inside the community. In all these activities, the teachers produced several objects/ideas that were either directly linked to the design of teaching (e.g. the final worksheets they designed) or acted as a starting point for a new negotiation of meaning. Despite the numerous elements that became a point of focus in the discussions during the exploration phase, in Table 4-17 I summarize the dominant elements linked to the use of Fathom as being negotiated, shared, and agreed when the two subgroups completed their design for the classroom.

Table 4-17 – Reified elements composing the CoP's repertoire in the exploration phase

	Role of Fathom	**Dominant feature**	**Dominant learning potentials**	**Dominant resources**

The task designed by Subgroup A	Determinant in the progress of the task	Students' direct engagement with data exploration in Fathom	• Connect to probability ideas • Engage with a modelling activity • Engage with data simulation • Familiarize with hypothesis testing procedure	• List of words • Classroom data • Simulated data • Difference in means • Probability • Visualization tools • Relative frequency • Guidelines for using Fathom • Worksheet for students
The task designed by Subgroup B	Supportive in the progress of the task	The teacher presents to students data distributions	• Connect to probability ideas • Verify conjectures • Conflict with the idea of proportionality • Find all possible combinations • Familiarize with the idea of typical statistical error	• School population survey • Mean value • Population • Sample • Combinations • Probability • Visualization tools • Sampling distribution • Media extracts • Worksheet for students

Next, I refer to some examples from the 7th meeting, which was crucial for both subgroups' designs of the classroom tasks for students. Particularly, subgroup A did a pilot study of the experiment aimed at its students, namely, to answer statistically whether listening or not to music can affect their capability to recall words. They collected data from the 11 participants in this meeting, then analyzed the data in Fathom and produced simulated data to test the hypothesis that "listening to music" does not affect their capability to recall words. This pilot study was catalytic for teachers in deciding what task to give to their students. Moreover, the fact that in this meeting the teachers had the opportunity to interact directly with Fathom and explore various tools in the software was also instrumental in helping them understand the possibilities of the software and the process of data simulation they had discussed extensively in previous meetings. The extract below is from the plenary discussion regarding the results from the data simulation in

Fathom and gives a picture of the plethora of reifications negotiated during the exploration phase.

Extract 5: from the 3rd episode of the 7th meeting (18th episode in the total of the 25 episodes)

Marcos:	There can be various parameters that result in an insignificant difference. So, I need to *check all the parameters if I want to conclude something* if listening to music has an effect or not with some confidence. If I get in the classroom the same results we got here, and eventually, we do as excellently as we did the experiment here, I am not really sure what I would tell my students. I mean *if the difference is significant, it can be easily didactically exploitable.* But now that the difference is so small, we need to ***discuss things that have to do with the design of the experiment****, things that are not mathematics.*
Researcher 1:	What difference would you consider as significant?
Dinos:	Let's say 3 for example, or so?
Marcos:	In that case, it would be meaningful didactically, I could **use the concept of *relative frequency to approach the probability***, to see *how we could calculate the probability of this difference to appear,* etc. But now it is pointless, I mean whether I choose mean values or medians, the difference is so often that it is pointless *to explore combinations that can give the same difference or a bigger one*. An alternative problem is *to explore words that have*

meaning and words that don't have meaning. In such a question, the difference is expected to be significant but in this case, you have other problems. I mean in this case, it *is difficult to convince students about the aim of the study*. Do we need mathematics to prove that the words that do not have meaning are more difficult to be recalled? I mean it is so obvious that you can't *support the rationale for the study*.

Researcher 1: But when your sample size is small there is always the risk that *the results will be out of your expectations*. I mean maybe you need to reconsider Athina's suggestion and **take a bigger sample, from the whole school, for a web database, if there is one, and then explore your data set in the classroom.**

Researcher 2: Do you think you can find data sets for this problem?

Marcos: I don't know, I need to search on the internet to see. Maybe we also need to consider that there is a possibility that the music actually doesn't affect our capability to recall words.

Researcher 1: In any case, *such a small sample always has great restrictions when you want to make conclusions.* But is it so important for the goals of the activity to **collect the data in the classroom**? I mean the students could **collect the data in the schoolyard**, **asking all the students of the school and then analyze them in the classroom**. Is it crucial to **collect the data from one classroom**?

Marcos: The thing is that the situation in Lyceum is rather difficult.

35 You need to deal with the national exams, I mean the last months of the school year, the students come to school only
36 occasionally, the population of the school is getting smaller and smaller, especially in the last grade level.

As we can see in this extract, the results (reified objects) from the pilot study conducted by the teachers, motivated negotiation regarding the design of the experiment, the involved statistical parameters, and possible modifications that could be considered. Moreover, we can also see a strong interplay among resources, features, and learning potentials in this extract although not directly linked to the actual teaching. More specifically, we can see that the negotiation of the produced data brought to the fore links among features, learning potentials, and resources (see for example Lines 11-22) which, by data explorations in Fathom, lead to new reifications, namely the alternative problem (see lines 16-17) which lead finally to the problem teachers actually gave to the students. This extract illustrates, on the one hand, the strong interplay among various elements of teaching, and on the other hand, how their own data explorations with STSI resulted in various modifications on their design (reconsideration of the central question, reconsideration of the sample size, of the data collection process, etc).

Similarly, the 7th meeting was also crucial in subgroup B's design. Particularly, this group had decided in the previous meeting to ask their 8th-grade students to answer statistically, "How much money on average do the school students spend in the school canteen per week?" The main statistical idea they wanted to link to this problem was an intuitive approach to the typical statistical error and how this is connected to the concept of probability. In this meeting, Dinos defended strongly the argument that students need to explore the idea of sampling with paper and pencil in a small population because if they do not take all the combinations they will not be able to connect the concept of probability with the typical statistical error. The extract displayed below is from the plenary discussion where the participants negotiated a paper and pencil, or a more technologically oriented activity for the students. This extract aims to show teachers' reliance on manipulatives when they focus on illustrating probabilistic ideas instead of using digital resources, something that was quite dominant during the exploration phase, generally, and particularly among the members of subgroup B.

Extract 6: from the 1st episode of the 7th meeting (16th episode in the total of the 25 episodes)

Dinos:	Here we want some help.
Researcher 1:	It seems somehow difficult for *students to manage two different contexts*. Of course, Dinos made a strong point when he said that 8th-grade *students cannot understand the combinations behind unless they* ***calculate the various combinations themselves***. What I suggest is to *take three different sample sizes and produce as many samples as they can for each size*, so that, **by the help of the software**, *they will gradually produce the mean distribution for each case*.
Dinos:	This is a good idea.
Researcher 1:	The aim then would be *to calculate the probability of being in a particular range of values each time*.
Dinos:	Ok, maybe this *can be visible to students*. Of course, I don't know the possibilities of the software, what we can do and how this can be seen in the software.
Researcher 1:	Since Marcos already uses Fathom now, he could probably show us tools that can assist this aim too.
Marcos:	This is not difficult. *You can choose a sample of size 10, for example, for each sample the software keeps the mean value. You can repeat the process, let's say 1000 times, and then you have the distribution of the 1000 means*. We

can do it now. I will show you next.

Researcher 1: So, you can find how many <u>means</u> are <u>within particular limits</u>.

Marcos: Yes, this is not difficult to find.

Researcher 1: But, what Dinos insists on, is that using the software, *they will not understand what we mean when we say <u>1000 samples of size 10</u>, <u>the different combinations</u> behind.*

Dinos: Exactly. When you take <u>10 values</u>, and then 10 more, and 10 more, how many such <u>sets</u> exist?

Researcher 1: Dinos, what if they start to **take <u>some samples</u> by hand** and then **they find the others in the software**?

Dinos: I don't know. I mean OK, *but they won't understand why the probability is as it is.*

Athina: Yes that is the point, *I mean OK they would choose some samples but then what*? **Then it will be the software that will take the others**.

As we can see in the extract above, it is Researcher 1 who made Fathom a point of focus for the subgroup B, and Dinos' insistence on giving students opportunities to find all possible combinations motivated the negotiation of whether it would be better for students to explore the central statistical ideas with paper and pencil, or with software tools. Moreover, we can see that Athina supports the Dinos'view (lines 30-31). This negotiation brought to the fore various resources, features and learning potentials and it was key in teachers' decision to engage students with both paper and pencil and digital explorations. Moreover, this extract indicates that throughout the exploration phase, there were several suggestions and links among resources, features, and learning potentials that became a point of focus for the community (see for examples researchers' suggestion in lines 2-10), but didn't necessarily influence teachers' final design.

To sum up, during the exploration phase the teachers had the opportunity to explore STSI tools in many ways. In some episodes, they talked about using STSI tools (e.g. in the 1st extract), other times they used a particular STSI tool and explored its possibilities (e.g. in the 4th extract) and other times they doubted the effectiveness of STSI tools in developing students' understanding (e.g. in the 5th extract). In all cases, there were many possibilities for negotiating meaning around the use of STSI tools in teaching and learning statistics and for developing links among the three elements of DSTR practice with respect to such tools. Moreover, STSI tools sometimes acted as the motive (e.g. in the case when Marcos wanted to illustrate the idea of doing data simulation using computer tools), and in other times as the means (e.g. when the teachers used Fathom to analyze the data they collected in the pilot study they conducted) for teachers to negotiate the meaning regarding statistical concepts or processes. In this way, they also had the opportunity to deepen statistical content and develop statistical tools (resources). This negotiation of meaning led to several links among the developed tools, features, and learning potentials that were also shared inside the community, although not always explicitly connected to the actual practice (e.g. 4th extract, lines 21-23). The continuous interplay among various resources, features, and learning potentials, as well as among links between them, led to the design of the two particular tasks (resources) teachers implemented in the classroom, in both of which Fathom had a specific role while linked to different features and learning potentials (see Table 4-17).

4.3.3. The Immersion Phase In The Re-Sourcing Process

The immersion phase occurred when STSI tools, and in particular Fathom software, were immersed in the actual learning environment and the participants had the opportunity to use them in the classroom and gain experience from their impact on students' learning. This phase contained four episodes from the tenth meeting where the teachers reflected on what they had done inside the classroom, and the last episode is from a further meeting from the discussion of Marcos' implementation.

During the implementation, the members had the opportunity to use in the classroom the tasks they had designed, and to observe students' reactions and difficulties. In this phase, the negotiation of meaning was mainly with regard to limitations and potentialities that seemed to be crucial in the students' learning. Similarly to the exploration phase, we will discuss the results of this phase with respect to the processes of participation and reification as observed in the five episodes.

Participation: Expressing Complementary Views Of Shared Experiences.

During the exploration phase, all the participants explored Fathom as a potential teaching tool and reached a consensus of how they would use it and why. In the immersion phase, the task designed by subgroup A was implemented in Marcos and Kimon's 12th-grade classrooms. Ria and Lia participated as observers of Marcos' implementation, Kimon video-recorded his lesson and shared it with the other members of subgroup A, while Lidea did not participate in the implementation. The task designed by subgroup B was implemented only in Dinos' 8th and 9th-grade classrooms and all the other members of this group were observers during the two implementations.

In this phase, we can see that the participants who shared the same experience regarding the implementation actually built on each other's words. The participation process here revealed more a consensus and less a debate on the issues that emerged during the implementation. Generally, this phase was characterized by a degree of complementarity in participants' views and observations.

Extract 7 below is indicative for illustrating this consensus. This extract is from the reflection of subgroup B on their implementation, and as we can see, four of the five members of subgroup B contributed to the discussion, adding and completing each other's words.

Extract 7: from the 1st episode of the 10th meeting

Dinos: The mean of the census data was 2.47. Ok, we had one column for the census data and in the column, on the right, we were *taking samples of 150 and calculated the mean.* Then I asked them to *note whether the result was between 2.47±0.3 or not. Sometimes it was between and there were times that the result was smaller than 2.17 or bigger than 2.77*. Then I told them, let's take 100 such results.

Sofi: Yes, and then we suggested to *take 1000*, and some students said, "How we are supposed to find all those samples?"

Dinos: And *we took 1000 samples* and I asked them *how many results are out*. And then I *created the diagram*. When **they saw the distribution they understood**.

Sofi: Someone said, "What is this mountain?" (laughs)

Athina: I was impressed because the students seemed quite familiar with the software. I mean, if I were in their shoes I would definitely have asked, "What is this thing doing now?", but they seemed OK. Were they indeed familiar or did they appear so because they trust you? (She addresses her question to Dinos).

Dinos They thought that OK, this is what this tool is doing. I had already told them that this *gives you random samples of 150*, yes we have discussed this from the very start, and then we *took the means of these samples* and then we put the results on the diagram.

... (some minutes later)

Athina: And they looked generally quite familiar with the stochastic context as well, at least much more than we actually are.

Dinos: This is true, this was impressive even for me. For example, the *data entered in the software* was entirely students’ responsibility and without being told or asked, they actually *did data cleaning* on their own.

Marcos: Can I add something? I was watching my students [12th grade] and I realized that they were very familiar with many probabilistic concepts. Of course, this is also due to the computer games they play. You see, many of these games work in an uncertain environment or you are told that *there is a certain probability for an object to be found*.

Chloe: Imagine this generation’s motivation. Now things are completely different, what they see, what they hear, the access on the internet, it is completely different.

As we can see, while Dinos was describing the process of the implementation, the other members of subgroup B, who were observers during the implementation, completed his words with examples of students’ reactions. Even in the case of Marcos, who was not in the same subgroup with the others in the design and implementation of the task, his contribution aimed at enhancing Athina’s argument that students seemed very familiar with the stochastic context (lines 21-22). Furthermore, we can see that participants based their words mainly on students’ attitudes and reactions (e.g. lines 12-16 and 26-30), as well as on STSI tools’ potentialities that seemed to have a strong impact on the learning process (e.g. line 10 for the power of the visualization). Particularly, students and STSI potentialities constituted two of the dominant resources that came into play during the immersion phase.

Reification: Developing Awareness Of The Links Among Features, Learning Potentials and resources.

The resources of the community that was central in the phase of immersion were the specific resources that have been negotiated and accepted by the participants in the previous phase, which were now reconsidered and subjected to possible modifications. The main elements were already defined during the exploration phase (see Table 4-17), however, the implementation of the task gave rise to interactions with new resources (e.g. the school context, the students' habits/reactions, management issues etc) that motivated the emergence of new elements or links among elements already reified. During the immersion phase, both participation and reification processes are characterized by complementarity and explicitness that was not so obvious in the previous phases. Table 4-18 displays the dominant elements linked to the use of STSI after teachers' experience from using Fathom in their teaching.

Table 4-18 – Reified elements composing the CoP's repertoire in the immersion phase

	Role of Fathom	**Dominant features**	**Dominant learning potentials**	**Dominant resources**
Subgroup A **Reflection on the task**	Determinant in the progress of the task	Students' direct engagement with data exploration in Fathom	• Connect to probability ideas • Familiarize with the environment of Fathom software • Explore different parameters of the problem	• Students' stance and habits • Stochastic parameters (e.g. results of simulated data/ impact of the number of words in the selected lists etc)

				• STSI
Subgroup B **Reflection on the task**	Determinant in the progress of the task	Students' direct engagement with data exploration in Fathom	• Connect to probability ideas • Develop distributional reasoning	• Students' stance and habits • Learning management issues • Classroom reality • STSI

Extract 8 is an illustrative example of the emergence of new elements and links in already reified elements.

Extract 8: from the 5th episode of the 10th meeting – discussion on Subgroup B

Dinos: I am very positive about the idea of ***students doing their own data explorations***. When you *engage them with sampling, they need to take samples on their own*. We could omit **the paper and pencil task** of *finding all possible samples of size 3 from a population of 5* and instead of this, we could **engage them somehow with technology**, to *allow them to make data explorations*. These explorations cannot be made without technology, technology is a prerequisite, students can **work in groups of 2-3 with a computer**.

Researcher 2: Do you mean to ***explore data in Fathom***?

Dinos: I mean Fathom or some relevant software. I liked Fathom, I think it is very useful for students. But OK, **when you present in Fathom**, you can give them things, but, you know, this is 30% of what you expect them to take. The thing is ***to explore the data* on their own**, *to take the samples*, to *explore, to observe*, but this… this requires time. But anyway, I prefer two **such activities from 2-3 weeks of traditional teaching**. I mean OK, you give them the formula for the probability, you also give them some applications from the textbook, and then what? How *can these ideas be connected*?

More specifically, as we can see in lines 10-18, Dinos linked directly the feature "Engage students in data explorations using Fathom" with the learning potential "Connect statistical investigations with probability". Although both elements have been observed in other phases, it is the first time, here in their reflection, that there is a direct link between them highlighting the importance of STSI tools in supporting both the learning of statistics and probability. Furthermore, although Dinos insisted very much during the exploration phase that students explore small populations with paper and pencil, in his reflection we see that he appreciated more the utilization of STSI tools (F) (line 1-8; extract 7) and he emphasized the importance of students making their own data explorations (L.P.). This was a point of agreement for all the members of subgroup B. As Dinos argued later, by using small populations you may create a conflict for students regarding the proportional reasoning, but it is STSI tools that can support the development of distributional reasoning. This was the first time someone from the community referred to the development of distributional reasoning.

Similarly, in subgroup A, in his reflection, Marcos highlighted the importance of students being familiar with the environment of the software they use (learning potential) since as he mentioned, "This is necessary in order for their activity to be more exploratory, to be able to take initiatives while they inquire. I mean it would be interesting if they could change on their own parameters and check what may happen in the results." This made visible new links between STSI and elements of the practice.

To sum up, we can say that the immersion phase was the phase that proved to be crucial in forming shared experiences from the actual practice in the community.

As regards STSI tools, we saw that teachers appreciated more its potentialities and linked it stronger with specific elements of the practice, and in particular with the aim to connect statistical and probabilistic notions in the teaching of statistics. Furthermore, the immersion of the STSI tools in the classroom reality gave particular insights into time management issues; ways of putting students into groups; communication norms; collaboration with teachers of other school subjects and those in leadership positions; school organization; and ethical issues. These experiences and insights also provided new meanings to the use of STSI in statistics teaching and revealed limitations connected to the particularities of the Greek educational context. Finally, participants who did not take part in the implementation and did not engage much with Fathom maintained a peripheral role in the discussion regarding reflections of the use of STSI in the teaching.

4.3.4. Summarizing The Results Of The Third Level

In this level of the data analysis, we aimed to deepen in the re-sourcing process of teaching statistics for developing statistical reasoning and thinking with respect to software tools for statistics instruction. Particularly, through the lens of the communities of practice theory (Wenger, 1998), we focused on the process of the negotiation of meaning through the duality of participation and reification with respect to STSI. This focus helped us gain insight into the particularities and characteristics of each phase in terms of who and what interacts with the resource (participation), as well as to why and how this interaction was realized in relation to the CoP practice (reifications).

As we have shown in the results, the fact that STSI became available to the members of the community in the emergence phase did not imply members' awareness regarding its potentiality or further links to other elements of teaching. In this phase, we could not identify any interaction among the various dimensions of the DSTR teaching scheme (Figure 2-11) regarding the use of STSI in the learning process. In contrast, during the exploration phase, there was a remarkable interplay among potential resources, features, and learning potentials connected to the use of STSI. An important aspect that influenced this strong interplay, as illustrated in the process of participation with STSI, was the collaboration among the teachers and the interplay of various human resources (e.g. statistical knowledge, everyday experiences, teaching experiences, familiarity with STSI) which motivated powerful challenges and justification of how STSI can be utilized in the practice. Moreover, the collective context in teachers’ work accentuated various elements and links in the triangle of DSTR practice (Figure 2-11) and led to a significant production of teachers’ own reifications (pilot data, drawings, Fathom files, draft worksheets). In this phase, all the participants offered rich possibilities for the negotiation of meaning with regard to STSI tools, either by sharing their content or technological knowledge (e.g. Marcos and Dinos) or by sharing their need to understand and learn more about the content and technology of STSI (e.g. other teachers). However, the process of negotiation of meaning seemed to develop differently during the immersion phase. This phase was characterized by less interaction among the teachers, which indicated a consensus regarding their shared experience of using STSI in the classroom. Although teachers’ interactions and emerging elements of the triangle of Figure 2-11, were not as rich as in the exploration phase, the experience of STSI use in the classroom helped teachers delve more deeply into the links among features, learning potentials, and resources and be more aware of STSI potentialities in DSTR practice. Moreover, it helped them in modifying the productions of the previous phase and make new productions that they considered more effective in the learning process (e.g. handouts, written guidelines for Fathom, etc).

5. Reflections On The Study

5.1. Discussion

The research questions that guided our study as presented in Chapter 3 are the following:

1) *What elements of the DSTR teaching were revealed and negotiated in the formation of the community's practice, with respect to the various professional development tasks in which the mathematics teachers were engaged?*
2) *Focusing on the resource of STSI tools, what phases seem to characterize the re-sourcing process inside the community?*
3) *How does the secondary mathematics teachers' CoP develop the DSTR practice through negotiating meaning with respect to STSI tools? How is the process of negotiation of meaning formed within the various phases of the re-sourcing process?*

Next, we will discuss how the findings of our study respond to these questions and also how they contribute to the general field of research regarding the teaching and the learning of statistics with a special emphasis on secondary teachers' professional development.

5.1.1. Responding To The First Research Question

Our first research question was:

> What elements of the DSTR teaching were revealed and negotiated in the formation of the community's practice, with respect to the various professional development tasks in which the mathematics teachers were engaged?

In the theoretical section of this dissertation, we reviewed various elements of the DSTR as have been reflected and discussed in the existing literature. Particularly, we discussed on the specific nature and the particularities of statistical activity and statistical thinking as well as the role of chance and uncertainty in the current guidelines for teaching and learning statistics (NCTM, 2000; MoE, 2007; Ontario, 2008; ACARA, 2008; New School, 2011). The reform movements in secondary mathematics curriculum form a new ground for the statistics teaching practice (Burril & Biehler) providing new resources which aim to support new learning goals and being facilitated by innovative teaching strategies within a DSTR learning environment.

In our conceptualization for the DSTR practice, we see the development of teaching as a shared history of learning in a CoP context. The development in this manner reflects the results of mutual engagement in negotiating what can be utilized to facilitate the development of statistics learning, why this learning is important and how this learning can be supported inside mathematics classroom. We see the DSTR practice in a dynamic triangle which connects three core elements of the teaching practice, namely the resources (what), the learning potentials (why) and the features (how) of the teaching in a DSTR learning environment (see Figure 2-11).

The open coding process helped us go deeper in these three elements of the teaching and get insight on their emergence and development during teacher's collaboration within a CoP and among various PD tasks. Particularly, we show the richness and ubiquity of resources in the formation of the DSTR practice and how the appearance of features is enhanced while we move to tasks closer connection with the classroom reality. Focusing on these elements of the DSTR practice, except their intensity in the various PD tasks, we also identified different categories of codes that characterize aspects of each element that influence the formation of the DSTR practice.

More specifically, the findings regarding the resources revealed both the abundance, in terms of differences in nature as well as in the frequency of appearance within the various PD tasks, and they also highlighted the specificities related to the statistical content. For example, the appearance of resources such as real data, simulated data, results of randomization processes, STSI tools or media extracts, which were very frequent in the formation of DSTR practice (see Table 4-

6), constitute innovative and challenging resources for mathematics teachers that are closely connected to the content of statistics. The important role of such resources have been highlighted by many researchers for their potentiality as teaching tools and their effectiveness in facilitating students' understandings of statistical ideas (MacGillivray & Pereira-Mendoza, 2011; Stohl Lee, Angotti, & Tarr, 2010; Merriman, 2006; Pfannkuch, 2011; Fitzalen & Watson, 2014). Our research helped further to associate these resources with particular PD tasks as well as with other resources. For example, we saw that in most PD tasks these resources were mainly associated with human resources and in particular to teachers' knowledge of statistical tools. However, during the design task, resources from various material and cultural categories came to play as well as human resources more related to the classroom reality (e.g. teaching experiences, knowledge for students, etc). Moreover, apart from resources closely related to the content of statistics, our study revealed the influential role of cultural resources (e.g. everyday use of statistical terms, educational policy, accessibility to data sources, classroom norms, etc), especially when teachers work in statistical inquiry tasks or in designing for statistical inquiry. Such resources, although have been acknowledged as important factors for statistics teaching (e.g. Wild & Pfannkuch, 1999), they have not been a point of focus among empirical studies. Similarly, empirical studies often focus on human resources related to aspects of knowledge and intuitions. But our study goes beyond that to provide evidence of how previous experiences from everyday life or knowledge about students' interest and everyday habits influence the formation of teachers' practices. The prominent role of the human resources in the formation of the DSTR practice is rather obvious in Table 4-6, where we can see that the appearance of codes related to human resources exceeded in frequency the codes related to statistical objects.

The second point of focus in the open coding process was learning potentials related to DTSR practice. Learning potentials refer to knowledge, skills, and competencies that are expected to be acquired in a DSTR learning environment.

This element of DSTR practice is widely discussed and explored in statistics education literature in terms of curriculum guidelines as well as of cognitive demands and students' difficulties that are associated with these learning potentials. Many of the emerged categories in our findings are also emphasized and studied by other statistics education researchers and curriculum designers. For example, the most central categories appeared in all PD tasks, are related to support students in developing a deeper understanding of statistical inquiry and inference as well as a deeper understanding of the conceptual connections between statistics and probability. As shown in Table 4-7 of the results and also discussed in the theoretical sections of this thesis (see section 2.4), these categories are formed by learning potentials that are widely highlighted in the statistics education literature (e.g.. compare or explore data distributions, Ben-Zvi, 2004; interpret results of statistical investigations, Watson 1997; interpret data graphs, Friel, Cursio & Bright, 2001; understand the meaning of a random sample, Lee, Starling & Gonzales, 2014). Due to their importance in the statistics education community, the emergence of these learning potentials was part of the researchers' design of the PD tasks and expected to be dominant in some extent. However, the discussions during the design phase and especially in the reflection phase indicate that these learning potentials are also viewed as important by the teachers themselves in the development of the CoP's practice. Moreover, some learning potentials that were out of the researchers' design were brought to the fore through the teachers' collaboration in the design and reflection phases. Particularly, we saw that students' familiarity with the environment of STSI tools as well as their comfort with manipulations related to probability and combinatorics were discussed and acknowledged by the teachers in both design and reflection.

Features constituted the third point of focus in our open coding analysis. As mentioned in section 2.4.5 "the features constitute particular teaching approaches, strategies, and means that shape the conditions within which the students are provided with the resources chosen by the teachers to achieve the defined learning potentials". The findings reveal seven categories of features among teachers' discussions. The frequency of occurrence of each category indicates a strong appearance of this element of DSTR practice during the design phase. Moreover, there was no particular feature that seemed to dominate the teachers' negotiations of meaning in the various PD tasks (see Figure 4-5) and about half of the emerging codes of features appeared during the design phase. During the design, the dominant categories, in terms of frequency of occurrence, were those most closely related to the defined learning potentials (e.g. engaging students with data simulations or encouraging the development of particular types of reasoning), and in this task, the frequencies of Features and Learning Potentials differ only for 8%. This strong interaction between features and learning potentials was also apparent in the reflection phase. This is reasonable since, during these two PD tasks, the features emerged to respond to particular learning potentials that were defined by the teachers, while in the other PD tasks, the features were closer to the content of the task as been defined by the researchers. The particular features associated with the seven emerged categories are also discussed in the existing literature, and similarly to our study, these features are usually related to particular learning potentials and resources. For example, the importance of engaging students with dynamic software tools have been related with supporting students in data explorations and inference (e.g. Ben-Zvi,2006; Fitzalen & Watson, 2014), with facilitating students' reasoning and argumentation (e.g. Pfannkuch, 2011) or with the development of students' understandings for randomizations processes (e.g Stohl Lee, Angotti, & Tarr, 2010). However, since features represent the 'how' of the DSTR practice and they reflect the locality and the content of the community's joint enterprise, there is a number of features that are related to the classroom context and particular norms that stem from this context. For example, codes such as the didactical management of unexpected statistical results or building on students' previous experiences constitute some features that are context-oriented.

Last, the findings revealed the prominent role of STSI tools for both facilitating students' learning and teachers' design for statistical learning.

Giving a synopsis of all the above, the results from the open coding in the first level of the data analysis, brought to the fore codes and categories with respect to the three core elements of the DSTR teaching, in terms of frequencies of occurrence among teachers' discussions in general, as well as, in terms of teachers' discussions in the various PD tasks in particular. The findings of this level respond to our first research question and give us a deeper insight for both the potentiality of the various PD tasks for the teachers' professional development and, of the different sources that seem to provide meaning when teachers negotiate for the development of DTSR practice. However, these findings are quite narrow to provide substantial evidence on how DSTR practice is developed in the context of a CoP, how these three elements are associated and interact with each other, and how the process of negotiation of meaning forms and broadens the shared repertoire of the CoP. Such evidence constituted the aims of the following levels of our data analysis. Specifically, in the levels of analysis that followed this level, we concentrated on a resource that seemed to have a notable role in all the three elements of the DSTR practice, namely the STSI tools. This focus allowed for deeper analysis with respect to the process of negotiation of meaning and provided stronger evidence for the formation of the CoP's shared repertoire and the development of the DTSR practice.

5.1.2. Responding To The Second Research Question

As we discussed below, the findings from the first level of the data analysis gave us an overall picture of how the resources, learning potentials and features, regarding the DSTR practice, were discussed and developed through the various PD tasks. In order to study deeper the development of DSTR practice with respect to these elements, we focused on a particular resource, namely STSI tools, and we trace this resource throughout the data. We view the development of the DSTR practice through the process of the integration of the resource in the practice of the community, namely the re-sourcing process (Adler, 2000) with respect to this resource. In order to study deeply this re-sourcing process, we were first interested in possible phases that characterize this process. So our second research question was:

Focusing on the resource of STSI tools, what phases seem to characterize the re-sourcing process inside the community?

The trace of the STSI tools in the thirty-one relative episodes revealed three distinct phases that characterize the re-sourcing process of the resource in the community's practice. Particularly we show that this process passes from the emergence phase to the exploration phase which may lead to the immersion phase. Although the immersion phase constituted the ultimate phase in our data, the final interviews indicate that this phase may indicate the consolidation of the resource to the shared repertoire of the community and consequently the development of the community's practice. As we highlighted in the chapter of the results the re-sourcing process is not linear, and the passage from the one phase to the other is neither guaranteed nor inconvertible. Particularly, we saw that STSI tools followed a linear three-phase re-sourcing process that seemed to end up to a shared acceptance of STSI tools as a determinant for DSTR practice. However, this result stems from the prominent role of STSI in both teaching and learning statistics and this linear sequence is not characteristic for the re-sourcing process of other resources. Moreover, we saw that although the chronological sequence of the PD tasks seemed to influence the three phases, the phases in the re-sourcing process are actually independent of the tasks. More specifically, we could focus on a re-source that emerged through the design, while teachers were exploring various teaching resources but never immerse to the practice of the community (e.g. because the teachers decided not to utilize this resources).

Summarizing the results of the second level of the data analysis, we can say that the identification of the three phases, namely the emergence, the exploration, and the immersion phase, set light to the re-sourcing process while teachers work to develop their practice inside the CoP. Although many researchers have studied the integration of a resource in the mathematics teaching or the use in practice of curriculum materials (Stein & Lane, 1997; Remmilard, Gueudet &Truche, 2009; Rowland, 2009), the focus is either on the learning outcomes of the integration (e.g. Stein & Lane, 1997) or on the resource's use by individual teachers (e.g. Remmilard, Gueudet &Trouche, 2009; Rowland, 2009). The identification of the three phases in our study gives a picture of the path followed by the resource in order to consolidate to the practice of the community and reveals the complexities that are associated to this process. Apart from the collective dimension of these phases, the concentration to a resource which is specific to statistics teaching and learning, namely STSI tools, constitute an added value to the findings of this study. As we saw these three phases are distinct but they don't follow a linear or sequential path. On the contrary, the passage from the one phase to the other seemed to be spiral and repetitive. The fact that STSI tools passed all the phases and seemed to consolidate in the practice of the community is indicative of the importance of this resource for the teaching and learning of statistics. Our final aim was to study deeper the negotiation of meaning with respect to STSI tools in the three phases and how the community passes from the one phase to the other.

5.1.3. Responding To The Third Research Question

Our third research question aimed to provide a thorough insight into the development of the DSTR practice within the community of the 11 secondary teachers while they collaborated to deepen on teaching and learning issues. This development was viewed under the scope of the negotiation of the meaning process (Wenger, 1998) through the study of links among the three core elements of the DSTR practice. These links became transparent within the community, other times explicitly in the discussions and other times implicitly, while teachers collaborated to form the practice. The particular research question that guided this level of analysis was:

How does the secondary mathematics teachers' CoP develop the DSTR practice through negotiating meaning with respect to STSI tools? How is the process of negotiation of meaning formed within the various phases of the re-sourcing process?

Particularly, we focused on the process of the negotiation of meaning through the duality of participation and reification with respect to STSI, in order to go deeper in each of the three phases identified in the second level of analysis, namely emersion, exploration, and immersion. This focus helped us gain insight into the particularities and characteristics of each phase in terms of who and what interacts with the resource (participation), as well as to how this interaction was realized in relation to the CoP (reifications).

As we have shown in the results, the fact that STSI became available to the members of the community in the emersion phase does not imply members' awareness regarding its potentiality or further links to other elements of teaching. In this phase, we could not identify any interaction among the various dimensions of the DSTR teaching scheme and the majority of the members retained their initial views regarding the use of STSI in the learning process. In contrast, during the exploration phase, there was a remarkable interplay among potential resources, features, and learning potentials connected to the use of STSI. An important aspect that influenced this strong interplay, as illustrated in the process of participation with STSI, was the collaboration among the teachers and the interplay of various human resources (e.g. statistical knowledge, everyday experiences, teaching experiences, familiarity with STSI) which motivated powerful challenges and justification of how STSI can be utilized in the practice. Moreover, the collective context in teachers' work accentuated various elements and links in the triangle of

DSTR practice (Figure 2-11) and led to a significant production of teachers' own reifications (pilot data, drawings, Fathom files, draft worksheets). However, the process of negotiation of meaning seemed to develop differently during the immersion phase. This phase was characterized by a mild interaction among the teachers, which indicated a consensus regarding their shared experience of using STSI in the classroom. Although teachers' interactions and emerging elements of the triangle of Figure 2-11, were not as rich as in the exploration phase, the experience of STSI use in the classroom helped teachers delve more deeply into the links among features, learning potentials, and resources and be more aware of STSI potentialities in DSTR practice. Moreover, it helped them in modifying the productions of the previous phase and make new productions that they considered more effective in the learning process (e.g. handouts, written guidelines for Fathom, etc).

In the current literature for studying resources, we reviewed various approaches in our theoretical section. All perspectives that refer to human interactions with resources highlight the importance of the ways teachers use the resources in their practice and the meaning they attribute to resources. In particular, we discussed Remillard's (2005) view about interpretation of and participation with resources, moreover, Gueudet and Trouche (2009) provides the idea of the scheme of utilization to include the underlying interactions among resources, teaching goals, expectations, and teaching strategies, while Adler (2000) uses the term resources-in-practice-in-context to also include sociocultural interactions in teachers' work with resources. Our study acknowledged all these interactions in the inquiry of teachers' work with resources and aimed to provide a methodological path for the empirical investigations of these interactions. More specifically, our conceptualization for the DSTR practice (see Figure 2-11) and the systematic analysis of the links among learning potentials, features, and other resources in the trace of STSI, seemed to be promising in gaining insight on how these interactions contribute to the integration of the underlying resource in the DSTR teaching, and consequently to the development of this practice. Moreover, using the CoP as the unit of analysis and focusing on the process of negotiation of meaning helped us acknowledge socio-cultural aspects in the re-sourcing process. For example, the productions of the reification process reflect the particularities of the CoP we studied and the history of learning inside their community. Thus, they are directly connected to the members' work and consequently, reflect their own difficulties and the limitations of the context they develop their practice.

Concerning the particular role of STSI in DSTR practice, we show that it was difficult for teachers to appreciate the potentiality of such tools in their teaching. Actually, as the results indicate, despite the rich exploration of such tools during the exploration phase, the appreciation of STSI and their potentiality to support statistical learning came after the immersion of Fathom in the actual teaching. The use of such tools seemed to be related with teachers' statistical background, their familiarity with data explorations and statistical software (e.g. we show that for Marcos, it was easy to appreciate the use of Fathom, while for his colleague Lia, it was very difficult to understand how simulation tools can facilitate statistical content). Particular learning potentials were also key (e.g. we show that Dinos initially focused mainly on mathematical learning potentials and had a view to STSI more for their potentiality to support students' visualizations, while after using Fathom, he linked STSI directly with the development of distributional reasoning and appreciated more its potentiality as a teaching tool for DSTR). Additionally, we also show the potentiality of STSI to support teachers' design. For instance, we show that the subgroup A teachers used simulation tools to simulate the experiment they designed for their students, and they used the results from the simulation to reconsider and modify their design. Moreover, as can be seen in the immersion phase, the teachers become more aware that the connection between probability and statistics is essential in supporting students in developing statistical reasoning and thinking and they also greatly appreciated the role of STSI in this respect. The essential role of STSI in supporting the connection between statistics and probability, and consequently in DSTR, which was evident in our empirical study, has also been highlighted by many researchers who suggest the strengthening of probability content within statistics (e.g. Borovcnik, 2011; Biehler et al.,2013).

5.1.4. Summary Of The Findings

The three levels of data analysis respond to the three research questions that guided our study. The PD tasks of our study aimed at motivating negotiation of meaning regarding the DSTR practice as well as facilitating the mathematics teachers to develop this practice. The findings of the data analysis in all the three levels indicate the development course in this community and particularly in relation to our conceptualization of the DSTR practice as displayed in Figure 2-11 (see also below).

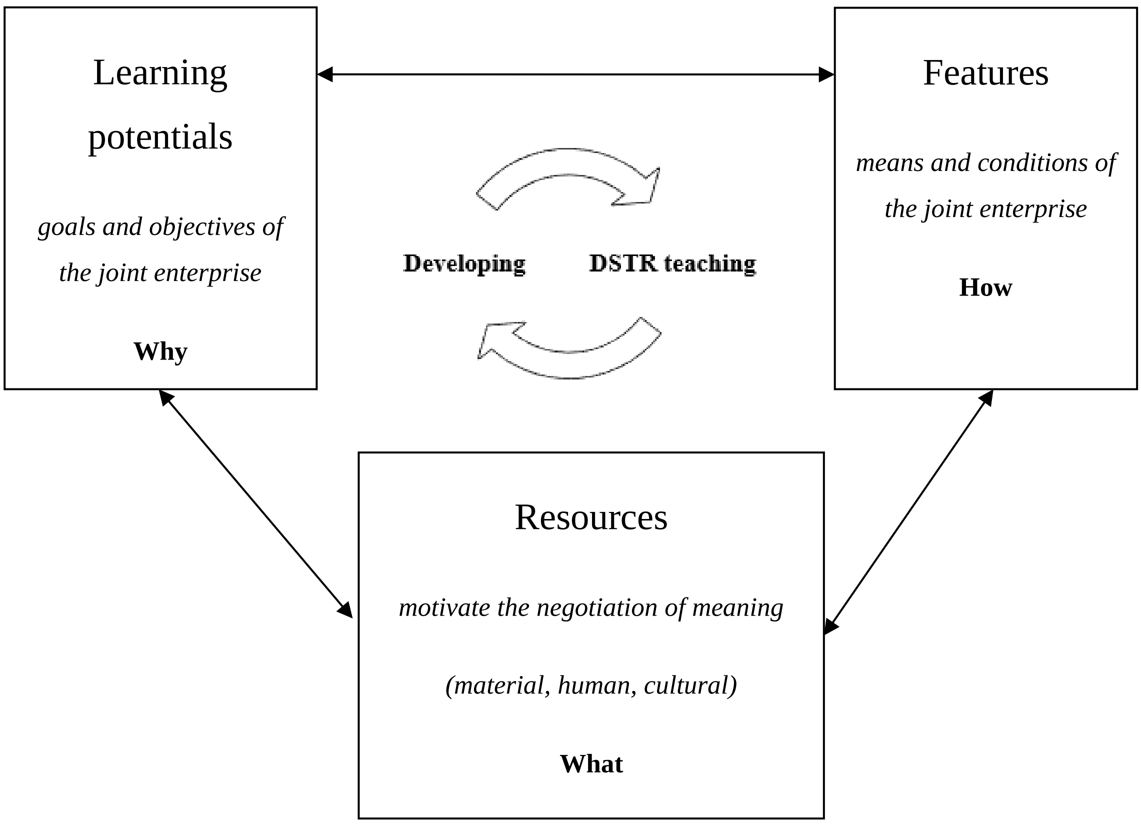

Figure 2-11 - The conceptualization of DSTR teaching scheme

Particularly, the findings of the first level focus on the appearance of the three elements in the various PD tasks, and provide an insight for both the potentiality of the various PD tasks for the teachers' professional development, and, of the different sources that seem to provide meaning when teachers negotiate for the development of DTSR practice. The focus in this level was in the three vertices of the triangle and the development is indicated by the intensity and the diversity of codes revealed in the various PD tasks. The second level of analysis aimed at relating the development of the DSTR practice with the dynamic character of the triangle, namely to the existence of phases that seem to characterize the links among the three vertices of the scheme. This level revealed three phases that seemed to characterize the developmental process of the teaching of the community regarding a particular resource, namely the STSI tools, that the first level of the analysis showed such tools to have a key role in the DSTR practice. In the final level of the data analysis, we built further on the idea of negotiation of meaning (Wenger, 1998) by searching for links among the vertices of the triangle. In particular, we viewed the process of participation in the members attempt to link the three elements of the dynamic triangle as they negotiated STSI tools, and the process of reification in the transparency of the emerging links, regarding the use of STSI tools, among the members of the community. The development of the practice in the findings of this level is apparent as we move from the one phase to the next and it is also informative for the particularities regarding the re-sourcing of the particular resource, namely the STSI tools.

5.2. Contribution Of The Study

In this thesis, I focused on the development of teaching statistics that assist learners in developing statistical thinking and reasoning (DSTR). This practice was studied in the context of a community of practice where eleven mathematics teachers collaborated in successive cycles of (a) exploring statistical tasks, (b) discussing educational material, (c) designing for their students, (d) implementing in the classroom and (e) reflecting on the implementation. The established community collaborated voluntarily for two academic years in exploring various resources that aim to support the DSTR practice. Our theoretical approach to this developing practice lays upon a dynamic triangle (see Figure 2-11) which capture three core elements of the teaching, namely the resources, the learning potentials, and the features and acknowledge links among them. The triangle of the DSTR practice is located in the heart of the community, and the interactions among the three core elements reflect the process of negotiation of meaning, in its duality of participation and reification. Particularly, the resources motivate the process of negotiation of meaning in terms of what is important to be achieved in the learning process and why, forming the goals of the community's enterprise (learning potentials), as well as, in terms of features that support the communally negotiated learning potentials. The contribution of the members of the community in linking the three elements of the dynamic triangle is related to the process of participation while the process of reification is located in the transparency and sharing of the emerging links. The practice develops by being re-sourced when new resources or new links become shared by the members of the community.

Our research approach to the development of DTSR practice includes three main characteristics that support the originality and novelty of this study. First, we investigated the development of statistics teaching in a collaborative context and in a long period of time. Such contexts are quite rare in statistics education literature and in the cases that they exist, they constitute short-term studies or parts of professional development courses (e.g. Visnovska, Cobb, & Dean, 2012; Zapata-Cardona (2014); Meletiou-Mavrotheris & Paparistodemou, 2013). The potentiality of such contexts for the development of teaching statistics and of teachers' professional development, has been highly recommended by many researchers and statistics educators (e.g. Shaughnessy, 2014; Eichler & Zapata-Cardona, 2016; Ponte & Noll, 2018), however the research in this field of inquiry still focus mainly on the professional development of individuals (e.g. Lee, et.al., 2016; Frischemeier and Biehler, 2014). The two academic-year collaboration of the eleven mathematics teachers, in combination with the volunteer character of their participation in the study, are indicative of the potentiality of the PD tasks that were developed in this study, on the one hand, and on the other hand, they constitute a rich source of empirical data which provide a deep insight on the formation of the DTSR practice.

The second characteristic is our theoretical conceptualization for the DSTR practice (see Figure 2-11). We locate the practice in a dynamic triangle which links three core elements of DTSR teaching, namely the resources, the learning potentials, and the features. The links among these three elements constitute a collective result produced by the community of secondary mathematics teachers while they negotiated the meaning of their teaching of statistics. This triangle seemed to be a useful tool in studying both the development of DTSR teaching and the complexities and particularities that are related to this practice.

The third characteristic is my methodological approach to the re-sourcing process. Particularly, in this work, a resource can be anything that can motivate the negotiation of meaning (a statistical tool, a shared experience, a curriculum material, a digital tool, the available time, the language, etc). In this sense, resources are located in the heart of the work of the community and they provide meaning and feedback to the formation of the community's practice. Viewing resources in this sociocultural perspective, we exceed our lens far from individuals to get insight on the collective work of the teachers with resources. The use of Wenger's concepts of participation and reification seemed to fit very well in analyzing empirical data of teachers' collaboration with respect to the re-sourcing process. Moreover, locating the participation and reification in the DTSR teaching scheme we viewed the re-sourcing process in the emerging links among the various elements of the scheme. These links helped us to gain access inside the scheme of the utilization of a resource (Gueudet & Trouche, 2009), contributing a methodological tool for the empirical study of this scheme in the field of the study of teaching resources in mathematics education.

Apart from the contribution of this study in terms of methodological tools, the investigation of the development of DSTR teaching revealed empirical evidence related to the specificities of the statistical content and of the role of STSI tools in the teaching and learning of this content. Particularly, the results from the first level of the data analysis formed a content for the three core elements of DTSR practice in relation to the various PD tasks, revealing the special role of cultural and human resources as well as of STSI tools. The results of the second level of analysis gave us insight into the re-sourcing process and on the importance of the immersion of the resource in the practice of the community in this process. The results of the last level revealed particular links between STSI tools and DSTR practice that are associated to the local reality of the CoP we studied as well as to the integration of DSTR practice in the mathematics teaching. The potentiality of STSI tools, for both learning statistics and design for statistics learning, was also an important finding of this study.

5.3. Limitations Of The Study

The limitations of this study stem mainly from the contextual aspects of the study. Particularly, this research was conducted in Greece where the teaching of statistics constitutes a very small and fragmentary part of the national mathematics curriculum. Moreover, the traditional teaching approach to statistics content puts an emphasis on formulas and procedures neglecting concepts and activities that aim to foster statistical reasoning and thinking. In the period that this study was about to start, a reformed curriculum was first implemented in secondary mathematics education which enhanced the role of statistics and put an emphasis on statistical reasoning and thinking. Although this situation is not rare in the international landscape regarding the teaching of statistics, it contains some specific limitations such as teachers' unfamiliarity to the statistical content and the stochastic context, the insufficiency of educational material especially designed for the teaching and learning of statistics, the teachers' unfamiliarity with digital resources for statistics instruction etc. These limitations, in combination with a general social tension towards statistics, form an innovative terrain for the DSTR practice to develop and provides great challenges to the practitioners of the teaching practice. Moreover, the particularities of the Greek context acted as a motive for the secondary mathematics teachers to engage in our study.

Except for the general educational context that was in a reform movement, the participants of our study had also a strong personal motive to learn and develop professionally. In particular, they were already graduates, or about to graduate from a post-graduate program in Mathematics Education so they were all open to teachers' collaboration and curriculum innovations. Moreover, they had an already shaped relationship with the researchers of this study from their post-graduate studies which acted also as a motive for their participation in the established CoP. All the above conditions constitute limitations since we cannot argue that the participants of this study are a typical group of secondary mathematics teachers. However, these particular conditions were catalytic in fostering the existence of this group for two academic years in a voluntarily base. Furthermore, since our focus was on the developmental process, and not on professional development as an outcome, the special conditions of this group of teachers were also profitable for the implementation of this study.

Additionally, the design and selection of the PD tasks that the teachers of our study were engaged in, were also subjected to some limitations. For example, the particular research papers we selected for being discussed inside the community, the statistical problems we gave to the teachers or the degree of the researchers' intervention in teachers' discussions may have also been considered as restrictions for the transferability of the findings of this study. But, since our interest is in studying the re-sourcing process of a particular resource, and we consider the PD tasks as resources that motivate the negotiation of meaning regarding the integration of specific resource in the communities practice, the special characteristics of the PD tasks are acknowledged in the formation of both the results and the extensions that stem from them.

5.4. Recommendations For Further Research

As we discussed in the limitations of the study, one of the main constraints in our approach was the particularities of the participants of the CoP. However, our research methodology seemed to have the potentiality to reveal contextual particularities and further to denote their role to the formation of the DSTR practice. For this reason, it would be of value to investigate communities of secondary mathematics teachers in different national contexts, which could have cultural differences in terms of teachers' familiarity to collaborative contexts as well as to statistical contexts related to DTSR. Research on multiple communities would give us deeper knowledge about the conditions and parameters that seem to be important for the consolidation of a resource in the re-sourcing process, as well as the potentiality of this study PD tasks in other contexts.

Another extension of this study would be the investigation of other teaching resources that seem to have a special role in the whole range of the data, in the two years collaboration of this CoP. For example, the trace of the concept of probability and how it emerges and forms the practice of the statistics teaching in this community. Since the conceptual connections between probability and statistics have been highlighted by many researchers and curriculum designers (Borovcnik, 2011; Biehler et al.,2013, Chernoff et.al., 2016), it would be interesting to see empirically how such connections are acknowledged and utilized by the secondary mathematics teachers while they develop their teaching.

Last, as we show and discussed earlier, the methodological tools we employed to study the re-sourcing process of STSI tools, helped us to focus on human resources and study their role and impact in the formation of the communities of practice. The researchers of the study constituted a major human resource for the development of the communities' practice. They provided material resources, motivated discussions within the community, provided particular PD tasks for the teachers and encouraging the collaboration among the participants. Their role was both distinctive and determinant, so it would be of value to study further this particular human resource in the formation of the DSTR practice within the community of the teachers.

5.5. Final Conclusion

The present dissertation contributes to the field of research related to the professional development of secondary mathematics teachers in the teaching of statistics. Viewing the DSTR practice on the boundaries of statistics education research, mathematics education research and secondary mathematics teaching we integrated three theoretical constructs, namely the CoP theory (Wenger, 1998), the resources-in-context-in-practice theory (Adler, 2000) and a conceptualization of the DSTR practice that captures the fundamental statistical ideas (Burril & Biehler, 2011) as well as an inquiry approach to statistical reasoning and thinking (Wild & Pfannkuch, 1999). This approach makes it feasible to connect research on the professional learning of mathematics teachers with the findings of statistics education research regarding the development of statistical reasoning and thinking. The methodological approach to facilitate the teachers' role as key-stakeholders (Shaughnessy, 2014) in the formation of the DSTR practice constitute the essence and the major confluence of this study.

6. Table of Figures

7. Table of Tables

8. References

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