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Gender and Computing Algorithms: The case of Stable Matching

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Summary

Algorithms - i.e. ordered steps to solve mathematical problems - are present in many aspects of our daily lives, and encapsulate the fundamental logic that guides the operation of computers. Algorithms lie hidden behind the computer interface and, essentially, are seldom questioned. On the contrary, as digitization becomes more widespread and effective, computers and their underlying operating mechanisms are perceived as increasingly trustworthy not only in performing scientific calculations but, also, in processing our personal data and supporting us on various facets of our public and private lives. However, this blackboxing of technology (alongside a heightened sense of trust in the objectivity of algorithms and mathematical logic) gradually brought about the belief that technology is error-free, unbiased and unaffected by social, political or other influences.

This thesis seeks to analyze one well-known problem and its algorithmic solutions in computer science education programs and real systems applications (such as the Evolution of the Labor Market for Medical Interns and Residents as referred in [5]) from a Science, Technology, Society (STS) perspective, particularly with regard to gender bias. The Stable Marriage Problem rests upon the marriage metaphor so that the concept of finding a stable matching between two equally sized sets of elements can be illustrated. This problem has been chosen as a typical example of how algorithmic language and narrative can implicate gender roles. In other words, it can demonstrate how certain social content can sustain ideologies and social conceptions through scientific/mathematical procedures.

The present study was focused on scientific literature discussing the stable marriage problem that was drawn from the digital libraries of the Association for Computing Machinery (ACM) and the Institute of Electrical and Electronics Engineers (IEEE). This research calls attention to the aforementioned literature's social and, especially, gender-related implications.

The findings of this thesis indicate that algorithms and algorithmic techniques are not as objective as one would believe. Stereotypes and conceptions were identified in the formation of the problem's solutions. Additionally, the language that scientists use to communicate their ideas was, on occasion, found to rely on stereotypical gender characteristics. Texts regarding this problem and its algorithms are laden with a social content that puts into question whether algorithms and, hence, computer technology are so unbiased and objective after all.

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Chapter 1
Introduction

Computer science and computers in general affect an evergrowing part of our lives. The way we work, communicate and live our everyday lives is wrapped around computers and digital technologies. But what are the inner workings of all these consequential technologies? How are they constructed and developed? How do the people who develop and apply them talk to each other?

Although computers have penetrated many aspects of our lives, we rarely wonder how they are really constructed or scrutinize the terminology and models that were brought into play so that a smartphone can end up in our hands. This thesis aims to go beyond screens and applications, in an effort to reach the point where algorithms mold the operation of our digital world.

Algorithms by their technical definition, are ordered steps that describe a procedure and are very strongly entwined with the way computers operate (i.e. multiple steps carried out very quickly). Algorithms are traditionally understood as mathematical processes for problem-solving [11]. Mathematics is considered to be the scientific field that is the most objective and clean from social interaction. Given such a presumption, algorithms are likewise thought of as objective and veracious if they are proven so. This consideration is so strong, that both math and algorithms find best their role as tools rather than sciences.

In computer science, algorithms are considered to be steps including math and assignment processes. During these steps, engineers and mathematicians developing algorithms and data structures habitually adopt social concepts to communicate the operations that an algorithm should perform.

One typical such example - on which this research focuses - is the 'stable marriage' problem, a matching problem of two sets that reaches back to 1962 [17]. The problem has to do with how two disjoint sets of the same size can form pairs. This problem might be found in many contexts: Evolution of the Labor Market for Medical Interns and Residents, the Student-Project Allocation problem [38], Network problems [37] and numerous two sided market problems. However, the social concept chosen for this problem's description is that of marriage, something that entails the assignment of the terms 'women' and 'men' to the sets. As this thesis argues, such practices tend to perpetuate gender biases.

In its attempt to 'open the black box' of one of the structural algorithms of computer science, this thesis reports to the analytical resources of the interdisciplinary field known as STS ('Science, Technology, Society' or 'Science and Technology Studies') and Gender Studies.

To understand the significance and reach of this 'gendered' algorithm it should be noted that it is widely used as an educational tool, so that students can grasp the notion of the algorithm and algorithm complexity analysis. It is deemed to be an easily understood indicative example of what an algorithm is. The way this is communicated through images and dialogues in an educational setting, is quite revealing and there is an amount of work done in this field [5]. This issue, however, is beyond the scope of this thesis.

Rather, what will be explored is how scientists communicate the development of new mathematical tools and applications regarding this problem, in formal published scientific texts. This area was signed out for examination because it is one of the most profound and, presumably, one of the most objective levels of this technology's development. Yet, it will be subsequently argued that it preserves and reproduces stereotypical and other social conceptions.

The primary material for this thesis was collected from the digital libraries of two of the most significant associations for computer engineers and scientists worldwide: the Institute of Electrical and Electronics Engineers (IEEE) and the Association for Computing Machinery (ACM). Both organizations were founded before computing became a separate academic field. Over the years, they attended to the development of computers and established themselves as two of the most historic scientific and professional organizations of relevance to computing.

Another reason for this selection is the awareness that has emerged within these two organizations regarding gender inequality. Both ACM's Council on Women in Computing (ACM-W) [https://women.acm.org/] and IEEE's Women in Engineering (WIE) network [https://wie.ieee.org/] are encouraging research on how gender biases are still perpetuated.

Of relevance to such issues is the current debate regarding the 'master-slave' terminology, which recently emerged on public discourse. This terminology and its corresponding model are a structural part of computer history (given their usage in the development of computers and for educational purposes). A discussion about how these terms are reproducing stereotypes and racism has been reprised. As computing terminology is under scrutiny, the present research could contribute to its reappraisal and shed some light on unnoticed stereotypical class/race/gender conceptions found in technical language.

Chapter 2

Theoretical Framework and Secondary Literature Review

The present chapter sets forth this thesis's theoretical framework. More specifically, it provides an introduction to STS's literature regarding algorithms and, then, outlines a number of texts that illustrate the gendered character of the problem in question. These are followed by an introduction to Gender Studies (as a field that will contribute to the analysis) and, lastly, by a review of an indicative bibliography from the intersection of STS and Gender Studies.

For STS and the related fields, an algorithm is not simply a technical term [14, 16]. As Ziewitz has wondered before [10], here it might be useful to pose the following question: "What is an algorithm?" For Ziewitz, algorithms have become an 'ontology' and cannot be used just as a technical term. Algorithms can be wrong, fair or not, discriminating or not. In the era of digital computing, there is a whole myth built around them. So, wondering what an algorithm is may seem like a straightforward question but, actually, has become difficult to address.

Algorithms are not a twentieth century breakthrough, as they were already used for many centuries [11] in several cultures and traditions - Western and non-Western. By now, they are a symbol of prestige for computer science and technology and, also, hold a huge communicative power as a word. Algorithms are typically assumed to be associated with justice or truthfulness [13, 15].

As advanced through the works of theorists like Scott [7] and Keller [3], Gender Studies focus, among other issues, on the relation between masculinity and femininity. Of particular use in this thesis is an article by Haig [8]. This work concerns the presence of the terms "sex" and "gender" in academic literature and provides an additional insight into how gendered discourse is embedded in scientific texts.

The 'gender' concept - which is central to this thesis - among other social construction issues, has to do with the relation between the masculine and the feminine and is linked to (but does not overlap with) the relationship between the male and the female, men and women. In *Gender: A Useful Category of Historical Analysis* [7] Joan Scott introduces gender as a useful analytic category.

Initially Scott presents how gender as a category arose through the work of feminist historians. The concept of 'gender' was gradually adopted as a means to ascertain the distance between the biological sex and the social role that is assumed by a person growing in a society with specific rules and identities. Over the course of time, 'gender' became a key concept for feminist studies. This separation between 'sex' as a biological attribute and 'gender' as a social construction meant that feminists now had a powerful analytical tool. Scott additionally illustrates the historical transition from feminist and women studies to gender studies and, also, highlights the introduction of gender into historical analysis.

Scott specifically shows how this transition occurred and how psychoanalytic and other theories contributed to the establishment of gender as an analytic category. The transition from women's history to an era where gender studies seem to be a respected academic field was hard and engendered alliances as well as rivalries. In Scott's work, this history is outlined and interpreted.

'Gender', for Scott, is a tool to observe and understand the actions and behavior of people in a society. Through these observations, one can better understand social structures and the therein behaviors on a more 'macro' level, beginning from the 'micro' or domestic one. Especially when it comes to the development of a person's identity, Scott makes a detailed analysis of how language is forming the way a person thinks of themselves and others (psychoanalytic theory), and, consequently, how the relationships between people of the same and different 'gender' (not 'sex') are formed. As she underlines:

> " [...] gender is a primary way of signifying relationships of power." [7, p.1067]

Next Scott examines how gender contributes to the understanding of power relations within the various levels of society - the market, politics etc. Scott further clarifies her analysis by citing McKinnon's dissection of sexual objectification:

> " "Sexual objectification is the primary process of the subjection of women. It unites act with word, construction with expression, perception with enforcement, myth with reality. Man fucks woman; subject verb object" *Catherine McKinnon, "Feminism, Marxism, Method and the State: An Agenda for Theory, Signs*, 7 (Spring 1982): 515, 541." " [7, p.1058]

"Subject verb object". This schema is extremely useful in identifying syntactic and grammatical mechanisms in the texts under study. Scott's understanding of the issue offers a

blueprint for the inspection of this kind of schemas and the identification of their prominent characteristics.

An additional analytical tool from gender theory can be found in Keller's article *Feminism and Science* [3]. The privilege of choice - especially in a problem of matching pairs where the active parts are people - is of great interest. The right of choosing a partner or not and how it is addressed to roles in an engendered concept, needs an interpretation that Keller's work offers:

> "Our early maternal environment, coupled with the cultural definition of masculine (that which can never appear feminine) and of autonomy (that which can never be compromised by dependency) leads to the association of female with the pleasures and dangers of merging, and of male with comfort and loneliness of separateness. [...] The values of autonomy are consonant with the values of competence, of mastery. Indeed competence is itself a prior condition for autonomy and serves immeasurably to confirm one's sense of self. But need the development of competence and the sense of mastery lead to a state of alienated selfhood, of denied connectedness, of defensive separateness? To forms of autonomy that can be understood as protections against dread? Object relations theory makes us sensitive to autonomy's range of meanings; it simultaneously suggests the need to consider the corresponding meanings of competence. Under what circumstances does competence imply mastery of one's own fate and under what circumstances does it imply mastery over another's?" [3, p.595-596]

In this part of the text, Keller exposes a connection between masculinity, autonomy, competence and mastery of one's own or over another's fate. These associations reserve for men certain affordances that are easily observed in how the roles are divided in the algorithm.

Emily Martin's *The Egg and the Sperm: How Science Has Constructed a Romance Based on Stereotypical Male-Female Roles* [4] has been particularly useful for this thesis. Martin's research is concerned with how biology's textbooks describe the "roles" of the egg and the sperm during their interaction. Her main argument is that they are portrayed in a manner which is closer to the social stereotypes vis-à-vis male and female roles than to what was actually observed in the lab. She describes how the sperm is presented as the penetrator - the active part - and how the egg is static and passive-aggressive. She also demonstrates how in the description of the two, social rules about sex and relationships are reflected and encapsulated for each gender. Her textual analysis is focusing on the language used to describe the entire fertilization process and the actions of the two parts.

She begins with the period when the prevalent view among scientists was that the sperm exhibits an aggressive behavior towards the egg, and acts like a conqueror. Thus, the egg was merely a passive receptor of the sperm's actions. This narrative was not based on any findings, but rather, was mostly a fable concocted to supposedly lay out how fertilization occurs. Here it is to interest (and also, provides the crux of Martin's argument) that when research on fertilization exposed some radically different egg and sperm behaviors, scientists insisted on using the 'roles' previously prescribed for the description of the process. They made no adjustments, even when actual scientific findings were begging for a revision of the issue.

Martin remarks that, eventually, the reassessment of how fertilization transpires brought about a change to the pertinent storytelling. The resulting description, however, was far from being neutral or free of stereotyping. Rather, Martin discerns the deployment of yet another stereotype: the egg's behavior was now linked to that of a *femme fatale* while the apposite male role was affixed to the sperm.

She concludes by emphasizing that this stereotypical hierarchical narrative does not seem to have escaped sexism even if unconsciously so. Martin tries to find an explanation for this in how biology is conceived and structured as a science. She sees a chance to avoid any hierarchical narrative only on condition that another conception of nature is embraced. For instance, she points to an understanding of fertilization and other processes in terms of a "cybernetic model". Such a model offers a dynamic perception of systems and their components. As Martin points out, this kind of conceptualization involves "feedback loops, flexible adaption to change, coordination of the parts within a whole, evolution over time, and changing response to the environment" [p.499]. Thus, studying the issue within a cybernetic framework could arguably prevent the proliferation of static roles and biased narratives.

Of particular help to this thesis were articles that examine the position of women and gender interaction within science and technology and focus on the social biases that result in such predicaments [1, 2, 3]. Through stereotypes regarding gender and science [3], advertisements [2] and the concealment of the role of women in technology [6, 1], these articles clarify why gender can serve as a useful analytic category in understanding how science and technology are shaped.

Some of these articles [2, 1] are raising a common set of questions. Why do women shy away from the technological and engineering fields as an educational path? Concerning those who do not, why are their career paths suboptimal? Rentetzi [1] argues that stereotypical gender roles are entrenched in the arrangement of relations inside the 'women-technology-men' nexus and, hence, define the female role within technology, as well as the applicable career opportunities.

Tympas et al. have studied computer advertisements in home technology publications so as to retrieve mechanisms of co-construction amid gender and computing artifacts [2]. They paid attention to images that depict men relaxed on their desks, talking on the phone, giving commands and using the computer mouse. In contrast, women are typing, printing, working, mirroring on the screen, while having both hands tied to the keyboard. Through these images and the patterns involved, Tympas et al. uncovered a hidden mechanism. Specifically, they argued that the personal computer was instantiated through the differentiation between analog parts that are controlled by men (e.g. the mouse) and digital parts which control the women (e.g. the keyboard).

As a preamble to the stable marriage problem, here it is useful to mention what Evelyn Fox Keller has asserted about scientific problems:

> "A slightly more radical criticism continues from this and argues that the predominance of men in the sciences has led to a bias in the choice and definition of problems with which scientists have concerned themselves." [3, p.590]

The problem of matching pairs figures prominently in economic theory and computer science. An important formulation of this problem can be found in the so called *Stable Marriage Problem*, while the algorithm that solves the problem is designated as the *Stable Marriage Algorithm* or *Traditional Marriage Algorithm* [5]. For quite some time, this has been an important problem in working with algorithms and educating about them. This problem and the algorithm connected to it are used extensively in computer science and are taught in engineering departments that offer a course on algorithms. As an indication of its widespread use, it should be noted that this algorithm is even utilized by doctors in hospitals.

To study this problem, according to Martin's model, this thesis has been focused on the examination of scientific texts. My aim was to address the following questions:

- Is there a gender bias in this problem and the algorithm related to it?
- How can stereotypical gender roles be reproduced through the algorithms that solve the problem?

Previous research on the problem was undertaken by Roy Wagner, who published an article on this in 2009 [5]. His work has been a valuable guide for this thesis and some of his insights have been used in the interpretation of the findings. The ensuing study can actually be read as an extension and specification of some of Wagner's work. In the following pages, a summary of his *Mathematical Marriages: Intercourse Between Mathematics and Semiotic Choice* [5] is presented, focusing on the parts of his text that were most useful for this thesis.

Wagner begins with an introduction to the history of the stable marriage problem. There, we learn that Gale and Shapley were not the first to define the problem. However, their definition and their algorithmic solution for a stable matching eclipsed all others, even though, as

Wagner argues, it was neither the simplest nor the best one. According to him,there were much earlier references to this specific problem, while algorithms that solved one-to-one matching problems were being used for decades prior to Gale and Shapley's publication. Notably, he mentions one definition that was devised three decades earlier, which also restored to 'girls-boys' sets in its description.

Next, Wagner focuses on one example of how this problem is taught and the terminology and language that a certain professor has used to communicate it to the classroom. Wagner notes that men are the ones who propose, whereas women are waiting and holding proposals. Women do not actually accept a marriage offer but stand by until the conclusion of men's actions. In his analysis, Wagner borrows certain tools from Gender Studies, like the passive-aggressive female-male differentiation observed by Martin in her research. He explains that inversing the roles does not eliminate the reproduction of stereotypes. Hierarchical, gendered relations are embedded in the formation of roles. For Wagner, inversing the roles does not alter their semantics, nor does it eliminate the gendered content they feed to the (still active) mechanism. Regarding the educational use of the algorithm, he substantiates the perpetuation of specific stereotypes by providing certain examples of the terminology and the educational material used in classrooms.

Wagner furthermore demonstrates that, over time, there were shifts in what was considered desired solution. The initial algorithm produced a result that was the best for the proposing set, which, bibliographically, is almost always that of men and the worst for the other set, women (*which now is a well-known theorem*). Variants of the problem, which asked for a more balanced result, were introduced nine years after Gale and Shapley gave their solution, while algorithms to achieve this appeared even later. Wagner argues that the conception of men and women as separate sets with different characteristics was somewhat abandoned in order to achieve a more balanced result. However, he refers to the exchange of spouses without delving into the language that goes along with this action.

Another of Wagner's observations has to do with how scientists frequently resort to a heterosexual hierarchical framework. Firstly, they endorse a certain ideology regarding the definition of a good marriage. A good marriage is a stable marriage that is not threatened by people who want to form pairs with anyone already paired. Secondly, their descriptions are, in many cases, sexist. As an example, he mentions a professor who, within the problem's context, characterized a woman as a "slut" because she rejected someone over someone else.

In addition, Wagner argues that matching problems fall back on canonical, conventional schemas. The algorithms may include actions that are unrealistic, which, remarkably, are not dropped (like men and women inversing roles during the algorithm). In fact, such actions are widely used. Yet, whenever a variation of the problem - if insisted in marriage narrative - would generate a real, non canonical schema (like a marriage between people of the same sex or a marriage between three people) the paradigm is changed. The marriage concept is good only so far as it refers to a stereotypical marriage. In light of these two observations, Wagner ascertains the distance between the world of mathematics and the real world. The world of mathematics not only preserves stereotypical schemas, but also, excludes out real cases (i.e., diversity) while it include unrealistic and non-existent scenarios.

Chapter 3
Methodology

The present chapter details the methodology followed in this research. At first, the relevant literature from STS and Gender Studies was identified and studied. The two guided articles were Martin's *Egg and Sperm*[4] and *Constructing Gender and Technology in Advertising Images* by Tympas et al.s [2]. Of special help was Wagner's *Mathematical Marriages: Intercourse Between Mathematics and Semiotic Choice*[5].

Regarding this study's primary sources, the starting point was the Gale and Shapley's *College Admissions and the Stability of Marriage* published in *American Mathematical Monthly* in 1962 [17]. Their paper designated the Stable Marriage Problem as such, despite the fact that this matching problem already existed, as Wagner has shown [5].

One of the two main sources of the primary research were the publications of the ACM Organization, which "is the world's largest educational and scientific society, uniting computing educators, researchers and professionals to inspire dialogue, share resources and address the field's challenges." [https://www.acm.org/, accessed in 01/03/2019]. The association was founded in 1947.

ACM's Digital Library platform [https://dl.acm.org/, accessed in 01/03/2019] was used as a search gateway, so as to conduct a search for texts containing the terms "stable" and "marriage" in their title. The search targeted the title of the texts because the Stable Marriage Problem is very popular and has many applications. The search resulted in 28 texts for the 1971-2020 period, which were narrowed down to twelve based on the criteria detailed below.

The IEEE "is the world's largest technical professional organization dedicated to advancing technology for the benefit of humanity." [https://www.ieee.org/, accessed in 01/03/2019]. Its roots go back to 1884.

The IEEE Xplore digital library "is a powerful resource for discovery of and access to scientific and technical content published by the IEEE (Institute of Electrical and Electronics Engineers) and its publishing partners." [https://ieeexplore.ieee.org/Xplore/home.jsp, accessed in 01/03/2019]. It was used as a search gateway for the IEEE texts. This search also looked for texts containing the terms "stable" and "marriage" in the document title (as was the case with the search in the ACM gateway). This resulted in 21 texts for the 1989-2019 period, nine of which were selected for examination.

The three criteria used were the following:

- Time representation: in order to show how the terminology was changed or not through the years, the first criterion for selection was time. For every two- or three-year period only one text was chosen.
- Number of citations: if for some period, multiple papers showed up, only the most-cited one was selected.
- Author representation: if, for some reason, the previous criteria lead to one or certain multiple authors more than once for every two- or three-year period, texts written by the already represented author(s) were disqualified.

By applying the above, twelve texts from the ACM and nine from the IEEE were signed out. On the whole - and including the Gale and Shapley's paper - 22 scientific texts were examined, covering a time period from 1962 to 2020. However, time representation criterion consistency exists only after 2005. These are the results for each source before 2005:

Gale-Shapley:	1962
ACM:	1971, 1987, 1989, 1999 (with the rest after 2005)
IEEE:	1989, 1995, 1999 (with the rest after 2006)

The intervals of this table could be partly explained by the overall number of papers published each year, which has grown considerably during the last two decades. Also, these figures are a result of computing's concurrent growth which prompted its ever-increasing application to other scientific fields (especially, when compared to its twentieth century rate of expansion).

Regarding the primary research, the main goals were two: a) to identify the most 'popular' texts(i.e., the most read and influential ones) and, b) to proceed to the most random selection possible. The latter goal was set in an effort to avoid results (and conclusions) that are author- or time-specific.

In order to analyze these texts, this study focused on three elements that were found in most of the texts. The first was the natural language used to 'tell the story' of the problem and describe the algorithmic steps or techniques. This is the higher level of the analysis, which also was an indicator of how the problem's description changes over time as it happens with gender and sex terminology (see the chapter).

Nonetheless, language (which, here, equals words) is not the only way that an algorithm is constructed. The second level of the analysis was oriented towards math notation and math language, where again gender bias was present in the form of symbols and naming choice by the authors. The last aspect examined is the most technical one and pertains to the use of data structures or math/algorithmic techniques and the way that conceptions about gender behavior were embedded in this technical level.

Chapter 4
Primary Research Results and Analysis

This chapter focuses on the mechanisms through which gender biases are reproduced in the texts that were studied. Specifically five mechanisms will be introduced. These range from more high-level and natural language related mechanisms to micro-level ones, which are related to technical language and schemas. These mechanisms are the following:

- The first locus where the social influences on the algorithm can be discerned is found at the problem's definition. Each author selects a different narration for it. In addition, within the definitions in the selected texts, an analogy between the disjoint sets and men and women has been established. The heterosexual concept of marriage and the use of the word "stable" regarding a marriage were attached to that analogy. These observations concern the definition of the problem - i.e., the way that mathematicians attempt to introduce and explain the problem. They try to make it more understandable by setting up a concept and constructing a reality that would be easier to grasp, compared to a description strictly in mathematical notation.

- The second mechanism is more general and concerns the use of natural language: that is the verbs or names used for men and women and the way that they relate to each other within a sentence.

- Still focusing on natural language, it is possible to spot how it changes over time. Especially regarding the use of the terms 'sex' and 'gender' and, also, the narrative regarding men and women to be equally able to 'perform any role' (proposing-responding).

- Switching to the mathematical language, we can observe the perpetuation of stereotypes through math notation, algorithmic procedures and data structures. The notation and symbols or names used involve certain hierarchical relations. Procedures and structures also contain gendered social content that affects the formation of roles.

- Finally, after an overview of all the above, a question emerges, along with one last hidden mechanism. The use of phrases like "members of a set" and "roles of men and women" and, also, the differentiation between the two sets should be examined separately. Their use, arguably, combines all the other mechanisms and, when closely examined, reveals that the line separating the social world from that of math is very thin.

4.1 Definitions and Disjoint sets

The first step in the analysis of the Stable Marriage Problem and the algorithms that intend to solve it requires an inspection of its definition. What is the Stable Marriage Problem and how is it described? Here it is useful to provide the very first definition of the problem that would, thereafter, become known as the Stable Marriage Problem even though the problem preexisted, as already mentioned [5]. The definition that Gale and Shapley gave to the problem in 1962 was the following:

> " [...] but there is another 'story' into which it fits quite readily.
> A certain **community** consists of n men and n women. Each person ranks those of the opposite sex in accordance with his or her preferences for a marriage partner. We seek a satisfactory way of marrying off all members of the community. Imitating our earlier definition, we call a set of marriages unstable (and here the suitability of the term is quite clear) if under it there are a man and a woman who are not married to each other but prefer each other to their actual mates." [17, p.11;emphasis added]

The problem in question is very popular and has many applications. It can be found in economic theory, computer science and game theory. Thus, one could expect a strict mathematical definition that resorts to specific symbols and rules. Instead, the problem is described in terms of a 'community', '[...] men [and] women', and the 'suitability' of the term 'stable' for a 'marriage'. A more scientific-like definition was given in 1971:

> "Consider two distinct sets A and B. An assignment of the members of A to the members of B is said to be a stable marriage if and only if there exist no elements a and b (belonging to A and B respectively) who are not assigned to each other but who would both prefer each other to their present partners." [18, p.486]

This definition refers to the exact same problem. Yet, while more formal, terms that allude to a men-women community ('partners') are not absent. Tellingly, in the same paper, another definition actually resorts to 'community' setting: "[a] stable marriage assignment in which all the members of the community are married is required." [18, p.487].

Here it is important to list some narratives regarding the definition of the problem and the issue of 'stability', in order to show how the social concept of marriage is set and described, and, thus, how it involves social customs.

> "Suppose all **the eligible bachelors and bachelorettes** in a **town** confide in **the town's matchmaker** their ideal **spouses**. [...] The matchmaker must arrange

marriages such that no one is tempted to **ask for a divorce**. In particular, the matchmaker must be sure that there is no pair of **young lovers** who prefer each other to their assigned spouses. Such a set of marriages is called stable, and finding a set of stable marriages is known as the stable marriage problem. [...] In a small town, every man knows every woman, [...]" 2005, [22, p.53;emphasis added]

"In the stable marriage problem (SM for short), each person expresses not only the acceptability but also a preference order of the members of the opposite sex, and an output matching must satisfy the stability condition, which intuitively means that there is no man-woman pair both of which **have incentive to elope**." 2008, [34, p.131;emphasis added]

"A match is said to be stable if there is no pair of matched couples in it containing individuals who would prefer to be matched to each other but are not. The thought is that if the couple here is unstable with regard to **the couple next door, divorce** and **remarriage** will (or at least may) ensue." 2010, [25, p.1283;emphasis added]

"In the stable marriage (SM) problem , the **set of agents** is partitioned into men and women; men and women express strict preferences over all their **counterparts**; and the aim is to find a stable matching in which men and women are matched to each other. [...] **In the frivolous marriage parlance, this can be interpreted as requiring permission for divorce.**" 2013, [26, pp.288-289;emphasis added]

"There are men and women, looking for partners, as shown in Fig. 1." 2011, [35, p.586], see figure 4.1.

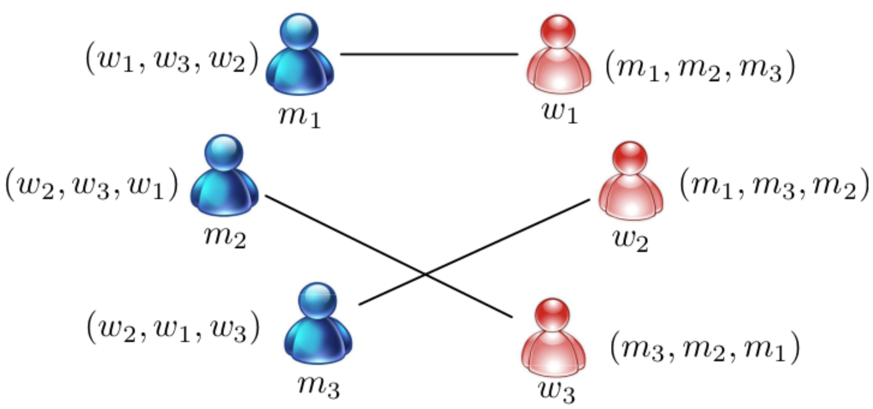

Fig. 1. A simple example of the marriage market. The matching shown is a stable matching.

Figure 4.1: Image from "Seen as Stable Marriages", IEEE 2011 [35, p.586]

Here, one first conclusion is that there are many ways to narrate this problem. Each one carries with it certain social influences regarding how people and, hence, scientists understand and experience the concept of marriage and the relationships between men and women. The variety of names and narratives offers a first that this problem is not strictly mathematical, but has some social aspects as well.

Men and women are called ‘spouses’, ‘young lovers’, ‘bachelors and bachelorettes’, ‘newlyweds’ [24], ‘agents’, ‘players’, ‘participants’ [30], ‘fiances’ [32] they are getting ‘engaged’ [32], ‘ask for divorce’, ‘have incentive to elope’ and live in a ‘town’ or a ‘community’ where a ‘matchmaker’ may also exist.

Taking all the above into account, one can see that mathematicians construct a real-like yet odd concept in order to comprehend and process this mathematical problem. They not only use ‘men’ and ‘women’ as elements and ‘marriage’ as their connection, but also construct a narrative that makes this problem more vivid, descriptive and, unavoidably, socially bounded.

During the research, the terms ‘disjoint’ (sets that have no common elements), ‘distinct’ and ‘bipartite’ (a graph that is separated into two parts, where the elements of each part communicate only with the elements of the other part) appear many times when the texts refer to the sets of men and women. A strict heterosexual relation between the two sexes and the formation of their gender roles based on this relation is the starting point of the definition for this matching problem. Thus instills into the mathematical narratives the prevalent view of how sex determines one’s social role and the heterosexual relation between the two sexes. The following quotes are representative parts of the texts in which this mechanism was detected.

> “**The term marriage is used because one set can be considered to be men and the other women.** […] if the sets are **disjoint** then at least one set of stable marriages exists. […] However, **the theory does require that the sets be distinct**.” 1971, [18, pp.486-487;emphasis added]

> “Two well-known problems that can be studied in this framework are the **stable marriage problem and its non-bipartite version**, the stable roommate problem. ” 1989, [20, p.514;emphasis added]

> “A **finite set** of “men” is given along with a set of “women” of the same cardinality. In addition, each person is assumed to have a ranking of the members of the opposite sex . A matching is a **bijection** between the two sets . We think of a matching as a set of n **monogamous marriages**.” 1999, [21, p.3;emphasis added]

> “In the **simplest** formulation of the Stable Marriage Problem, there are two **disjoint sets** of n men and n women.” 2008, [24, p.1223;emphasis added]

> “Consider a **bipartite graph** $G = (U, V, E)$, where U and V are sets of vertices and E is a set of edges. A matching M in G is a subset of E such that each vertex appears at most once in M . **Bipartite matchings are sometimes interpreted as marriage between men and women**: U and V represent the set of men and women, respectively, and an existence of an edge between $m \in U$ and $w \in V$ implies that m and w are acceptable to each other.” 2008, [34, p.131;emphasis added]

> “The stable marriage problem is **prototypical** of **two-sided matching problems**[…]” 2010, [25, p.1283;emphasis added]

> “There are **two disjoint sets** of agents, $M = m_1, m_2, ..., m_n$ and $W = w_1, w_2, ..., w_n$, men and women.” 2011, [35, p.587;emphasis added]

Similar examples can be found in almost all of the examined texts, because the definition of the sets’ characteristics is very crucial to achieve clarity and consistency in mathematical texts.

The property of the two sets of the problem as strictly ‘distinct’, ‘disjoint’ (or, in graph terms, ‘bipartite’ - i.e., not connected to each other) is crucial.

Requiring that the two sets possess this structural property and, at the same time, naming them ‘men’ and ‘women’ not only designates them as two completely different ontologies, but also asserts that only heterosexual relationships are permitted.

> “Then, in any IR partition π in game (N, v), the pigeon-hole principle ensures that **two members of the same sex are never together in the same coalition**. […] If a coalition were of size more than two, then **members of the same sex** would be present which makes the coalition **unacceptable**.” 2013, [26, p.290;emphasis added]

Additionally, when the problem does not demand two separate sets but one, the marriage paradigm is replaced by a roommate relationship (i.e., no need for a heterosexual relationship restriction). The same applies to the hospital/residents narrative when the relationship is not monogamous.

> “The stable roommates problem (SR for short) is a non-bipartite extension of SM … […] The hospitals/residents problem (HR for short) is a many-to-one extension of SM, where we consider men as residents and women as hospitals.” 2008, [34, pp.133-134]

> “**The stable roommate problem (SR) is the unisex generalization** of the stable marriage problem in which roommates are paired with each other in a stable matching.” 2013, [26, p.288;emphasis added]

> “In this article, we mainly consider **the traditional two-sided one-to-one stable marriage setting**.
> […]
> The college admission problem (also called hospitals/residents problem) is **a many-to-one extension of the original stable marriage problem**.
> […]
> Stable roommates problem is a **nonbipartite extension of the stable marriage problem** originated from the problem that focuses on assigning a set of $2N$ players into N twin rooms according to their preference. **In this problem, all players belong to a single set.**” 2016, [37, pp.39-43;emphasis added]

As Wagner observes:

> “The variations of the marriage problems include what we might call ‘alternative families’. For example, instead of a matching based on gender division, we may be given a set of people each of whom can be matched to any other. The literature never refers to this as the ‘homosexual marriage problem’. Rather, it is called the ‘roommates problem’[...]” [5, p.303]

This observation precisely shows that the heterosexual conception of the two disjoint sets cannot be overseen or changed, making the definition a key locus: this is where the mechanisms under scrutiny initially emerge.

This section sought to show how, from the very beginning, the definition of the problem contains signs and, mechanisms that convey and perpetuate some ‘traditional’, stereotypical settings. The utilization of the ‘marriage paradigm’ in the definition of the specific matching problem is, in itself, one such mechanism. Within the various narrations, it is easy to corroborate that, through the adoption of marital concept, social conceptions affect the way mathematicians write about the problem.

4.2 Natural Language

This section elucidates the mechanism that arises from the use of specific terms to address certain actions. Here, the scientific text is mostly treated as specimen of natural language, detecting phrases and words that carry social content and form ‘peculiar’ sentences.

The focus has been placed on sentences where verbs are specifically used to associate men with women (within a phrase) and are accompanied by certain nouns and other terms. This analysis will be performed for each gender role in the position of the subject and will be complemented by some observations that will be reprised in the the final conclusions.

Table 4.1: Men verbs and women names as objects.

Year and Reference	Verbs	Objects
1971[18], 1999[32], 2005[22], 2020[29], 1962[17]	propose to	first/next/higher/worse/preferable choice/girl/woman
1971[18], 1999[21]	can obtain	woman
1987[19], 1999[32]	pause/continue	condition
1987[19]	exchange	his partner for
2011[35]	makes proposal to	preferred partner
2019[38]	provides	a preference list of women
1989[30]	is matched with	highest preference
1989[30], 1995[31], 2008[34], 2008[24]	prefers *woman/the partner* to *woman/the partner/his match*	woman/his match/their current wives
1989[30]	gives ranking to	woman
1962[17], 1999[32]	is refused/rejected/accepted	— (no reference from whom)
1999[32]	take/do action for proposing to	woman
2008[34]	prefer to exchange	their partners
1995[31]	receives	the better/poorer of his partners
2008[24]	swap	their wives
2010[25]	is paired with	a woman
2008[24]	has	one of his first choices
2005[22]	is rejected by	women
2005[22]	names	women
2020[29]	will marry	one woman
2008[24]	may/can choose *these* to be/as his favorite	neighboring women/any adjacent pair of women
2008[24]	choose the order of	three points
2020[29]	is assigned with	a woman

Above, the findings of the research regarding verbs that associate men and women in the same sentence are presented. First, the 'men-towards-women' verbs are listed, followed by the various terms that described women (Table 4.1). Table 4.2 is the counterpart of Table 4.1, as it lists the 'women-towards-men' verbs and the terms that described men.

Through the review of the tabulated results, a group of mechanisms regarding natural language can be detected.

- **Active/passive voice:** The results indicate that the passive voice is used more frequently (if not exclusively) to describe women which, again, frames men as the active element. When, in some examples, the passive voice is used for men, women were absent from the sentence. In addition, male activity engenders the use of phrases or verbs like 'take actions' or 'do'.

Table 4.2: Women verbs and men names as objects.

Year and Reference	Verbs	Objects
1971[18]	hold in suspense	their propoer
1971[18]	has the choice of/chooses	favorite men
1971[18], 1989[30], 1995[31], 1999[32], 2005[22]	reject/likes better/prefers *someone* in favor of/to *someone else*	man, man/fiance/her partner/her current assignment
1971[18]	has had a proposal	—
1971[18], 1999[32], 2005[22], 2020[29]	accepts	him, man
1987[19]	agrees to consider/hold for consideration	a proposal
1987[19], 1999[32], 2011[35], 2005[22] 1962[17]	receives/rejects/holds/ refuses/accepts/ chooses	a proposal
2011[35]	holds	preferred offer
2019[38]	is most preferred/chosen by	the man
1989[30]	is removed from	preference list of men
1989[30], 2005[22]	is matched with/to	man
1989[30], 2005[22]	has	partner/ husband
1999[32]	can be matched with	number of men
1999[32]	can accept/able to accept	men
1999[32]	likes/prefers him as	a man of rank *number*
1999[32]	was proposed by	man
1999[32]	selects	man
2005[22]	divorces	her husband
2005[22]	listed by	man
1999[21]	dumps	her current fiance
2020[29]	get	offer
2020[29]	choose	man higher on list

- **Dominating/possessive verbs:** The verbs 'exchange', 'swap', 'name' and 'obtain' (which were used for men towards women) indicate a certain mindset: women are men's property or can be treated as objects.
- **Proposals instead of men:** Going through the texts, when women are the subject of a sentence, quite often it is the proposals of men that are found in the position of the object instead of the men themselves. As a result, this 'women-towards-men' relation is rendered indirect. It is an interaction of women with men's actions. This phenomenon was not observed with reversed genders.
- **Names:** If one looks closely at the objects of 'men-towards-women' sentences, women are oftentimes referred to as 'choices', 'partners' or 'girls'. These terms are also linked with a possessive pronoun and, sometimes, with a comparative phrase (e.g., **the better of** *his* partners). On the other direction, possessive pronouns are absent or not that commonly used and, when men were used as objects in sentences (mainly in the passive voice), they are referred to as 'men' or 'partners'.
- **'Hold'/'pause'/'continue'/'divorce'/'dump':** This group of words could be analyzed under the passive-aggressive schema, but such analysis would be beyond the goals of this section. A similar argument has been advanced by Wagner [5].

The main conclusion is that the natural language that encompasses mathematical text establishes a mechanism which reproduces the active-passive stereotypical gender roles, the objectification of women and the stereotypical hierarchical relation between men and women (especially within a marriage).

4.3 Math

Mathematical notation is considered to be one of the most neutral things in science. It is a world of letters, symbols and numbers that does not have any 'intentions' other than solving a system of equations. But what happens when typical mathematical notations, such as subscript, is used to describe a relation between certain ontologies within a mathematical algorithmic problem? In this chapter, examples of mathematical notation and data structures from the examined literature are introduced in an effort to demonstrate that notation and structures imply certain relations between letters and ontologies and, hence, some kind of hierarchy. It is also shown how these structures actually do not introduce some new discrimination, but simply stay on at the same path, translating natural language discrimination and other relations into 'objective', discriminating math.

From the preceding textual analysis, it became clear that the concept of marriage and the applicable gendered roles strongly affect the way that the problem and its solution are structured and communicated. When scrutinizing the level of pure, technical mathematical notation and data structures, things become more ‘black-boxed’. We reach a level where everything is just letters and structures: a level that is hard to translate and highly complicated. Yet, when this part of the scientific texts is considered from a gender perspective, the usual schemas are reproduced.

What follows is a walk-through of the algorithm’s mathematical parts, from the simplest to the more complicated ones. This will illustrate how, in all of the algorithm’s levels, the mathematical parts contain stereotypes about men and women. Firstly, it will be demonstrated how men and women are represented by letters and how they are ‘named’ when there is a need to describe the behavior of the algorithm. Next, examples of men and women being assigned to elements of several data structures and to the actions inside them (such as graphs, trees and rotations) will be highlighted. Then it will be shown how a structural element of the stable marriage problem (the preference lists) is shaped and transformed in the variants of the problem for each gender.

Finally, there will be a formulation of some conclusions regarding the perpetuation of the dominant relation between men and women and, also, regarding any potential static roles when it comes for each side.

4.3.1 Math Notation

After the definition of a problem, that transition into the world of math requires the emergence of symbols that describe each part of the algorithm. Concerning the ‘men’ and ‘women’ sets in the scientific texts, several variations were observed. Some of them are completely balanced, resorting to analogous letters and notation for them. However, when there is a need to associate the two sets, the notation becomes ‘men-centric’. To this point, two cases of mathematical notation that are defined by a hierarchical relationship between men and women will be presented. This relation has been translated into and advanced through mathematical symbols.

Starting with two particular texts [5, 24], it has been noticed that capital letters are used mostly for men and lower case for women, while ‘men’-letters are always followed by ‘women’-letters. This happens even when there is no need to assign upper and lower case letters to identify whether a letter stands for a man or a woman.

> “In a given marriage M, define a woman’s suitor to be her favorite of the men that prefer her to their current wives. As the marriage is stable, every woman prefers her husband to her suitor. A man-improving rotation is a sequence $\rho = (M_1, w_1, \cdots, M_{\mathrm{r}}, w_{\mathrm{r}})$ such that M_i is married to w_i and M_{i+1} is the suitor to w_i , where subscripts are modulo r.” [24, p.1225]

In this case, M=man and w=woman. Later in this specific paper, there is an explanation for some of the text's examples, where the letters do not suffice to indicate whether an element (here letter) is a man or a woman $(a, b, c...)$.

> "We use capital letters to represent men and lowercase for women throughout." [24, p.1228]

'Women'-letters are smaller, standing under 'men'-letters.

Moving on to more complex examples, occasionally women are found in a position where they do not have their own notation or index. In certain texts, to identify or name a woman it is necessary to know whom she is married to, and in which marriage instance (i.e., which matching).

> "[...] a further operation called *breakmarriage*. This operation consists of breaking the marriage of **a selected man** i, in **a stable marriage** S, and forcing him, therefore, to take a poorer choice in his list. Similarly, **the woman** i_S (who was married to man i) has the opportunity of getting a better choice. [...] Let the wife of man i in any stable marriage solution M be denoted by i_M." [18, pp.488,4891;italics in original;emphasis added]

In this example, the woman is defined and identified from the man i and the marriage S. A woman in this text has no independent notation. To specify the symbolism for a woman through the algorithm, one needs to know the specific matching instance as well as her husband and, then, combine the two to have a woman's husband - and matching - dependent mathematical name. Under this formula, it could be argued that, effectively, she has no name. She is a wife of a man with a name. Similar examples were also found in other texts [19, 31].

In another case, a woman is again identified through her husband and marriage, by means of her subscript, which is also depending upon the husband and the marriage instance.

> "A stable marriage is a complete matching $M = \{(m_1, w_{i_1}), (m_2, w_{i_2}), ..., (m_n, w_{i_n})\}$ with the property that no unmatched man-woman pair (m_i, w_j) exist where both m_i and w_j prefer each other to their partners in M." [30, p.129]

Without any definition for this notation, when indexed in a pair, women have a different subscript from that of men. This does not affect anything and is not even explained. All men are put in order and their indexing follows pairs ordering (i.e., first pair, second pair...). Here, note that men and pairs have the same index. Man 1 is at the first pair, man 2 at the second etc. Yet, the women, apart from them forming pairs, are not designated as woman 1, woman 2 etc. For the first pair, for example: she is woman w_{i_1} of man m_1, not woman w_1, who belongs to a pair. She is merely identified as the man's wife.

The mathematical name for a woman seems to be strictly dependent upon the name of her husband. The concept of marriage according to its stereotypical form imposes this kind of relation between men and women. A man within such a conception of marriage is hierarchically superior to a woman. The man identifies the marriage, while the marriage strongly ties the woman to the man.

Women sharing the name of their husband is a common social tradition. Here the subscript could indicate exactly that. Woman i_S is the property of man i under the condition of marriage S. Woman w_{i_1} is the wife of man 1. Man i is just man i. Similarly, in Figure 4.2 [31] the men's ordering is aligned with pairs ordering and put in front of women numbers. An exact same example of this figure was found in another text [33].

$$M_0 = \{(1,5), (2,1), (3,2), (4,3), (5,6), (6,8), (7,7), (8,4)\}$$
$$M_1 = \{(1,5), (2,1), (3,2), (4,3), (5,7), (6,8), (7,6), (8,4)\}$$
$$M_2 = \{(1,2), (2,1), (3,5), (4,3), (5,6), (6,8), (7,7), (8,4)\}$$
$$M_3 = \{(1,5), (2,1), (3,2), (4,3), (5,8), (6,7), (7,6), (8,4)\}$$
$$M_4 = \{(1,2), (2,1), (3,5), (4,3), (5,7), (6,8), (7,6), (8,4)\}$$
$$M_5 = \{(1,2), (2,1), (3,3), (4,5), (5,6), (6,8), (7,7), (8,4)\}$$
$$M_6 = \{(1,2), (2,1), (3,5), (4,3), (5,8), (6,7), (7,6), (8,4)\}$$
$$M_7 = \{(1,6), (2,1), (3,5), (4,3), (5,7), (6,8), (7,2), (8,4)\}$$
$$M_8 = \{(1,2), (2,1), (3,3), (4,5), (5,7), (6,8), (7,6), (8,4)\}$$
$$M_9 = \{(1,6), (2,1), (3,5), (4,3), (5,8), (6,7), (7,2), (8,4)\}$$
$$M_{10} = \{(1,6), (2,1), (3,3), (4,5), (5,7), (6,8), (7,2), (8,4)\}$$
$$M_{11} = \{(1,2), (2,1), (3,3), (4,5), (5,8), (6,7), (7,6), (8,4)\}$$
$$M_{12} = \{(1,3), (2,1), (3,6), (4,5), (5,7), (6,8), (7,2), (8,4)\}$$
$$M_{13} = \{(1,6), (2,1), (3,3), (4,5), (5,8), (6,7), (7,2), (8,4)\}$$
$$M_{14} = \{(1,3), (2,1), (3,6), (4,5), (5,8), (6,7), (7,2), (8,4)\}$$

(b) All Stable Matchings

Figure 4.2: "Genetic Algorithm for Sex-fair Stable Marriage Problem", IEEE 1995 [31, p.510]

Also, in the Figure 4.3 preference lists [32], men are always static and stable, as rows or columns. In women's preference list, men are statix and women are evaluating them, and not sorting them by their preference. On the other hand, women are put in order as men's choices (i.e, first choice, then second...), in men's preference lists. In both cases, men are stable.

Rank of woman evaluated by man:

	1	2	3	4
Man 1:	3	2	4	1
Man 2:	2	4	1	3
Man 3:	2	4	3	1
Man 4:	4	2	3	1
Man 5:	1	3	2	4
Man 6:	1	2	4	3
Man 7:	4	2	1	3
Man 8:	1	3	4	2

Fig.2. Men's preference list. (The each number under the bold bar in the list is woman's identity no. that man prefers.)

Man's identity no.:

	1	2	3	4	5	6	7	8
Woman 1:	5	8	3	7	3	1	5	1
Woman 2:	6	7	1	7	4	3	4	1
Woman 3:	8	6	2	6	4	1	5	2
Woman 4:	5	7	2	8	2	2	6	1

Fig.3. Women's preference list. (The each number under the bold bar in the list is man's rank evaluated by woman.)

Figure 4.3: "New Fast Iteration Algorithm for the Solution of Generalized Stable Marriage Problem", IEEE 1999 [32, p.VI-1052]

4.3.2 Names

An outstanding case regarding whether a woman is qualified to have a name or not can be found in the paper *Marriage, Honesty, Stability* [22] authored by Nicole Immoricla and Mohammad Mahdian ιν 2005.

In this paper, in order to describe the actions of a 'man' or a 'woman', one has to be 'fixed'. This means that throughout the text, the authors conceptualize a random fixed man or woman who takes action during a certain step, indexing all other men or women. Due to this, in this text women are fixed multiple times:

> "Fix a woman $g \in W$. We want to bound the probability that g has more than one stable husband." [22, p.58]

This is the way that a man is fixed when the authors need to refer to his actions:

> "Fix a man, say $Homer$. We want to bound the probability that $Homer$ names at least $k+2$ women before the number of different women he has named reaches k." [22, p.60]

When authors needed to 'fix' a man, they assign him a name. Combining this text with the mechanism of notation found above, it is clear how women are understood as a complementary part of men and, hence, have no name. In addition to this, it should be noticed that at random woman is assigned to a letter, g, stands for girl.

During this research, multiple passages where men interact with 'girls', times when women are referred to as 'girls' have been detected. Yet, this was not necessarily in a context where men were referred to as boys [18]. Even in Wagner's paper, using both 'girls-boys' and 'women-men' terminology, in some parts of the text, 'men' are negotiating with 'girls' [5]. In some paragraphs, one set is referred to as 'men' and the other as 'girls'. This indicates that while speaking or writing this schema occurred spontaneously. The 'boy-woman' combination of names was not found in any text. This indicates that in the examined texts women are considered to be inferior to men. By referring to them as girls, they are portrayed as younger, immature and weak when compared to men.

4.3.3 Data structures and Math actions

Up until now, various cases where letters are assigned to men and women have been mentioned. In this section, it will be shown how men and women are represented within some classic data structures that were used in the examined literature. Firstly, each of these data structures and math actions need to be detailed. Then, it will be demonstrated how the elements are assigned to men and women and the manner in which that this is described. It should be noted that, in computer science, data structures are used to place data in a certain topology and put into practice connections and hierarchies, in order to make calculations easier and quick.

Trees and Lattices

In computer science, a tree data structure is a hierarchical tree structure connecting elements-points-nodes. The basic node is the root node, which, hierarchically, has other nodes beneath it. These in turn, have other nodes beneath them and so on. The nodes that have no nodes beneath them are usually to be called leaves.

A lattice, on the other hand, is a more complex data structure, within which nodes are connected to each other, following some strict rules of paths (so that the paths to one node lead to the same result). In such a structure, the use of top and bottom terms are defining the flow of the lattice.

In the texts that were studied, these two data structures have been used to represent and illustrate the connection between the stable marriage that is best for men (the matching that is supposed to favor men and is the worst for women) and the stable marriage that is best for women (the matching that favors women and is the worst for men). These two stable marriage cases were explained in the 'Theoretical Framework and Secondary Literature Review' chapter. They are named male/man-optimal solution (which, at the same time, is the female/woman-pessimal solution) and female/woman-optimal solution (which, at the same time, is the male/man-pessimal solution).

In Figures 4.4 and 4.5 below, we can see the tree and lattice structures used to display all stable marriages (i.e., meaning all the matchings that meet the stability criteria as defined), accompanied by a text explaining the structures.

Fig. 1. An example of the tree-structure for the stable marriage solutions. The full size numbers represent the marriage that was broken to get from the higher solution to the lower.

In order to illustrate the process of *breakmarriage*, the nine solutions S_1, $\cdots$, S_9 have been drawn as a treelike structure in Figure 1. The figures on the branches between two stable marriages indicate which marriage is broken to get from the higher to the lower stable solution. S_1, the male optimal stable solution, is at the top of the tree. The female optimal stable solution (S_5 in this example) will always be at the bottom left-hand corner.

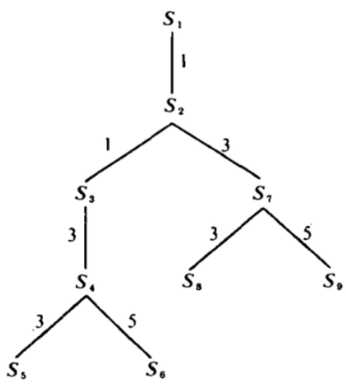

Figure 4.4: Tree of stable matching solutions, "The stable marriage problem", ACM 1971 [18, p.490;emphasis added]

In the first case (with the tree data structure) [18], the men's optimal solution is put at the top of the figure (the root of the tree). On the other hand, the women's optimal solution, is placed at the bottom left-hand corner (or leaf). In tree data structure, the root has the highest hierarchy. It is the most important node: it is unique and, as a result, put on the top. Thus, in order to reach any other node one needs to begin a path from the root. The leaves are numerous. They are the nodes that are put at the bottom. Their existence is not a prerequisite for the initiation and the existence of a tree.

This tree illustrates the mathematical logic developed in the specific text, where one begins from the male-optimal solution (the female-pessimal solution) and gradually moves to stable marriages that are worse for men and better for women. This is a popular way of thinking in the examined texts. There, to find all marriages, one begins from the male-optimal one. [31, 18, 19, 24]

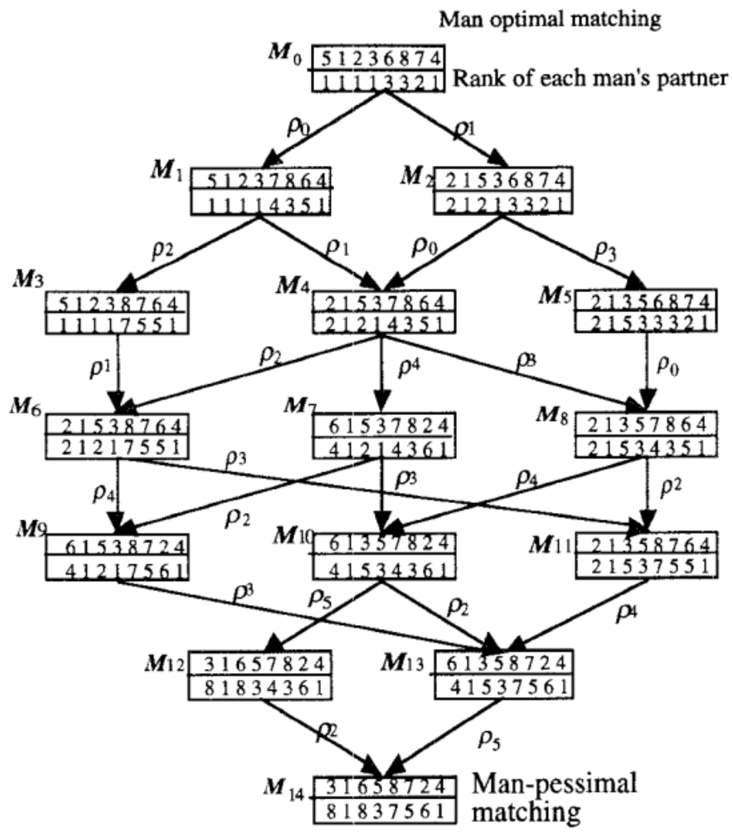

Fig. 2 Lattice structure of the instance in Fig.1

Figure 4.5: Lattice of stable matching solutions, "Genetic Algorithm for Sex-fair Stable Marriage Problem", IEEE 1995 [31, p.511]

This logic is also represented in the lattice of Figure 4.5. Again, the initial point is the man-optimal solution, which is put on top. Another detail in the representation of a lattice in this [31] is that the bottom point (the ending point) is not designated as the female-optimal solution but as the man-pessimal solution. This provides two mechanisms which will be explained at the end of the section. The one means that men are the active parts of the algorithm. Hence, the whole text, definitions and structures are wrapped around them. The other means that man-optimal solutions are put at the top, bestowed with the highest hierarchy and, consequently, importance and priority.

Additionally, between the parts-nodes of a lattice there is a relationship that indicates the flow, as well as a hierarchy between the two nodes. This is detected in the definition too, as shown in Figure 4.6 below, where dominance is established through men's preferable choices.

Definition 2 Stable matching M is said to dominate stable matching M', represented by $M \preceq M'$, if every man either prefers the partner in M to in M' or has the same partner.

Figure 4.6: Definition for lattice structure, "Genetic Algorithm for Sex-fair Stable Marriage Problem", IEEE 1995 [31, p.510]

Graphs

Graphs are a very popular data structure in computer science. They consist of a set of points-nodes and a set of connections-edges, which can exist between any two nodes (or not exist at all). If these connections are directed from one point to another, then the graph is called a 'directed graph'.

During this research, examples representing both men and women as nodes have been found, but in a very different way. In the following example [19], a graph is constructed with men as nodes.

> "Let S be a stable marriage, and assume that the preference lists have been reduced for S. Then for each man m we denote by $s(S, m)$ the woman who is second (if there is one) on m's reduced list, and by s'(S, m) her mate in S. We further denote by $G(S)$ **the directed graph consisting of n nodes, one for each man, where for every man m_i, there is a directed edge from node m_i to node s'(S, m).**"
> "It is clear that each directed cycle in $G(S)$ specifies a rotation exposed in S, and vice versa. In particular, **any directed cycle in $G(S)$ exactly specifies a set of men** in some rotation p exposed in S; **rotation p is completely defined by these men and their mates in** S, where **the pairs are ordered by the circular order of the men in the cycle**." [19, p.540;emphasis added]

In this paper, men are the nodes, and the directed connections between them signify the inclination to marry someone else's wife in the next step of the algorithm. Women enter the game only as something that a man would desire in the next step of the algorithm and, hence, would have an edge pointing to the man who now owns it. Women do not act and, are presented as powerless objects. Over the narrative of the algorithm, they gradually vanish as active (or, at least, re-active) parts of the problem. Rules are set only for men and women do not participate. Ironically, as before with the trees and lattices, the acts performed by the set of men are meant to be forming a better result for women, even though they begin from the man-optimal solution. Men define the graph, the rotation (by them and their mates, not women, who again are absent as independent units) and the cycle. Also, in this example, men are static and stable, while the ‘wives’ are moving around (‘exchanged’ as shown in the section of rotations below).

Things are quite different when a node is constructed with women as nodes. In a 2020 paper [29], there is proportion where men are the students that register for some college and women are the ‘seats of colleges’. Men are moving and deciding, women are static and accept or reject.

Another example that needs further investigation, appears in a book review found in the ACM Digital Library, though not in the work it reviews [21]. There, women are again addressed as nodes. Due to this text being only a review, not much can be extracted from it. However, there is a reference to a chapter of the reviewed book that uses the Dijkstra algorithm and an analogy between women and the nodes of the algorithm. In the Dijkstra algorithm, nodes are ‘visited’ in each step of the algorithm from ‘the one that uses the algorithm’ until all nodes are ‘visited’ Women again are assigned to a static role, waiting to be visited.

In both cases, when women or men are assigned to nodes, this implies a static behavior. However, as it will be discussed in the conclusions of this chapter, for men the term ‘static’ denotes being stable, acting and/or ‘exchanging’ their wives. Meanwhile, for women the term ‘static’ denotes waiting for someone. Also, in the men graph example, men as nodes act: the graph is active, as it involves the execution of processes. On the contrary, in the women graph examples, graph is inactive: it does not involve the execution of any processes. In the last case, the actions are performed by the ‘ones that execute the algorithm’, not the nodes themselves.

Rotations

The ‘rotation’ technique is typical in many mathematical fields. It refers to a procedure where elements rotate according to the rules of the structure they belong to. The idea of rotation was found in some of the studied texts [19, 24, 31]. All of them refer to the rotation between pairs, in an assumed graph structure. Here ‘rotation’ signifies the formation of a cycle of pairs and the therein rotation of the partners. In the first of these texts [19], this procedure is explained as follows:

> “The concept of a rotation, [...] The chief significance of such a rotation lies in the fact that if, in the male optimal solution, **each** m_i **exchanges his partner** w_i **for** w_{i+1} $(i + 1 mod r)$ then the resulting matching is also stable.” [19, p.535]

In another of these texts [24, p.1227], the phrase ‘pairs of men swapping their two wives/partners’ is used. The third text [31, p.511] resorts to this formulation: “each man m_i appearing in ρ matches $w_{(i+1) mod r}$”.

The concept of rotation (as set in these texts) is supposed to detect sets of pairs and form new pairs when the initial pairs are arranged in a circle according to certain criteria. For example, every man forms a new pair with the woman at his right, and every woman forms a new pair with the man at her left. The issue here is that the rotation does not occur among pairs. The women are rotated around men, who stay static and stable. Moreover, in order to describe this the authors construct a concept where men exchange and swap their wives. In addition, regarding the exchange of wives, there is even a variant of the stable marriage problem, defined as follows [34]:

> "Man-Exchange Stable Marriage for the classical SM (Stable Marriage), a new stability definition, man-exchange stability was defined. This stability requires, in addition to the original stability, the property that no two men prefer to exchange their partners." [34, p.134]

On the other hand, a more balanced description of rotation was found in one of the other texts [25].

A last comment regarding to rotation has to do with how an "action" in the algorithms is performed in favor of the welfare of women. The initial pairing is the best solution for men (male-optimal), who are forced to exchange wives in a rotation of pairs, in order to achieve a more "fair" pairing for both men and women. The narrative describes women as passive objects even when the algorithm is working in their favor. Wagner refers to the algorithms that use the rotation by stating that "more balanced algorithms start with the Gale-Shapley algorithm, and manipulate it by exchanging spouses in controlled ways" [5]. This is true. But the way this is described is not balanced at all. It may reflect the stereotypical conception of women as weak and men as protecting leaders, who are forced (or not) to retreat in order for women to have a better result.

This schema is used again in the "*breakmarriage*" [emphasis in text] procedure, which is discussed in the notation section.

> "This operation [breakmarriage] consists of breaking the marriage of a selected man i, in a stable marriage S, and **forcing** him, therefore, to take a poorer choice in his list. Similarly, the woman i_S (who was married to man i) **has the opportunity** of getting a better choice." [18, p.488]

Here too, women have that opportunity because men are forced to retreat. Again, men are the ones who act.

Unit Ball

A unit ball is a math structure, where all elements-points are placed in a specific distance from the center. In a paper where this structure is used [24], women are assigned to points. Figure 4.7 is the illustration provided in the original text.

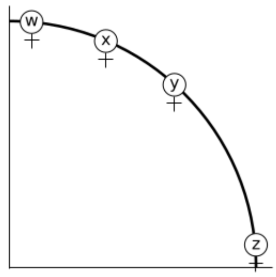

Figure 8: An example where, although adjacent, no preference list may begin with $\{x, y, z\}$.

Figure 4.7: Unit Ball, “Sampling Stable Marriages: Why Spouse-Swapping Won’t Work”, ACM 2008 [24, p.1229]

The text below is a proof of a theorem, using the unit ball structure.

> “[…] **we** will **place** all women on the quarter unit ball. That is, $\forall w, \| \mathbf{v}_w \| = 1$. For any three equidistant neighboring women on the ball, **a man may choose** these to be his favorite; he simply finds a vector that is a convex combination of the three women’s locations, and lets his function correspond to that vector. Furthermore, he can arbitrarily choose the order of those three points by letting the average weigh more heavily towards his first and then second choice. […] In actually placing the women, we need only put the large rotation in a circle, and the women for the other rotations near their neighbors […]” [24, p.1228-1229]

The concept of rotation was introduced in the previous section. Three points in this text form a certain conception of the roles of men and women and the structure. The unit ball is constructed by some ‘we’ – the authors – who are ‘placing’ the women in space. Women are placed as objects in space and, just like in the graph cases, they are static and passive. ‘A’ man chooses among all women, acting on the structure. The first point concerns the passive and active roles of women and men. The second point has to do with the fact that all women are put in order and only one man chooses (which gives him a full choice on the set of women). Finally, the way the authors use the term ‘we’ in the text indicates how they consider themselves to be ‘active’ individuals within the problem’s context. A latent identification with the man role in the algorithm is conceivable.

4.3.4 Preference Lists

The last mathematical topic that will be addressed concerns the handling and transformation of the preference lists. The stable marriage problem has many variants. When algorithms are applied to real world concepts, each concept has specific characteristics. If these characteristics are identified and the corresponding algorithmic parts get transformed accordingly, this might lead to a more clever (i.e. quicker and easier) algorithm for special cases of the general problem.

Regarding the stable marriage problem, such transformations usually pertain to the preference lists of the two sets. It is of interest how these changes happen and to which set the new type of preference lists is addressed. It has been observed that when a preference list is changed to a more strict and fixed type, usually this is the women's list. When the change allows more choices or freedom, usually the altered list is that of men.

The first category of such cases has to do with two articles where the choices of the women's preference list are very restricted or absent. In one of these [30], the preference lists of the two genders are given in matrices. Figures 4.8 and 4.9 present these matrices for, respectively, women and men.

For the canonical instance C, its women's preference matrix WP_C is defined by the function $WP_C[i,j] = j$, as illustrated in Figure 1. Lemmas 1 and 2 establish two important properties of WP_C.

$$\begin{pmatrix} 1 & 2 & \dots & n \\ 1 & 2 & \dots & n \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & \dots & n \end{pmatrix}$$

Figure 1. Women's preference matrix WP_C .

Figure 4.8: Women's preference matrix, "Lower Bounds for the Stable Marriage Problem and its Variants", IEEE 1989 [30, p.130]

At first glance, one can see that the preference matrix of men is much more complicated than that of women, and, also, bigger in size. On closer inspection, one observes that all women have the same preferences and that the men's preferences, while calculated, are still complicated and different from each other.

$$
\begin{array}{c|ccccccccccccccccc|}
 & 1 & 2 & 3 & 4 & 5 & \dots & i-2 & i-1 & i & i+1 & i+2 & \dots & n-4 & n-3 & n-2 & n-1 & n \\
\hline
1 & \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{\dots} & & & & & & & & & & \underline{n-2} & \underline{n-1} & \underline{n} \\
2 & \underline{2} & \underline{n} & \underline{1} & 3 & 4 & \dots & & & & & & & & & & & n-1 \\
3 & 1 & \underline{3} & \underline{n} & \underline{2} & 4 & \dots & & & & & & & & & & & n-1 \\
\vdots & \vdots & \vdots & & & & \ddots & & & & & & & & & & & \vdots \\
i & 1 & 2 & \dots & & & & i-2 & \underline{i} & \underline{n} & \underline{i-1} & i+1 & \dots & & & & & n-1 \\
\vdots & \vdots & \vdots & & & & & & & & & & \ddots & & & & & \vdots \\
n-2 & 1 & 2 & \dots & & & & & & & & & & n-4 & \underline{n-2} & \underline{n} & \underline{n-3} & n-1 \\
n-1 & 1 & 2 & \dots & & & & & & & & & & & n-3 & \underline{n-1} & \underline{n} & \underline{n-2} \\
n & \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{\dots} & & & & & & & & & & \underline{n-2} & \underline{n-1} & \underline{n}
\end{array}
$$

Figure 2. Men's preference matrix MP_C.

Figure 4.9: Men's preference matrix, "Lower Bounds for the Stable Marriage Problem and its Variants", IEEE 1989 [30, p.131]

The fact that all women have the same preferences is quite crucial. It means that their opinion actually does not matter for the algorithm. In fact, women are accepting the one and only man that will propose to them, due to their type of preference list and the fact that men propose in order, from man 1 onwards. Thus there is no way a woman will receive a better proposal.

This process can unfold as follows: men are ordered according to some criteria (one could think corresponding social criteria to this concept, like strength, money, prestige etc). The higher-valued man picks first. Women rank men according to these criteria as if they are all the same, which is tantamount to having no choice. Men just pick, with the lower-valued man picking last. Women have no say in this process. They can be 'lucky' and get picked from a high-valued man or be 'unlucky' and get picked last.

A similar but inverted example [24] reinforces the above point: "If all the men agree that some woman is best, she gets her first choice in any stable marriage."

The idea that one set has no preference and, hence, no opinion, is not unusual. Actually, in one article [34], it was described as a variant of the problem:

> "[One-Sided Preference Lists] There are some matching problems in which only one party (say, men) have preference lists over the other. […] A rank-maximal matching (or a greedy matching) is a matching that matches the maximum number of men to their first choice partners, and subject to this condition, the maximum number of men to their second choice partners, and so on. […] For two matchings M_1 and M_2, if the number of men who prefer M_1 to M_2 (in terms of the rank of his partner) is greater than that of men who prefer M_2 to M_1, we

say that M_1 is more popular than M_2." [34, p.134]

Elsewhere [29], the side that has no preference is, again, women.

> "But what about one-sided preference lists? Here, only one side has preference over other. [...] Let us say two men likes same woman. Which man should be paired up with the woman? It is difficult to do because, women have no preference here and selecting one man beside other without any reason seems unfair. In this case Irving chose general approach. Let us say there are two matchings M_1 and M_2. If number of men who prefer MM_1 to M_2 is larger than number of men who prefer M_2 to M_1, then M_1 is selected." [29, p.2]

In all cases displayed, women have no preferences, given or not a preference matrix. Despite this assumption, the problem's paradigm is not altered (as in the case of not disjoint sets). Here, the stereotype of women having no say in the selection of a marriage partner is apparent (because the concept is still marriage and not object allocation). "Which man should be paired up with the woman?" or the valuable object?

In a more complicated preference list variant, one set's list does not need to contain an ordering of all the other set's elements. This means that one side picks a certain number of the others which would prefer to be married than stay single. If they cannot be matched with some of them, they stay single. This property is usually assigned to only one of the two sets, and, for the most part, to the men's set. The set whose preference lists do not need to be complete preference lists has the choice and the right to stay single.

On the other hand, women's preference lists are complete and the possibility of staying single is communicated with a more negative sentiment than when the same possibility refers to men staying single. The latter is communicated as a freedom of choice for men: it is a choice of a man who just prefers to stay alone. Some indicative excerpts from two of the articles are the following:

> "By the fact that when a man provides a short list of acceptable women, preference lists of some women can be empty. This means that the women are unacceptable for any man." [38, p.109]

> "[...] man m_1's first choice of partner is w_4 , second w_2 and so on, until at some point **he prefers to remain single** (i.e. matched to the void set).
> [...]
> When no further proposals are made, the algorithm stops and **matches each woman to the man (if any)** whose proposal she is holding." [35, p.587]

In a final variation of this mechanism [32], one might observe that women can be matched to more than one man, but in no solution do they stay single. In addition, instead of resorting to the expression ‘to stay single’, this text mentions that a man ‘remains free’ [32, VI-1051].

4.3.5 Conclusions

All the above lead to three main conclusions:

- Relations within mathematical language: in some of the papers studied, the relations between men and women are hierarchical/dominating. Examples of this have been identified in the following instances: in the notation used for men and women, the way the naming of men and women is handled, their position and interaction within data structures and their different manipulation within the algorithms and the problems’ definitions. All these establish inequitably gendered relations between the two sets.
- Static positions: ‘static’ positions in the preceding mathematical structures convey the gendered active-passive model. In men’s case, being static is associated with stability, while, when this refers to women, it denotes passive waiting.
- Right of choice: the last conclusion has to do with the way men and women act and whether they possess the right to choose and/or have opinions. Preference lists in the examined papers indicate that the relevant perception is slanted, benefiting men sets. In addition, the absence of activity for the women sets (for instance in rotations) conceals the fact that they also have opinions and preferences. In these texts, women are often gradually disappearing from the narrative, while men hold the active role almost to the point that, in some algorithms, women are treated as mere objects.

4.4 Other mechanisms

Up until this point, the analysis of the texts has been focused on the language that was used for the description and the narration of the problem, the clarification of the relations between women and men and on the mathematical tools used. In this section, some additional and, arguably, valuable observations/mechanisms will be detailed.

4.4.1 Change of terminology - Sex or gender?

The inspection of the texts, showed that the terms ‘sex’ and ‘gender’ were used in the description of men and women sets. Notably, the two words were swapped even in the same text. The chronological ordering of the papers, reveals that, initially, the term ‘gender’ was absent and that the term ‘sex’ was used instead. In more recent papers, ‘gender’ gradually begins

Table 4.3: Occurrence of the terms 'sex', 'gender' and 'roles' in the examined texts.

Year of publication	'sex'	'gender'	'roles'
1971[18]	YES	NO	YES
1987[19]	YES	NO	YES
1989[20]	NO	NO	NO
1989[30]	YES	NO	NO
1995[31]	YES	NO	YES
1999[21]	YES	NO	NO
1999[32]	YES	NO	NO
2005[22]	YES	NO	NO
2006[33]	YES	NO	NO
2007[23]	YES	NO	YES
2008[24]	YES	YES	YES
2008[34]	YES	YES	YES
2010[25]	NO	YES	NO
2011[35]	NO	YES	YES
2013[26]	YES	NO	NO
2013[36]	NO	NO	NO
2015[27]	NO	NO	NO
2016[37]	NO	NO	NO
2018[28]	NO	NO	NO
2019[38]	YES	NO	NO
2020[29]	NO	YES	NO

appearing and, in some texts, completely replaces 'sex'. In others, the two coexist. Table 4.3 indicates whether a text utilizes on or more of the terms 'sex', 'gender' and 'roles'.

The table does not specify what happens when both 'sex' and 'gender' are used. Having said that, in most of the texts, where both terms are present, 'sex' is dominant especially in the main part of the text and 'gender' is mainly used in the abstract and the introduction.

In this table, it also becomes clear that the term 'gender' has only recently emerged, although it did not replace 'sex'. However, the use of the term 'roles' is present among both other terms, even in the earliest of the examined texts. It is noteworthy that the two of the papers that contain none of the three terms are, nevertheless, applying the stable marriage algorithm to another scientific field [36][37].

Regarding the terms 'sex' and 'gender', Haig [8] has explored their use in academic texts. His research provides the trends in certain disciplines (including, among others, the social sciences, the arts and the humanities) as regards the presence of the term 'gender' in the titles of the academic text. With the exception of the social sciences, the arts and the humanities, all other scientific fields have been slow to adopt the term 'gender' (without replacing 'sex' with it). A similar behavior can be observed in this thesis' limited sample of papers. Haig's more comprehensive article [8] illustrates how historically, 'gender' originated in feminist discourse and gradually spread within academia yet with most STEM fields falling behind.

> "This distinction is now only fitfully respected, and gender is often used as a simple synonym of sex."[8, p.87]

In the present study, this seems to be the case with the examined material as well. The two terms are not strongly defined or separated, while they are even swapped within the same text. Additionally, in definitions like 'male/female-optimal solution' and 'sex-equality' the same term transformation is not applied. Thus, possibly this is mostly a subliminal mixed use of 'gender' and 'sex', rather than a deliberate informed decision.

Regarding these results, two things should be pointed out.Firstly, the use of these terms seems to be spontaneous.

This could be due to mathematical language being directly affected by the general sex/gender redefinition. Secondly, the term 'roles' is always present. This fact will be analyzed in detail later in this chapter.Regardless of any specific terms, the conception of the sets as 'women' and 'men' with 'roles' brings to light a striking narrative and mindset.

4.4.2 Politically correct footnote

Another instance of the impact of public discourse on the pertinent literature can be found in an annotation from the 2008 article *Sampling stable marriages: why spouse-swapping won't work* [24], as shown in Figure 4.10.

[1] We restrict to a heterosexual context throughout this paper. Moreover, any stereotypes and/or politically incorrect statements are intended solely for clarity of exposition.

Figure 4.10: Annotation, "Sampling stable marriages: why spouse-swapping won't work", ACM 2008 [24, p.1224]

This article analyzes the stable marriage problem and applies it to a dating site. Given its positioning in the text, the footnote seems to be exclusively referring to the dating site and not to the mathematical language and notation. If this strictly concerns the application, it is remarkable how there is a perceived need for a disclaimer regarding any real-life implication while the analogous predilection in the text's mathematical portion remains unacknowledged given that both resort to the same paradigm and terms. In any case, this shows how in the twenty-first century a declaration of political correctness is a necessity, even in mathematical and scientific texts.

In these two sections (4.4.1 and 4.4.2), the examples drawn were intended as an elucidation of how changes in the public discourse have affected and have been reflected in the mathematical language of the examined texts. Here, it became obvious that mathematics can be socially influenced: over the last fifty years, the terminology and the approach of the authors of such papers has been adjusting to the era they lived in.

4.4.3 Roles and members of sets

In most of the examined texts two phrases/tropes were pervasive. The first, frames women and men as members of some sex/gender and the second refers to their characteristics and actions as a role. Some examples which cover the whole time span of the primary material are the following.

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- "It is evident that, by reversing **the roles of the men and women** and letting the women propose to the men, the female optimal solution can be obtained using these algorithms." [18, p.488;emphasis added]
- "In addition, each person is assumed to have a ranking of **the members of the opposite sex**." [21, p.3;emphasis added]
- "An instance of the stable marriage problem consists of N men, Nwomen, and each person's preference list. A preference list records the order of preferences (allowing ties) over a subset of **the members of the opposite sex.**" [23, p.2;emphasis added]
- "Likewise, **reversing the roles of men and women** determines the men's preference lists." [24, p.1224;emphasis added]

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- "An instance of the stable marriage problem of size n involves n men and n women. Each participant ranks all **members of the opposite sex** in order of preference." [30, p.129;emphasis added]
- "The stable matching found by this algorithm is extremal among many (for the worst case, in exponential order) stable matchings in that **each member in the man group** gets the best partner over all stable matchings, then each woman gets the worst partner." [33, p.5500;emphasis added]

- “The woman-proposing version works in the same way by swapping the **roles of men and women**.” [35, p.587;emphasis added]

- “In many situations, men and women are not required to rank **members of opposite sex** in a strict order. They also can find some **members of opposite sex** unacceptable.” [38, p.107;emphasis added]

Both phrases set a distance between the elements of the algorithm (men and women) and their characteristics. If someone is a member of a set, they just adopt the identity already embedded in that set. The term ‘member’ arguably supports this kind of distance. The use of the term ‘roles’ is more straightforward. Tellingly as shown in Table 4.3 the later term is present even in the earliest of the examined articles. Within the context of the stable marriage mathematical concept, the sets of men and women, having none of their social characteristics, ‘act’ as their ‘roles’ indicate. However their roles might precipitate certain additional actions. At this point, things get more complicated. Gendered mathematical roles are not clearly differentiated from real-world ones, as it seems like the former relay what their names indicate in the real world, into the way the authors think about roles, names and the algorithm.

It should be noted that, when the authors state that ‘if the roles of men and women are reversed’ or ‘interchanged’, such statements carry no additional information or mathematical value. This is different from cases that call for the allotment of ‘proposal’ or ‘respond to proposal’ actions to both gender identities, which would entail the exploration of how the members of each set will act through the steps of the algorithm. Still, there were some papers with more complex and diverse roles while the problem can also be formulated in a way where both genders propose or respond in some step of the algorithm. But this is a different case. Yet, given that, regarding the concept of marriage, men and women in the examined narratives bound to their ‘traditional social roles’ and, beyond these, have no further social or other characteristics, it is astounding how mathematical notions are that much linked to social ones especially when this has no actual scientific value.

Men and women are just placeholder names that accompany the actions of proposals and responses. Claiming that the reversal of these names would lead to symmetric results, does not change or add anything. The only thing that a neutral mathematical text should be concerned with is the algorithm’s roles per se and not the names that identify these roles. When women assume an algorithmic role that would otherwise be assigned to men, there is no reason to contemplate whether they would achieve the same results. Ii is at this exact point that, when mathematicians refer to women they do not solely recognize the name of a variable but, also, any possible social connotations the use of the word ‘woman’ might have. Hence, it seems valuable to them to state that women would achieve the same results if they were the ones proposing.

In order to put this more simply, it is important to revisit the problem's definition: the elements of two distinct sets of the same size need to be matched. In this problem's narration, scientists came up with the concept of marriage, men and women (as already exposed in Chapter 2). Therefore, the initial conceptualization requires two specific sets: a set of men and a set of women, whose members have no characteristics other that their designation. Even so, it was the Gale-Shapley algorithm that introduced the 'proposal' and 'response' actions. From that point on, mathematicians leaned towards men being the ones proposing and women the ones responding, with no other characteristic being attached to the two sets. There is no mathematical text that needs to contain the following clarification: "in $(a - b)(a + b) = a_2 - b_2$ if a and b was the other way around the same result would happen for b and a reversed".

Elaborating upon this point, the next excerpt makes clear that the authors' concern for symmetry is not motivated by an inclination for specificity and inclusivity. It includes a subtle distinction that can pass unnoticed: women are not women but, instead, the name of a set with characteristics that form a role. A practical example of this is the following:

> "Property 3. If every man is paired with the first woman on his shortlist, then the resulting matching is stable; it is called the male optimal solution, for no man can have a better partner than he does in this matching, and indeed no woman can have a worse one.
>
> Property 4. If the roles of males and females are interchanged, and if every woman is paired with the first man on her (female-oriented) shortlist, then the resulting matching is stable; it is called the female optimal solution, for no woman can have a better partner than she does in this matching, and indeed no man can have a worse one." [19, p.534]

It might seem like this passage is quoted for the sake of completeness. However, closer inspection brings forward this understanding of the mater: "whichever set's members are paired (through making proposals) with theirs first choices, achieve the best result for the set". We could state that 'reversing the roles' equals 'replace the names'. This is also explained in the following excerpt:

> "The woman-proposing version works in the same way by swapping the roles of men and women. It is then observed that which side proposes in the algorithm has significant consequences. Specifically, the algorithm finds the two extremes among the set of stable matchings. The man-proposing version yields a man-optimal outcome that every man likes at least as well as any other stable matching, and the woman-proposing version a woman-optimal one. This is referred to as the polarization of stable matchings." [35, p.587]

Here, it is not argued that definitions such as the 'female optimal solution' or alternative modes of calculation are useless or incorrect. Rather, in this section it has been attempted to establish that such statements, which were found at most of the texts that pertain to the problem (and not to the application of the solution/algorithm), are mostly (if not exclusively) social in character.

Additionally, given all the above, the 'inverse the roles' trope suggests that roles are attached to their initial set: i.e., that men and women have specific roles. Thus, this phrase, except from reversibility, affirms that the roles are attached to their initial 'sets'.

An additional observation that follows from this analysis has to do with the constant utilization of men and women as ontology for the algorithm's roles - which implies certain characteristics. Potentially, this is indicative for a specific conception of society on the authors' part. This comes to light when they are forced to insert social concepts into a 'mathematical' language. The insight gender studies provide us with leads to one critical observation. In the world of mathematics, it appears valid to refer to men and women behaviors as 'acts' or 'roles'. It also seems valid to 'inverse the roles'. However, mathematical language tend to be very strict and laconic, as it typically involves only what is necessary for the construction of a mathematical concept, in a quest for the most orderly of expressions. Yet, the fact that, in the transmutation of marriage from a social to a mathematical concept, the gender narrative remained intact in the form of roles is remarkable. This probably quite telling of the way we understand society, roles and the relations between genders even though the attitudes of question only apply to two gender identities, their traditional social roles and the heterosexual relationship that can be formed between them. The possibility that the need for such the roles is, in fact, generated by a need to inverse them perhaps merits further research. In any case, the naming of certain behaviors as roles and the ability to inverse them are aspects that should probably be revisited in order to better understand the causes behind them, as their presence in the relevant literature has not been questioned.

4.4.4 Reversing the roles

An noteworthy case regarding the algorithm's workings has been presented in the 2013 paper *Stable Marriage and Roommate Problems with Individual-based Stability* [26] and seems to be of particular interest for the field of Science and Technology Studies. In this paper, the author suggests an algorithm that includes the step "[n]ow run the **woman-optimal version** of the Gale-Shapley algorithm..."[26, p.291;emphasis added], which means that women propose and end up with their best possible choices as husbands.

Arguably, such formulation has no additional meaning. It is incomprehensible why this could not be simple expressed as 'run the Gale-Shapley algorithm with optimal results for set x' (where 'set x' can be either of the two), instead of presupposing a special, female 'version' — which, perhaps inadvertently, also implies that masculine configurations are the mathematical and, more generally, scientific ‘normal’/‘default’. Be that as it may, the trend of ‘reversing the roles’, or referring to ‘women and men’ instead of ‘men and women’ indicates a modicum reflexivity. Nevertheless, here, granted a thematic adjustment, Martin’s conclusion from her research on egg and sperm representations is quite applicable: “[all of the discussed] revisionist accounts of egg and sperm cannot seem to escape the hierarchical imagery of older accounts.” [4, p.498].

Given all the above, the following question seems timely: is making the roles ‘reversible’ enough or should one be concerned with the hierarchical relations, and the stereotypically traditional characteristics of femininity and masculinity that are perpetuated within and through these algorithms?

4.4.5 Role-free model case

Another interesting case can be found in the 2010 article *On Heuristics for Two-Sided Matching: Revisiting the Stable Marriage Problem as a Multiobjective Problem* [25]. In this text, the stable marriage problem is approached from a different perspective: “[u]sing evolutionary computation and an agent-based model heuristics, this paper investigates the stable marriage problem as a multiobjective problem...” [25, p.1283]. For the purposes of this thesis it suffices to say that this case also involves two sets, only this time both of them perform the same actions towards the other, in a random order. In the article’s description of the algorithm, no ‘feminine’ or ‘masculine’ characteristics are identified in either of the sets. However the terms ‘men’ or ‘women’ are still used.

These observations lead to two conclusions. Firstly, even when ‘behaviors’ and ‘roles’ are absent the heterosexual framing of the problem is not dropped and the paradigm is not changed despite the fact that this could be done easily and with no repercussions. Secondly, when new approaches are applied to old problems, deciding whether to preserve or revise the terminology, narration and construction of a problem can be difficult. Which path leads to new ways of thinking and computing?

In the present and the previous section, the two examined cases illustrated how the understanding of this problem is thoroughly bound to the conventionally accepted paradigm and the attendant terminology so much so that, even when it is mathematically pointless, this outlook lingers on and, eventually, affects the mathematical formulations.

Chapter 5
Conclusions

Having dissected the stable marriage problem and the related algorithms into three levels (definition, natural language, and math notation/data structure) and, subsequently, having presented some initial findings, this chapter collects the results for each level and puts forth the ensuing conclusions.

In the primary research results, an overview of the definitions and the accompanying narration—as found in several of the examined papers—are set out. Next, through an inspection of the natural language that was utilized, the assortment of verbs and nouns used for mathematical procedures was displayed. Lastly, by highlighting how the related terminology underwent changes over time, it was shown that mathematical language (and the manner in which mathematicians write) can vary and be affected by the social context of a given period.

All the above, promptly lead to one first conclusion: society exerts a significant influence on this problem. Proof of this can be found in the social conceptualizations within the narration, the variety of the words used and the fact that the applicable language changed over time.

Moving on to the second conclusion, in the papers that were studied, the role of women was much more passive than that of men. This can be observed in the language used, the structure of algorithmic processes/steps and the data structures themselves. Men are the most active set of the algorithm, while women the most passive. Additionally, on occasion, women gradually disappear from the narrative.

Here, an interesting finding was how the 'static' property was presented for each set, especially in data structures and algorithmic steps. For men, staying 'static' means that one holds his position. For women, this means they are being 'exchanged' around or that the pair is matched with the man's position and forms a 'static' place. Hence, 'static' for men is closer to a stable position. For women, staying 'static' means waiting or being put in some order for men to choose. As 'static' parts, women anticipate men that 'visit' or 'pick' them. Thus, their purpose and destiny is to passively wait for someone.

These conclusions were inscribed into mathematical notation and data structures, i.e., pure mathematics. This can be extended to a 'lower' level instead of reserving it for the explanation of behaviors within the algorithm. Such analyses have been detailed in Wagner's work [5]. The treatment of mathematical notation and data structures as another kind of language is not far off from this concept.

The third conclusion concerns how, in sentences where both ontologies are present, women find themselves in the place of an object: they are even treated as syntactic subjects when the passive voice is used. In relation to this objectification mechanism, mathematical notation, algorithmic narration, verbs that can be reasonably understood as possessive and dominating, possessive pronouns and data structures position woman as an object that is picked, exchanged, ordered, has no preference and is largely absent from the narration and the action.

The objectification of women is present in every level of the algorithm. This might be due to the combination of social conceptions alongside mathematical 'necessity' for one set to be more active than the other. However, natural language analysis reveals that stereotypical conceptions of gender roles and the resulting objectification of women are the main reasons affecting the perpetuation of certain predicaments within the concept of marriage meaning, within the relationship of women and men. The passive/active model and the objectification of women engender a hierarchical conception of said relations within the algorithm. A man 'has the right to own' a woman, and this is engraved in every level of the algorithm.

The next conclusion stemming from the research findings focuses on the right of choice. This right's meaning is twofold, as it denotes: a) the right of expressing a preference opinion over the possible partners and b) the right to prefer, at some point, to remain single. It has been shown how the preference lists of the two sets were manipulated whenever the problem became more specific and, also, how the two lists differed in the examined papers. An easy explanation for this could be limited to the problem's definition. Yet, given that this is a repeated phenomenon (supplemented with the use of specific language and notation) it is easy to recognize the analytical applicability of the two motifs that Keller has detailed: "mastery of one's one fate" and "mastery over another's" [3].

Men's preference lists sometimes afford them the right to remain single (or '*free*') instead of marrying someone they do not want. On the other hand, women's preference lists can be missed, or configured in a way that renders them meaningless. In any case, women have to get married. If they do not, this is communicated as a failure rather than a choice or a freedom as would be in the case for men in such a scenario. Thus, freedom of choice and of staying single as a masculine characteristic is embedded and reproduced via preference lists.

The final conclusion, has been explicated in the last chapter. The social context of the terms used in the stable marriage problem generate mathematical necessities that are not exactly scientific. This particular problem (which, essentially, is a matching of two sets) is bound to the paradigm of marriage and, therefore, to gendered roles, that go beyond mathematical notation and clarity. Men and women ought to merely be names for the two sets. However, as the analysis of the narrative indicated, the terms saturate the sets with other properties and conceptions, which clearly exceed the uncomplicated demand for a more vivid description. Lastly, even when the terms ‘men’, ‘women’ and ‘marriage’ were meant to simply serve as an illustration that entails no other differentiations or social context, is has been plain to see how this matching process could not evade the heterosexual context of traditional marriage.

A closing note

Here, it should once again be mentioned that the examined material was meant to be as diverse as possible. The conclusions arose from this specific selection of material and there was no extended research in mathematical schemas (e.g., preference lists) from a bibliography that would be mathematically consistent.

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