Consumer Choice, Market Concentration, and Wealth Distribution in Connected and Disconnected Economic Structures: An Agent-Based Modeling Approach

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Abstract

This thesis employs agent-based models (ABMs) to investigate the dynamics of consumer choice, market concentration, and capital distribution in artificial economies, aiming to provide insights into the micro-level behavior and its influence on macro-level outcomes. We depart from classical demand theory by incorporating heterogeneity among agents, bounded rationality, and multiparametric demand functions, in contrast to the representative agent framework. Through simulation-based approaches and incorporating various economic theories and concepts, such as firm dynamics, central banking, price formation, and profit maximization, we analyze the complex interactions between consumers, firms, and market conditions. By studying economies with multiple commodities and agents heterogeneous in terms of preferences and income, we derive relations of a dynamic nature and gain a deeper understanding of how individual choices impact the overall system. Our findings contribute to the literature on agent-based modeling in economics and offer valuable insights for policymakers and researchers interested in understanding the complex dynamics of economic systems.

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1 Introduction

In the evolving landscape of computational economics, the influence of Agent-Based Modeling (ABM) is becoming increasingly prominent (Macal (2010)). Before we delve into the myriad advantages and applications of ABM, it's crucial to anchor ourselves in its foundational principles (Gilbert (2008)). At its essence, ABM represents a computational methodology, meticulously crafted to simulate and analyze intricate economic systems (Tesfatsion (2006a)). This is achieved by modeling individual agents, each governed by predefined or adaptive algorithms (Epstein, Axtell (1996)). The true marvel lies in observing the collective results of their interactions, which frequently unveil profound insights into economic behaviors (Bonabeau (2002)).

The decision to incorporate ABM in computational economics is backed by compelling reasons (Farmer, Geanakoplos (2009)). For one, while traditional models have a penchant for treating agents as homogenous entities, ABM breaks this mold (Axelrod (1997)). It champions the cause of diversity and heterogeneity, enabling the programming of agents with distinct algorithms or characteristics (Bonabeau (2002)). This approach mirrors the rich diversity we observe in real-world economic actors (Tesfatsion (2006a)). Additionally, ABM shines in its capacity to capture emergent phenomena Miller, Page (2007). These aren't the result of explicit design but naturally arise in computational systems, reflecting patterns or economic phenomena that organically emerge from the chaos of agent interactions (Epstein (1999)). Then there's the challenge of complexity. Economic behaviors, interactions, and dependencies are anything but straightforward Arthur (1999). ABM, with its computational backbone, is adept at navigating these intricacies, offering a depth of analysis that often surpasses traditional models (Farmer, Geanakoplos (2009)). Furthermore, it's pivotal to acknowledge that economic interactions aren't imprisoned within static boundaries. They thrive in dynamic environments (Axelrod (1997)). ABM, powered by advanced computational platforms, can emulate a plethora of environments, from networks to grids, aligning more closely with our lived realities (Bonabeau (2002)).

A pivot towards ABM isn't merely academic; it addresses pressing challenges in computational macroeconomics (Colander (2006)). A longstanding quandary has been the chasm between micro behaviors and macro outcomes. ABM bridges this gap with remarkable finesse, illuminating how seemingly inconsequential micro-level behaviors can converge to sculpt monumental macro-level shifts (Kirman (1992)). In a world still nursing the scars of global financial meltdowns, there's a palpable demand for tools that can simulate and deconstruct a spectrum of scenarios (Helbing (2010)). ABM rises to this occasion, facilitating diverse simulations that shed light on how economic systems grapple with external shocks (Battiston et al. (2016)). Policymaking, often likened to treading unknown terrains, stands to gain immensely from ABM (Tesfatsion (2006a)). With its prowess in assessing policy impacts at a granular level, ABM provides a vantage point, offering clarity on potential macro-level repercussions (Farmer (2015)). Markets, in their ever-evolving nature, demand tools that can keep pace. ABM offers nuanced insights into market dynamics, capturing the nuances of shifting market structures and power dynamics (LeBaron (2000)). Beyond the cold calculus of numbers, economics is a testament to human behaviors. By weaving behavioral economics into computational models, ABM offers a more rounded, holistic perspective, sidestepping the sometimes unrealistic confines of the "rational agent" paradigm (Gintis (2009)). Lastly, in an age where economic disparities are under the microscope, ABM's intricate agent tracking casts a spotlight on wealth and resource distribution, unearthing potential avenues to redress these imbalances (Epstein (2006)).

While the problems addressed by ABM are manifold, its advantages in economic modeling are equally compelling ()Axelrod (1997). As economies burgeon in complexity, the models that seek to represent them must evolve in tandem. ABM is a beacon of scalability, with the capability to simulate a gamut of systems, from localized economies to sprawling global behemoths (Tesfatsion (2006a)). The dynamic nature of economics demands tools that are both flexible and adaptable. ABM's design, which allows for the simulation of diverse scenarios by fine-tuning agent rules or system parameters, ensures it remains both relevant and agile (Bonabeau (2002)). The litmus test for any model lies in its fidelity to reality. ABM, in its meticulous design, captures the essence of real-world behaviors and scenarios, aspects that often elude or get oversimplified in conventional mathematical models (Gilbert (2008)). Moreover, the fluidity of the economic landscape and our evolving understanding necessitates tools that can adapt. With its capacity for iterative refinement, ABM remains evergreen, updating and evolving as fresh data or theories come to the fore (Gilbert, Troitzsch (1995)).

In conclusion, the discourse isn't about the feasibility of integrating Agent-Based Modeling into economics. Instead, the real question is discerning the opportunity cost of not doing so. With its unparalleled granularity, adaptability, and fidelity to the real world, ABM is poised to redefine our grasp of complex economic systems, promising clarity where traditional models might only proffer ambiguity (Macy, Willer (2002)).

1.1 Thesis chapter analysis

In **Chapter 2** we delve on the theoretical underpinnings of the research topics we examine in Chapters 6-11. In particular we give the bare essentials of the theory of consumer choice we use in Chapter 6, the rudiments of the demand theory we use in Chapter 7, the basics of utility theory we use in Chapters 6 and 7, the essentials of rationality and uncertainty in economics we use in Chapter 8, the importance of wealth inequality we use in Chapters 9 and 10 and 11.

In **Chapter 3** the problem addressed in this thesis is stated in greater detail along with the underlying motivation. The pros of the agent-based method are outlined and the key problems which addressed in this research are summarized.

In **Chapter 4** the tools and methodologies used in this research are presented. In particular, Monte Carlo simulations, Machine learning methods such as Gaussian Process and Random Forest, Markov processes and Stochastic Processes, random walks, economic indices in computational economics and agent-based modeling, networks, small-world properties and agent-based modeling in economics are discussed and analyzed.

In **Chapter 5** Literature review relevant to the problems addressed in Chapters 6 to 11 is given.

With this motivation, in a bottom up agent-based approach, we address the problem of understanding macro-level outcomes through micro-level behaviour. As expected this problem has many aspects and facets. Some of these facets and aspects are addressed in chapters 6, 7, 9, 10, 11, and 12.

In **Chapter 6** we study an agent-based model of consumer choice. The main advantage of this type of modeling is that it allows for heterogeneity between agents compared to the 'representative agent' framework of classical demand theory. Agents are divided into heterogeneous groups. Hence, there does not exist a single representative agent. This constitutes a significant departure from classical demand theory. The sources of heterogeneity are linked to the preferences of the individuals and their income. Each group consists of agents who are more or less homogeneous. The choice of commodity bundles is serially optimal. This implies the existence of a corresponding optimal orbit associated with an initial commodity bundle. The rationality assumption embedded in classical demand theory is retained here. In particular, each agent is unboundedly rational, maximizing his utility. At the terminal commodity bundle the income of the agent has been exhausted. We so obtain entire trajectories of optimal consumption bundles as opposed to single optimum points in commodity space which is the case in classical demand theory. In addition, one can observe the expenditure path for a group of homogeneous agents as they gradually exhaust their income in each step of the algorithm as well as the average expenditure path. These are features that are unique to the agent-based approach. Furthermore, one can relate the various heterogeneity factors between the groups of agents and extract intuitive relations. The generation of these results is a feature of the simulations that characterize agent-based modeling.

In **Chapter 7** we study an agent-based model of consumer demand. In particular we calculate demand functions of a heterogeneous set of consumers and of a market consisting of different types of products. Consumers are characterized by income and by utility functions that reflect their preferences whereas market is characterised by the prices of products. We calculate demand functions of these products by using the prices of the products and the preferences and incomes of the consumers and interpolate these data with random forest interpolation. The demand functions we calculate are multiparametric. We depict them graphically in various forms by using alternatively the type of consumer, the price of the product, and a range of incomes of the consumers. We construct Engel curves and we draw conclusions about the market and the economy we simulate. The data are constructed by us; however our data-driven method can be applied to any relevant given set of data. Therefore, we can provide a reliable model for forecasting the demand preferences of a group of consumers given some characteristics of the consumers and the products' prices.

In **Chapter 8** we introduce an agent-based method in order to generate data with Monte Carlo and then we interpolate the data with machine learning methods in order to derive multi-parametric demand functions. In particular, the model we construct is implemented in a simulated economy with 1000 consumers and two products, where each consumer is characterized by a unique set of preferences and available income. The demand for each product is determined by a stochastic process, incorporating the uncertainty in consumer preferences. By interpolating the data for the demands for various scenarios and types of consumers we derive poly-parametric demand functions. These demand functions are partially in tension with classical demand theory since on certain occasions they imply that the demand of a product increases as its price increases. Our proposed method of generating data from discrete agents with Monte Carlo and of interpolating the data with machine learning methods can be easily generalized and applied to the assessment of economic theories and to the derivation of economic laws in a bottom-up approach.

In **Chapter 9** we study an agent-based model for dynamic distribution of firms in a closed (supply) economy. Our model economy consists of N non-interacting firms. The Capital of each firm is driven by a multiplicative random walk with a drift. The initial capitals of the firms are homogeneously distributed on a closed interval. The dynamics is determined by two universal constants, the capital productivity and the expenditure coefficient, which are the same across the firms of the economy, and the price of the unit produced by the companies which undergoes stochastic fluctuations in a supply economy. These stochastic fluctuations produce business cycles even in the absence of interactions between the firms. We run Monte Carlo simulations in a finite time horizon and we find that the distribution of the final capitals of the firms not only it is not homogeneous but it may lead to monopolies. What is crucial in the dynamics of our economy is the interrelationship between the universal constants and the stochastic process which drives the unit price. This relation determines the phases of the business cycle undergone by the firms of the economy as well as the final distribution of the capitals of the firms the economy. In order to explicate better the dynamics of our model we subsequently consider different sectors in our economy where the capital productivity and the expenditure coefficient cease to be universal but vary from sector to sector. We use various metrics to relate the final distribution of the capitals with the capital productivity, the expenditure coefficient and the probability distribution of the unit price.

In **Chapter 10** we use an agent-based model in order to simulate firm dynamics, market concentration, capital distribution and the critical role of banking capital in economic markets. Our model incorporates firm profit stochastic fluctuations allowing for firm entries and exits in economic markets. Our results show how shifts in economic parameters influence capital allocation and market concentration. One key result of our study is that under quite general assumptions even when wealth is initially evenly distributed among all participants, as time progresses, a small number of agents accumulate a substantial portion of the total wealth, while a large number of agents have a comparatively small share. An issue of critical relevance in today's world is economic inequality. Our findings reflect the prevalent real-world trends of escalating inequality in capital distribution over time, adding another layer of complexity to our understanding of market dynamics.

In **Chapter 11** we construct an agent-based model in order to understand money distribution in a closed economic system. We assume that the system of economic agents forms a network, where each agent exchanges money only with the agents adjacent to him. As usual, we model such a network as a graph whose vertices represent economic agents and the edges signify possible monetary transactions. What is more, we utilize Quantum Random Networks in order to model the mechanism of money exchange between agents. Random walks have been used to simulate various system dynamics. However, such models assume independent and identically distributed random steps, often leading to a Gaussian distribution over time. This linearity and predictability fall short in encapsulating the non-linear complexities and inherent uncertainties of economic transactions. In contrast, quantum random walks introduce an additional layer of randomness, owing to the fundamental principles of quantum mechanics. The probabilistic nature of quantum mechanics aligns closely with the stochastic character of economic transactions, and thus, presents a more accurate tool for modeling money exchanges. In our approach, as the system cannot reach an equilibrium distribution, we utilize the Gini coefficient in order to measure the distribution of money among the agents in the network. Our primary purpose is to examine how the topology of the network affects the distribution of money. The main results shows that when the connectivity of the networks increases, the inequality (represented by the Gini coefficient) decreases.

In **Chapter 12** we discuss the results and we outline prospects for future research.

2 A bottom up theoretical analysis of the research topics

The study of consumer behavior, decision-making, and wealth inequality is a crucial aspect of economics, providing insight into how individuals allocate their limited resources in the marketplace and how this allocation affects the distribution of wealth within an economy. The intersection between consumer choice, wealth inequality, and agent-based modeling is particularly relevant in today's dynamic and complex economic environment (Scheller et al. (2019)). Agent-based models are computational models that simulate the interactions of individual agents, such as consumers, in a system. This approach to modeling has become increasingly popular in recent years, as it offers a more nuanced and realistic representation of consumer behavior and wealth distribution compared to traditional models (Lo (2005)).

Consumer choice theory, a fundamental concept in economics, attempts to understand how individuals make decisions about what goods and services to purchase, taking into account various factors such as their preferences, budget constraints, market prices, and market structure. The concept of utility, which is a measure of the satisfaction a consumer derives from a good or service, plays a central role in consumer choice theory. The classical economists, such as Adam Smith and David Ricardo, laid the foundation for the theory of consumer behavior by developing the concept of utility and considering how consumers make choices based on their preferences and the prices of goods (Desjardins (2005)).

Wealth inequality, on the other hand, deals with the unequal distribution of assets, resources, and opportunities within an economy. As consumer choices directly impact the accumulation and distribution of wealth, understanding the complex interplay between consumer behavior and wealth inequality is essential for designing effective policies to address this issue. Agent-based modeling can provide valuable insights into the effects of consumer choices on wealth inequality and help identify potential drivers of economic disparities (Huang, Chen (2021)).

The use of machine learning algorithms in economics, consumer choice, and wealth inequality research is also becoming increasingly widespread, as they can help identify patterns and relationships in data that might otherwise be overlooked (Sharma, Osei-Bryson (2020)).

Given the importance of consumer choice, wealth inequality, and the benefits of agent-based modeling and machine learning, this thesis aims to explore the intersection between consumer choice, wealth inequality, agent-based modeling, and machine learning. The goal is to provide a deeper understanding of consumer behavior, decision-making, and wealth distribution, with a focus on how these computational models can help us better understand the complexities of the real-world marketplace and inform policies to address wealth disparities (Retzlaff, Zöttl (2021)).

2.1 Utility theory

The early concepts of utility theory can be traced back to the late 18th century with the works of Jeremy Bentham. Bentham introduced the concept of "utility," which he defined as the degree of pleasure or satisfaction that a person derives from consuming a good or service (Bentham (1843)). He argued that consumers make choices based on the utility they expect to receive from different options, and that the utility of a good or service depends on its characteristics and the individual's preferences.

In the early 18th century, Bernoulli Daniel further developed Bentham's theory by introducing the concept of expected utility (Levin, Milgrom (2006)). He suggested that consumers do not simply make choices based on the utility they expect to receive from different options, but also take into account the degree of risk or uncertainty associated with each option.These early concepts laid the foundation for the development of utility theory, which has since been expanded and refined by many economists and researchers over the centuries. For example, later researchers introduced the concept of cardinal utility, which measures the absolute magnitude of utility, and ordinal utility, which ranks options based on the relative utility they provide to the individual (Bogue (1981)).

Many researchers studied their work and made important contributions in the field. Adam Smith and David Ricardo, two of the classical economists, made important contributions to the theory of consumer behavior and the development of the concept of utility (Kurz, Salvadori (2010)). Adam Smith is considered one of the fathers of economics, and in his book "The Wealth of Nations," he introduced the concept of utility as a measure of the satisfaction a consumer derives from consuming a good. Smith believed that consumers make choices based on the expected utility of different options and that the market price of a good reflects its utility to consumers (Hall, Hitch (2011)).

David Ricardo, another classical economist, expanded on Smith's ideas and developed the theory of comparative advantage. In his work, Ricardo argued that consumers make choices based on the relative prices of goods and the utility they derive from them. He believed that trade between countries could be beneficial for both parties if each country specializes in producing the goods in which it has a comparative advantage (Kurz (2015)).

"The greatest improvement in the productive powers of labour, and the greater part of the skill, dexterity, and judgment with which it is anywhere directed, or applied, seem to have been the effects of the division of labour."

William Stanley Jevons was an English economist who made important contributions to the theory of consumer behavior and the concept of utility in the late 19th century. Jevons is most famous for introducing the concept of marginal utility, which argues that the satisfaction a consumer derives from a good decrease as they consume more of it (Cho, Hudik (2000)). Jevons believed that consumers make choices based on the expected marginal utility they will receive from consuming a good. In other words, consumers will continue to purchase a good as long as the additional satisfaction they receive from consuming one more unit (the marginal utility) is greater than the cost of that unit (Stigler (1950)).

The concept of marginal utility, introduced by William Stanley Jevons, can be represented mathematically as $MU = \frac{\Delta U}{\Delta Q}$, where MU is the marginal utility, ΔU is the change in total utility, and ΔQ is the change in the quantity of the good consumed.

"A man's purchases are limited by his needs and desires, which increase faster than his means of satisfying them."

Alfred Marshall was an English economist who made significant contributions to the development of the field of economics, including the theory of consumer behavior. In his influential textbook, "Principles of Economics," he introduced the concept of marginal utility, which refers to the additional satisfaction or pleasure that a person derives from consuming one more unit of a good or service (Ekelund, Hébert (2002)). Marshall argued that consumers make choices based on the marginal utility they expect to receive from different options. According to his theory, consumers will choose the option that provides the highest marginal utility, until the marginal utility of each additional unit of that good becomes equal to the marginal utility of the next best option. At that point, the consumer will switch to the next best option, and so on (Clemes, Mollenkopf (2010)).

" The consumer's demand for a commodity is a function not of how much of it he has, but of how much he would like to have."

Vilfredo Pareto was an Italian economist who made significant contributions to the field of economics, including the theory of utility. He extended the concept of utility by introducing the idea of Pareto efficiency, which is a fundamental concept in welfare economics (Wood (1999)). According to the theory of utility, individuals make choices based on their preferences and the marginal utility they expect to receive from different options. Pareto efficiency builds on this idea by examining the efficiency of resource allocations in society (Faravelli (2007)). Pareto efficiency states that an allocation of resources is efficient if it is not possible to make one person better off without making someone else worse off. This means that the allocation is considered optimal if there is no way to improve the well-being of one person without decreasing the well-being of another person (Bishop, Heberlein (1993)).

"Efficiency means doing things right, while effectiveness means doing the right things."

Irving Fisher was an American economist who made significant contributions to the field of economics, including the theory of consumer behavior and the theory of utility. He is best known for introducing the concept of indifference curves, which is a fundamental tool in microeconomics analysis (Fisher (1921)). The theory of utility is concerned with how consumers make choices based on their preferences and the marginal utility they expect to receive from different options. Fisher added to this theory by introducing the concept of indifference curves, which graphically represent the combinations of goods that provide the same level of satisfaction to the consumer (Hicks (1940)). An indifference curve shows all combinations of two goods that the consumer is indifferent between. This means that the consumer is equally satisfied with all points on a particular indifference curve, and is willing to exchange one good for another until they reach the highest possible indifference curve (Hill (1988)).

Indifference curves, introduced by Irving Fisher, represent combinations of goods that provide the same level of satisfaction to the consumer. Mathematically, an indifference curve can be expressed as a function of two goods, $U(X_1, X_2) = k$, where X_1 and X_2 are the quantities of the two goods, and k is a constant that represents the level of satisfaction.

"Utility is that attribute of things which tends to render them useful or desirable."

John Hicks was a British economist who made important contributions to the field of economics, including the theory of consumer behavior and the theory of utility. He is best known for introducing the concept of compensation tests, which are used to determine the relationship between two goods in the context of consumer behavior (Puttaswamaiah (2018)). The theory of utility is concerned with how consumers make choices based on their preferences and the marginal utility they expect to receive from different options. Hicks added to this theory by introducing the concept of compensation tests, which are used to determine whether a change in the price of one good affect the demand for another good (Freeman, Taylor (2014)). Compensation tests are used to determine whether two goods are substitutes or complements. For example, if a decrease in the price of good A leads to an increase in the demand for good B, then goods A and B are substitutes. On the other hand, if a decrease in the price of good A leads to a decrease in the demand for good B, then goods A and B are complements. Compensation tests are important because they provide insights into the relationship between different goods, which can be used to better understand consumer behavior and to analyze the effects of changes in consumer preferences or market conditions (Nicholson, Snyder (2012)).

Paul Samuelson was an American economist who made significant contributions to the field of economics, including the theory of consumer behavior and the theory of utility. He is particularly known for introducing the concept of revealed preference (Samuelson (1950)). The theory of utility is concerned with how consumers make choices based on their preferences and the marginal utility they expect to receive from different options. Samuelson expanded this theory by introducing the concept of revealed preference, which states that a person's preferences can be revealed by their choices in the marketplace (Hands (2014)). Revealed preference theory has important implications for understanding consumer behavior and for designing economic policies that reflect consumer preferences. By using revealed preference theory, economists can analyze consumer choices in the marketplace to determine consumer preferences and to predict how changes in market conditions, such as changes in prices, will affect consumer behavior (DeShazo, Fermo (2002)).

Paul Samuelson's concept of revealed preference can be represented mathematically using the Weak Axiom of Revealed Preference (WARP). If a consumer chooses a bundle (x_1, x_2) over another bundle (y_1, y_2), then the consumer reveals that (x_1, x_2) is preferred to (y_1, y_2). WARP can be expressed as $(x_1, x_2) \succsim (y_1, y_2) \Rightarrow p_1x_1 + p_2x_2 \leq p_1y_1 + p_2y_2$, where p_1 and p_2 are the prices of goods 1 and 2, respectively.

2.2 Demand theory: contemporary perspectives

Demand theory continues to evolve as researchers incorporate contemporary insights and real-world data to refine the underlying models. Some of the recent developments in demand theory include the incorporation of behavioral economics, the analysis of the sharing economy, and the impact of new technologies on consumer behavior.

2.2.1 Behavioral economics and demand theory

Behavioral economics has emerged as an important field of study that combines insights from psychology and economics to better understand consumer decision-making (Kahneman (2011)). Traditional demand theory is based on the assumption that consumers are rational and utility-maximizing agents. However, behavioral economics has revealed that consumers often deviate from rational decision-making due to cognitive biases, limited information, and other psychological factors.

For example, research on the "endowment effect" has shown that consumers tend to value goods they already own more than similar goods they do not own (Thaler (1991)). This can lead to consumers being less willing to sell their possessions at market prices, affecting the demand for used goods. Similarly, the concept of "nudging" has been successfully applied in various contexts to influence consumer behavior by altering the presentation of choices, such as changing the default option for organ donation or retirement savings (Thaler, Sunstein (2008)).

2.2.2 The sharing economy and demand theory

The sharing economy, characterized by the rise of platforms like Uber, Airbnb, and TaskRabbit, has significantly altered the way goods and services are consumed (Buczynski (2013)). The sharing economy allows consumers to access goods and services on a temporary or as-needed basis, rather than purchasing them outright. This shift in consumption patterns has important implications for demand theory.

For instance, the growth of ride-sharing platforms like Uber and Lyft has led to a decrease in demand for traditional taxi services and potentially affected car ownership patterns Rayle et al. (2016). Similarly, the rise of Airbnb has impacted the hotel industry, with research showing that hotel revenues have declined in markets where Airbnb is present (Zervas et al. (2017)). These real-life examples demonstrate how the sharing economy can lead to substitution effects and alter the demand for certain goods and services.

2.2.3 Technology and consumer behavior

Rapid advances in technology, such as the rise of e-commerce, digital payments, and social media, have transformed the way consumers access and purchase goods and services (Lu (2018)). These technological innovations can have significant implications for demand theory.

For example, the widespread adoption of e-commerce platforms like Amazon has made it easier for consumers to compare prices and product features, leading to increased price sensitivity and potentially altering demand elasticities (Bronnenberg et al. (2012)). Additionally, the popularity of social media influencers has led to the rapid dissemination of information and influenced consumer preferences, affecting the demand for certain goods and services, such as fashion and beauty products (Djafarova, Rushworth (2018)). Furthermore, the rise of digital payments and subscription models, such as Netflix and Spotify, have changed the way consumers perceive and allocate their budgets, which can also have implications for demand theory (Van Hove, Vuchelen (2018)).

In summary, contemporary developments in fields such as behavioral economics, the sharing economy, and technology have led to a deeper understanding of consumer behavior and its implications for demand theory. By incorporating these insights and real-life examples, researchers can develop more accurate and nuanced models of demand, which can inform business and policy decisions.

2.3 Consumer choice

The concept of consumer choice is a central idea in microeconomics and is concerned with how consumers allocate their limited resources, such as money and time, among various goods and services. The theory of consumer choice is based on the assumption that consumers act rationally and try to maximize their satisfaction or utility, given their limited resources and the prices of goods (Farber (2002)).

Consumer choice is analyzed through the use of budget constraints, which represent the limitations on a consumer's ability to purchase goods and services, and utility functions, which describe how a consumer's satisfaction or utility depends on the goods and services they consume. The combination of budget constraints and utility functions allows economists to analyze how changes in prices, income, and other factors affect a consumer's behavior and the demand for different goods and services (Thompson (2005)). In order to make these choices, consumers consider the trade-off between the benefits they receive from consuming a good and the costs associated with it. For example, a consumer may choose to purchase a more expensive car, like a Tesla Model S, because the additional features and comfort it provides is worth the higher cost to them, compared to a more affordable Toyota Corolla.

Utility theory, which is a key part of consumer choice theory, provides a framework for understanding how consumers make these choices. Utility refers to the satisfaction or happiness a consumer derives from consuming a good or service. According to utility theory, consumers attempt to maximize their total utility by choosing the combination of goods and services that provides them with the highest level of satisfaction, given their budget constraints (Prasad (2014)). Consumer choice theory also considers how changes in market conditions, such as changes in prices or consumer income, affect consumer behavior. For example, if the price of a good increases, consumers may choose to purchase less of it or switch to a substitute product. In the case of rising gasoline prices, consumers may opt for public transportation or carpooling to reduce their fuel expenses.

The following factors play a role in determining a consumer's choices:

1. Heterogeneity in consumer choice refers to the idea that different consumers have different preferences, income levels, and demographic characteristics, which influence their purchasing decisions (Liechty et al. (2001)). For instance, younger consumers might prefer to spend more on technology and entertainment, such as purchasing the latest smartphone or attending music festivals, while older consumers may prioritize spending on healthcare and home improvements. Incorporating heterogeneity into consumer choice models allows for a more realistic representation of consumer behavior, including capturing the impact of different preferences, income levels, and demographic characteristics on consumer behavior (Acquisti, Varian (2005)).

2. Budget constraint is a fundamental aspect of consumer choice theory, as it represents the limiting factor that restricts the purchasing power of consumers. For example, a college student with limited income may prioritize purchasing essential items like food and textbooks, while forgoing more discretionary purchases like expensive clothing or dining out at restaurants. The budget constraint plays a critical role in shaping the choices made by consumers and influences the trade-offs they make between different goods and services (Kornai (2003)).

3. The market prices of goods and services influence the choices that consumers make. For example, during sales events like Black Friday, consumers may take advantage of lower prices to purchase items they might not have otherwise been able to afford, such as electronics or appliances Thaler (1980). This demonstrates how price changes can influence consumer behavior, as described by the law of demand (Carvalho (1995)).

4. The market structure, such as competition, can also influence consumer choice. For example, the introduction of low-cost airlines like Southwest and Ryanair has forced traditional airlines to offer more competitive prices, providing consumers with a wider range of options and more affordable air travel (Yenipazarli (2019)). In a monopolistic market, however, firms may have less competition and be able to charge higher prices, leading to a reduction in consumer surplus and potentially leading to reduced consumer choice. For instance, in areas with only one broadband internet provider, consumers may face limited options and higher prices due to the lack of competition (Inderst, Valletti (2009)). Furthermore, consumer characteristics and preferences also play a crucial role in consumer choice. These can include factors such as age, income, education level, personal values and beliefs, and cultural background (Yin et al. (2018)). For example, a consumer who is environmentally conscious might prioritize purchasing eco-friendly products, even if they are more expensive, due to their personal values and beliefs. All of these factors can interact and shape the consumer's decision-making process.

Consumer choice is influenced by a variety of factors, including heterogeneity in consumer preferences, budget constraints, market prices, market structure, and individual characteristics. By understanding and incorporating these factors into consumer choice models, economists and market researchers can gain a deeper understanding of the motivations and factors that drive consumer decisions, and they can use this information to inform marketing strategies and public policy decisions.

2.4 COVID-19 and its Impact on Consumer Choice and Wealth Inequality

The COVID-19 pandemic has led to significant changes in consumer behavior and has exacerbated wealth inequality across the globe. This subsection will provide a literature review of the key findings and real-life examples related to the impact of the pandemic on consumer choice and wealth inequality.

During the pandemic, there has been a considerable shift in consumer behavior, as individuals have prioritized health and safety over other factors. For example, (Baker et al. (2020)) found that consumers increased spending on essential items, such as groceries and healthcare products, while reducing expenditures on non-essential items like clothing and entertainment. Additionally, the pandemic accelerated the adoption of e-commerce, with online shopping platforms like Amazon and Alibaba experiencing a surge in sales (Kapoor et al. (2020)). Many brick-and-mortar retailers, particularly small businesses, faced significant challenges due to lockdown measures, decreased foot traffic, and changes in consumer preferences (Carvalho et al. (2020)).

The pandemic's economic impact has not been evenly distributed, further exacerbating wealth inequality. Low-income workers, particularly those in service and hospitality industries, experienced higher rates of job loss and financial instability compared to higher-income individuals who were able to work remotely (Atkeson (2020)). Additionally, the wealth of online retail giants' owners and shareholders, such as Jeff Bezos of Amazon, increased significantly during the pandemic, while small business owners and employees faced financial hardships (Alstadsæter et al. (2020)).

Governments worldwide implemented various fiscal and monetary policies to mitigate the economic fallout of the pandemic, such as stimulus packages, unemployment benefits, and interest rate changes (bal (2020)). Agent-based models (ABMs) have been used to assess the effectiveness of these policy interventions in supporting economic recovery and reducing wealth inequality

(Guerrieri et al. (2020)). For example, ABMs have been employed to examine the impact of temporary income support measures like the United States' stimulus checks on low-income households and their spending behavior (Coibion et al. (2020)).

Some researchers and policymakers have called for more targeted policies to address the growing wealth inequality exacerbated by the pandemic. For instance, progressive tax reforms, direct financial assistance to small businesses, and investments in social safety nets have been suggested as potential interventions (Auerbach et al. (2020)). Agent-based modeling can be used to evaluate the effectiveness and equity of these targeted policies by simulating their impact on different socioeconomic groups, thereby guiding evidence-based decision-making (bal (2020); Dosi et al. (2020)).

Moreover, the pandemic has highlighted and exacerbated the existing digital divide, with individuals and households lacking access to high-speed internet and digital devices being at a significant disadvantage in terms of remote work, online learning, and accessing essential services (Nicola et al. (2020)). This digital divide has further widened wealth inequality, as those with better access to digital resources have been more resilient in adapting to the pandemic-induced changes, while those without access have faced greater challenges in maintaining their livelihoods and education (Fernon, Slatina (2021)).

The shift to remote work has also led to changes in consumer preferences, such as increased demand for home office equipment and a heightened interest in relocating from urban centers to suburban or rural areas, where housing costs are typically lower and more space is available (Barrero et al. (2020)). These changes have implications for the real estate market, local economies, and the distribution of wealth among different regions and population groups (Florida (2020)).

Furthermore, the pandemic has had significant mental health consequences, with increased levels of stress, anxiety, and depression observed among the general population (Pfefferbaum, North (2020)). The mental health effects of the pandemic may indirectly influence consumer choices and spending patterns, as individuals experiencing mental health issues may change their preferences, have reduced motivation to engage in economic activities, or face difficulties in managing their finances (Holingue et al. (2020)). This could contribute to further disparities in wealth and well-being between different population groups.

2.4.1 Discrete choice

Discrete choice refers to the situation where consumers have a limited set of alternatives or options to choose from, and they must make a decision based on the available options. This is in contrast to continuous choice, where consumers have a continuous range of alternatives and can choose any quantity they desire. In the context of consumer choice, discrete choice models are used to analyze and predict the choices made by consumers in situations where the alternatives are limited, such as choosing between different brands, products, or services. These models are based on the idea that consumers make choices by weighing the attributes and features of the different options and selecting the one that provides the most satisfaction (Lancsar, Louviere (2008)).

Discrete choice models often use probabilistic methods, such as logit models or mixed logit models, to estimate the probabilities of consumers choosing different options. The parameters of these models are estimated based on survey data or observed market choices, and the results can be used to make predictions about consumer behavior in similar situations. These models can also incorporate different assumptions about consumer behavior, such as the degree of substitution between options, the presence of brand loyalty, and the impact of external factors such as advertising and price promotions (Revelt, Train (1998)).

Recent advances in discrete choice modeling include the integration of machine learning techniques and the incorporation of behavioral insights from psychology and neuroscience. For example, researchers have started to use machine learning algorithms, such as decision trees and neural networks, to improve the predictive accuracy of discrete choice models (Gopinath, Chintagunta (2018)). These approaches can help to identify complex patterns and interactions between variables that may not be easily captured by traditional econometric methods.

Moreover, the growing field of behavioral economics has provided new insights into the factors that influence consumer decision-making in discrete choice settings. These insights include the role of cognitive biases, such as loss aversion, anchoring, and mental accounting, in shaping consumer preferences and choices (Thaler (2015a)). By incorporating these behavioral factors into discrete choice models, researchers can develop a more accurate and nuanced understanding of consumer behavior.

Discrete choice models can provide valuable insights into consumer behavior and can be used by businesses and policymakers to make informed decisions about product design, marketing strategies, and pricing policies. One common discrete choice model is the multinomial logit model, which assumes that consumers make choices based on the maximum utility they can attain from each option. This model is widely used in fields such as transportation, marketing, and environmental economics, and its applications have expanded to areas like healthcare, energy, and urban planning.

2.4.2 New methods of studying the consumer choice model

Agent-based modeling (ABM) has emerged as a powerful tool for modeling complex systems, capturing the behavior of individual agents, and examining their interactions to understand system-level outcomes. In the context of consumer choice and supply theory, ABM has been applied to study a range of issues, such as individual consumer behavior, market demand, firm creation, production decisions, and the emergence of aggregate patterns in the economy. They have been increasingly used in economics and other social sciences to capture the complexity and heterogeneity of real-world systems. These models allow for the examination of emergent patterns and dynamics that arise from the interactions of individual agents with their environment and with other agents. They can also account for non-linearities, feedback loops, and other features that are not easily captured by traditional models (Kohler (2018)).

Through these methods we can study a wide range of economic phenomena, including market dynamics, financial stability, innovation diffusion, and environmental policy. They have also been used to investigate social phenomena such as segregation, social norms, and cooperation. While agent-based models have been criticized for their complexity and computational requirements, they offer a powerful tool for exploring complex systems and generating insights that may not be obtainable through other methods. As such, they are likely to continue to be an important tool for economic research and policy analysis in the future (Weimer (2016)).

Traditional economic models often relied on aggregating the behavior of numerous individuals or firms into a single representative agent, which limited the ability to capture the heterogeneity and complexity of real-world consumer and firm behavior (Turrell (2016)). ABM addresses these limitations by representing individual consumers and firms as distinct agents with unique preferences, behaviors, and interactions. By employing ABM, researchers have gained insights into the impact of individual-level factors, such as heterogeneity in preferences and production capabilities, on aggregate outcomes. This has led to a more nuanced understanding of consumer choice and firm behavior, including the emergence of collective patterns, market dynamics, and the investigation of various policy questions related to consumer and firm behavior (Heppenstall et al. (2016)).

Moreover, ABM allows researchers to study consumer choice and supply theory in dynamic and evolving contexts, like changing market conditions, and to capture the non-linear interactions between agents and their environment. This has helped to address significant limitations of traditional models, which frequently relied on static and deterministic assumptions, opening up new avenues for research into consumer choice and supply theory (Negahban, Smith (2014)).

As data sets have grown larger and more complex, researchers have increasingly turned to machine learning methods to analyze consumer choice and firm behavior. Machine learning algorithms excel at analyzing large and complex data sets, identifying patterns, and making predictions that would be challenging or impossible to uncover through traditional econometric methods.

Machine learning is becoming more prevalent in economics, including in the study of consumer choice and supply theory. The increasing availability of large and complex data sets has enabled machine learning methods to uncover patterns and relationships in these data. Machine learning algorithms can analyze consumer and firm behavior and predict future choices and production decisions based on past behavior. For instance, they can be used to predict product demand, the impact of prices and marketing efforts on consumer behavior, and the effects of economic changes on consumer preferences and firm production decisions (Athey (2018)).

Machine learning can also help address the limitations of traditional models of consumer choice and supply theory, such as the assumptions of rationality and complete information. Machine learning algorithms can learn from data and incorporate non-linear relationships and interactions between variables, resulting in more accurate predictions of consumer and firm behavior. The use of machine learning in economics, consumer choice, and supply theory is still an evolving field, with considerable room for future development and refinement of these methods (Dixon (2020)). However, it is clear that machine learning has the potential to revolutionize our understanding of consumer and firm behavior, providing valuable insights for policymakers, businesses, and individuals.

2.5 Supply Theory

Supply theory is a fundamental aspect of microeconomics, focusing on the behavior of firms in response to changes in market conditions, particularly prices (Greenwald, Stiglitz (1993)). It examines how firms determine the quantity of goods and services to produce and offer for sale at various prices. This section provides an in-depth discussion of supply theory, its key concepts, and assumptions, supported by relevant economic literature. The law of supply is a basic principle of economics that states that, all else being equal, the quantity of a good or service supplied by firms tends to increase as the price of the good or service rises (Mankiw (2007)).

This positive relationship between price and quantity supplied is driven by the profit-maximizing behavior of firms, as higher prices generally lead to higher revenues and profits, encouraging firms to produce more (Thatcher (2001)). Market supply refers to the total quantity of a good or service supplied by all firms in the market at various prices. The market supply curve is derived by horizontally summing the individual supply curves of all firms in the market (Hobelsberger (2021)). In perfectly competitive markets, the market supply curve exhibits a flat or horizontal shape, as each firm is a price-taker and adjusts its output to equate marginal cost with the market price (Card, Ashenfelter (2018)).

Several factors influence the supply of a good or service in the market, including:

1. **Input Prices:** Changes in the prices of factors of production, such as labor, capital, and raw materials, can impact the cost structure of firms and their willingness to supply goods and services at various prices (Fernández (2018)).

2. **Technology:** Technological advancements can increase productivity and reduce production costs, leading to an increase in the quantity of goods and services supplied at each price level (Autor (2015)).

3. **Government Policies:** Policies such as taxes, subsidies, and regulations can affect the production costs and incentives faced by firms, thereby influencing the supply of goods and services in the market (Tobua (2010)).

4. **Market Structure:** The structure of the market, including the number of firms, the level of competition, and the degree of market power, can impact the supply behavior of firms and the overall market supply curve (Day (2002)).

Supply theory is based on several key assumptions:

1. Profit Maximization: Firms are assumed to be rational and profit-maximizing, seeking to minimize costs and maximize revenues (Narver (1971)).

2. Production Function: Firms are subject to a production function that represents the relationship between the factors of production and the resulting output (Banker et al. (2003)).

3. Input Prices: The prices of inputs are taken as given by firms, which then determine the optimal input combination to minimize costs and maximize profits (Baumol (1970)).

4. Market Conditions: Supply theory assumes that firms respond to changes in market conditions, particularly prices, adjusting their production levels accordingly (Lucas (1967)).

Supply theory distinguishes between short-run and long-run supply based on the time horizon and the flexibility of input adjustments. In the short run, at least one input (typically capital) is fixed, and firms can adjust only the variable inputs, such as labor. This results in diminishing marginal returns to variable inputs and increasing marginal costs. The short-run supply curve is derived from the marginal cost curve, showing the quantity of output supplied at each price, given the fixed inputs (Lucas (1967)).

In the long run, all inputs are considered variable, allowing firms to adjust their production processes more flexibly in response to changes in market conditions (Milgrom, Weber (1990)). This allows firms to achieve economies of scale and reduce average costs as they expand production. The long-run supply curve is typically flatter than the short-run supply curve, reflecting the greater responsiveness of firms to price changes due to their ability to adjust all inputs (Tybout (2001)).

The behavior of firms and the resulting supply curves can vary significantly depending on the market structure. In perfectly competitive markets, firms are price-takers, and the market supply curve is derived by summing the individual supply curves of all firms in the market. Under perfect competition, the market supply curve reflects the marginal cost curve of the industry (Chen (2008)).

In monopolistic or oligopolistic markets, firms possess market power, allowing them to influence prices and potentially restrict output to maximize profits (Borenstein et al. (1999)). In these market structures, the supply curve may not be as clearly defined, as firms strategically interact and consider the behavior of rivals when determining their production levels. As a result, the supply behavior in imperfectly competitive markets can differ substantially from that in perfectly competitive markets (Limpaitoon, Polidano (2014)).

Supply theory is a crucial aspect of macroeconomic analysis, examining the behavior of firms in response to changes in market conditions and the determinants of the quantity of goods and services supplied at various prices. This in-depth discussion of supply theory has covered the law of supply, supply curves, market supply, determinants of supply, elasticity of supply, and the implications of different market structures on supply behavior, drawing on foundational economic literature to provide a comprehensive understanding of the topic (Browning, Zupan (2020)).

Supply shocks are unexpected events that lead to sudden changes in the supply of goods or services in the market. These shocks can be positive or negative, depending on their impact on the production process. Positive supply shocks, such as technological advancements or favorable weather conditions, increase the quantity supplied at each price level, resulting in a rightward shift of the supply curve. Conversely, negative supply shocks, such as natural disasters, labor strikes, or geopolitical tensions, decrease the quantity supplied at each price level, leading to a leftward shift of the supply curve (Kilian (2008)).

They can have significant implications for market equilibrium, as they alter the balance between supply and demand, potentially leading to changes in prices and output levels. Policymakers and businesses need to be aware of the potential impacts of supply shocks on the economy and develop appropriate strategies to manage these risks and mitigate their adverse effects.

2.6 Contemporary Developments in Supply and Demand Interaction

The interaction between supply and demand remains a critical area of study in economics, with contemporary research shedding new light on the dynamics of market equilibrium and resource allocation. Recent developments in areas such as globalization, digital transformation, and environmental concerns have added new dimensions to the analysis of supply and demand interaction.

2.6.1 Globalization and international trade

Globalization has led to an increasing interconnectedness of markets, allowing for the exchange of goods, services, and capital across national borders. This has significant implications for the interaction of supply and demand, as international trade can influence the domestic market equilibrium through changes in both supply and demand conditions (Krugman et al. (2018)). For instance, the integration of emerging economies like China and India into global trade has led to shifts in the supply of labor and capital-intensive goods, affecting the relative prices and quantities of these goods in international markets (Rodrik (2018a)). Furthermore, global value chains have allowed firms to optimize their production processes by sourcing inputs from different countries, enhancing efficiency and reducing costs (Antràs (2015)). Understanding the implications of globalization for supply and demand dynamics remains an important area of research, particularly as trade policies and geopolitical developments continue to evolve.

2.6.2 Digital transformation and market dynamics

The digital transformation of economies has led to the emergence of new market structures and business models, which can have significant implications for the interaction between supply and demand. The rise of e-commerce, online platforms, and the sharing economy has changed the way goods and services are supplied and consumed, potentially altering market equilibria (Goldfarb, Tucker (2019)). For instance, digital platforms can facilitate price discrimination and dynamic pricing strategies, which can impact the elasticity of supply and demand for certain goods and services (Chen, Mislove (2018)). Moreover, the increasing availability of real-time data and advanced analytics has allowed firms to better anticipate and respond to shifts in supply and demand, enhancing market efficiency and resource allocation (Provost, Fawcett (2013)). Research into the implications of digital transformation for supply and demand interaction continues to be a key area of inquiry, as new technologies and business models continue to reshape markets and industries.

2.6.3 Environmental concerns and green growth

Environmental concerns, such as climate change and resource depletion, have become increasingly prominent in recent years, with policymakers and businesses alike seeking to transition towards more sustainable models of growth (Stern (2007)). The shift towards green growth has significant implications for the interaction of supply and demand, as it involves changes in both consumer preferences and production technologies (Hallegatte et al. (2012)). For example, the growing demand for renewable energy and electric vehicles has spurred innovation in clean technologies, leading to shifts in the supply of energy-related goods and services (Agency (2019)). Additionally, the introduction of carbon pricing and other environmental policies can influence the relative prices of different goods, potentially affecting the market equilibrium and the allocation of resources towards more sustainable outcomes (Goulder, Schein (2013)). Understanding the implications of environmental concerns and green growth for the interaction of supply and demand remains a crucial area of research, as economies continue to grapple with the challenges of sustainable development.

In summary, contemporary developments in globalization, digital transformation, and environmental concerns have added new layers of complexity to the analysis of supply and demand interaction. By incorporating these insights, researchers can develop a more nuanced understanding of market dynamics and resource allocation, which can inform policy and business decisions in an increasingly interconnected and rapidly changing world.

2.7 The issue of rationality and uncertainty in economics.

In economics, rationality is often defined as the ability of an individual to make logical, self-interested decisions based on their preferences and beliefs. However, the concept of rationality can be subjective, as individuals may have different preferences and beliefs. Uncertainty refers to the lack of complete information or knowledge about a particular situation or outcome. In economics, uncertainty often arises in decision making, as individuals must make choices based on incomplete information or a lack of certainty about the future. This uncertainty can impact the rationality of decision making and lead to sub-optimal outcomes (Miller (1992)).

The terms "rational consumer" and "irrational consumer" have been used in economics since the advent of microeconomic theory. The concept of rational consumer choice, or the idea that consumers make choices based on a consistent and well-defined set of preferences, was first introduced by economists such as Leon Walras and Vilfredo Pareto in the late 19th and early 20th centuries (Dall (1995)). It refers to the idea that consumers make decisions based on rational self-interest, taking into account the costs and benefits of different choices and maximizing their utility. The concept of irrational consumers was first introduced by behavioral economists such as Herbert Simon, Daniel Kahneman, and Amos Tversky in the 1970s and 1980s (Camerer et al. (2007)). These economists challenged the traditional assumption of economics that consumers are rational and make choices based solely on their self-interest and based on the complete and accurate information. The idea of irrational consumer behavior, or the idea that consumers may make choices that deviate from what is predicted by the standard economic models of rational choice, has more recently been explored in the field of behavioral economics (Posner (1997)).

Behavioral economists have studied this phenomenon and have developed models that incorporate the effects of uncertainty on consumer behavior, making the consumer choice model more realistic and useful in real-world applications. One of the key insights of behavioral economics is that people's decisions are often influenced by factors such as biases, emotions, and heuristics (mental shortcuts), rather than solely by self-interest and a perfectly rational calculation of costs and benefits. This has led to a more nuanced understanding of how people make choices, including in areas such as savings, debt, and investments (Twomey (2002)).

In the consumer choice model, uncertainty plays a crucial role as it affects the decisions made by consumers. Uncertainty refers to the lack of information or knowledge about the future outcomes of different options, which can lead to risk and ambiguity. This uncertainty can impact consumers' preferences, beliefs, and decision-making processes. As a result, consumers may make decisions that deviate from the standard rational choice model, leading to a departure from the predictions of traditional economic models (Politi (2007)).

According to this concept, consumers have well-defined preferences, complete information about the goods and services available to them, and the ability to make optimal choices based on this information. The assumption of a rational consumer is used to build mathematical models of consumer behavior, which are then used to make predictions about how consumers will respond to changes in prices, income, or other factors (Politi (2007)).

Critics of the "rational consumer" concept argue that it oversimplifies the decision-making process, ignoring the impact of psychological, emotional, and social factors that can influence consumer behavior. This has led to the development of behavioral economics, which incorporates insights from psychology and sociology to understand the ways in which real-world consumers deviate from the ideal of rational decision-making (Wright, Ginsburg (2011)).

On the other hand, the term "irrational consumer" refers to a consumer whose behavior is not guided by rational decision making, meaning they do not make choices that are based on the best possible outcome given their limited resources, such as time and money. Instead, their behavior may be influenced by emotional factors, biases, or cognitive limitations that prevent them from making the most optimal decisions. This concept has been studied in behavioral economics, which examines the ways in which people deviate from rational decision making in real-world situations (Hanson (1999)).

John Maynard Keynes: *"The peculiar character of the problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we must make use never exists in concentrated or integrated form but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess."*

To back up the previous concept of "irrational consumer" bounded rationality came to help us pass from the fully rational consumer to a more irrational. It is an economic concept that acknowledges that individuals and organizations have limited cognitive abilities and resources to make decisions. In the context of consumer choice, this means that consumers may not always make perfectly rational decisions and may make mistakes or face limitations in their decision-making process (Ellison (2006)).

Bounded rationality and uncertainty can lead to sub-optimal decision-making and can result in consumers making choices that do not align with their preferences or goals. This highlights the importance of considering these factors in the context of consumer choice, and of developing models and theories that take into account the limitations and biases that can affect decision-making (Kahneman, Tversky (2014)).

Herbert Simon: *"The decision-making of a firm's management cannot be treated as fully rational. Rationality is limited by the information the decision maker has, the cognitive limitations of his mind, and the finite amount of time he has to make the decision."*

These concepts play a significant role in shaping economic theory, informing models of consumer behavior, and guiding policy decisions. For example, if policymakers believe that consumers are rational, they may design policies that assume consumers will make the best decisions for themselves. If, on the other hand, policymakers believe that consumers are susceptible to irrational behavior, they may design policies that aim to guide or correct consumer decision-making.

Real-life examples and studies that illustrate the impact of rationality and uncertainty in consumer behavior and the importance of considering bounded rationality, uncertainty, and irrational behavior when studying consumer choice and developing economic models and policies:

- **Loss aversion:** A well-known example of how irrational behavior affects consumer choices is loss aversion, where individuals tend to place more weight on potential losses than on potential gains. This phenomenon was first described by Kahneman and Tversky (1979) in their seminal paper on prospect theory (Kahneman, Tversky (1979b)). Loss aversion can lead to suboptimal decision-making, such as holding on to poorly performing investments or avoiding risks that may have positive expected outcomes.
- **The endowment effect:** Thaler (1980) introduced the concept of the endowment effect, which states that individuals tend to value an item more once they own it, compared to when they do not (Thaler (1980)). This can lead to irrational behavior, such as demanding a higher price to sell an item they own than they would be willing to pay to buy it. The endowment effect has been demonstrated in various experiments, including the classic "mug experiment" by Kahneman, Knetsch, and Thaler (1990), where participants who were given a mug demanded a higher price to sell it than those who were not given a mug were willing to pay to buy it (Kahneman et al. (1990)).
- **The sunk cost fallacy:** The sunk cost fallacy is a type of irrational behavior where individuals continue to invest time, money, or effort into an endeavor because they have already invested resources in it, regardless of the current or future value of the investment. A study by Arkes and Blumer (1985) demonstrated the sunk cost fallacy by showing that participants were more likely to attend a non-refundable event they had paid for, even if they no longer had an interest in the event, compared to a free event (Arkes, Blumer (1985)).
- **Information overload:** Bounded rationality suggests that consumers can be overwhelmed by the amount of information available to them, leading to suboptimal decision-making. Schwartz et al. (2002) conducted a study on retirement plan choices and found that when employees were presented with a larger number of investment options, they were less likely to participate in the plan and tended to make less diversified investment choices (Schwartz et al. (2002)).
- **Status quo bias:** Samuelson and Zeckhauser (1988) introduced the concept of status quo bias, which refers to the tendency of individuals to prefer the current state of affairs over alternative options, even when the alternatives may be more beneficial (Samuelson, Zeckhauser (1988)). This can lead to suboptimal decision-making, such as consumers sticking with their current service provider or product, even when switching could save them money or provide better quality.

2.8 Contemporary Developments in Banking and their Impact on Supply and Demand

The banking sector continues to evolve in response to technological advancements, regulatory changes, and shifting economic conditions. These contemporary developments have significant implications for the role of banks in facilitating the interaction between supply and demand in the economy.

2.8.1 Fintech and digital banking

The emergence of financial technology (fintech) and digital banking has revolutionized the way financial services are provided and consumed. Fintech firms and digital banks have introduced innovative products and services, such as mobile payments, peer-to-peer lending, and robo-advisory, which have the potential to increase competition, improve efficiency, and expand access to financial services (Frost et al. (2019)). These innovations can stimulate demand for goods and services by offering consumers and businesses more convenient, affordable, and personalized financial solutions. Furthermore, the increasing use of data analytics and artificial intelligence in credit assessment and risk management can enhance banks' ability to extend credit to a wider range of borrowers, potentially supporting investment and production activities in the economy (Bholat et al. (2019)). However, the rise of fintech and digital banking also presents new challenges for regulators and policymakers, who must strike a balance between promoting innovation and ensuring financial stability, consumer protection, and fair competition (Buchak et al. (2018)).

2.8.2 Financial regulation and macroprudential policy

Following the global financial crisis of 2007-2008, financial regulation and supervision have undergone significant reforms aimed at strengthening the resilience of the banking sector and reducing the likelihood of future crises. These measures include stricter capital and liquidity requirements, enhanced risk management practices, and the introduction of macroprudential policy frameworks to address systemic risks in the financial system (Claessens et al. (2014)). While these reforms have contributed to the stability of the banking sector, they may also have implications for the supply of credit and the interaction of supply and demand in the economy. For instance, tighter regulations may constrain banks' ability to extend credit, particularly to riskier borrowers, potentially dampening aggregate demand and investment (Acharya et al. (2017)). However, a more stable and resilient banking sector can also support long-term economic growth by reducing the likelihood of financial crises and their associated costs.

2.8.3 Central bank digital currencies

The growing interest in central bank digital currencies (CBDCs) represents another contemporary development with potential implications for the role of banks in the supply and demand interaction. CBDCs are digital forms of central bank money that can be used as a means of payment and store of value, offering an alternative to traditional bank deposits and cash (Bindseil (2020)). The introduction of CBDCs could lead to increased competition in the banking sector, as consumers and businesses may choose to hold a portion of their funds in CBDCs instead of bank deposits. This could affect banks' ability to extend credit, potentially influencing the supply of credit and aggregate demand in the economy (Bordo, Levin (2021)). Moreover, the implementation of CBDCs could impact the transmission of monetary policy, as central banks may gain new tools for influencing interest rates and credit conditions (Boar, Wehrli (2020)). The implications of CBDCs for the banking sector and the interaction of supply and demand remain an area of ongoing research and debate among policymakers and academics.

In conclusion, contemporary developments in the banking sector, such as fintech, financial regulation, and central bank digital currencies, have significant implications for the role of banks in facilitating the interaction between supply and demand. By adapting to these changes and addressing the associated challenges, banks can continue to support the efficient functioning of markets and the allocation of resources in the economy.

2.9 The Crucial Contribution of Econophysics to Economic Science

A New Perspective in Economic Analysis Econophysics, an interdisciplinary field established in the 1990s, combines principles from physics, particularly statistical mechanics and complex systems theory, with economic research. This groundbreaking approach has significantly advanced our understanding of financial markets and economic systems. We will discuss the importance of econophysics in economic science, explore its practical applications, and explain the relationship between quantum phenomena and economics (Mantegna, Stanley (2000a)).

Analyzing Complex Economic Systems

Econophysics enhances economic science by offering novel analytical tools and models that dissect the complexity of economic systems. Traditional economic theories, such as the efficient market hypothesis or the random walk model, are based on oversimplified assumptions, including rational behavior and linear relationships. However, these assumptions often fail to accurately represent real-life economic systems characterized by non-linear dynamics, emergent properties, and feedback loops.

Using principles from statistical mechanics, like power-law distributions and phase transitions, econophysics models the behavior of economic agents and markets. This approach succeeds in capturing the complexity and dynamism of real-world financial systems, resulting in a more accurate and comprehensive understanding of their behavior (Stanley et al. (1996)).

For example, econophysicists have applied Ising spin-inspired models to study financial contagion and systemic risk in banking networks. By simulating interactions between financial institutions, they have revealed the mechanisms driving cascading failures that can escalate into widespread crises. This knowledge has significant implications for regulatory policies and maintaining financial stability (Gai, Kapadia (2010)).

Deciphering Market Anomalies and Forecasting Behavior

Econophysics' ability to uncover anomalous patterns and predict market behavior is another essential aspect of its contribution to economic science. By examining large-scale financial data from a physics perspective, econophysicists have identified scaling laws and long-range correlations that traditional economic models frequently miss.

A notable example is the discovery of the "inverse cubic law" governing financial markets. This law, which emerged from analyzing stock price fluctuations, asserts that the distribution of returns follows a power-law with an exponent close to -3. This observation challenges the long-held belief that financial returns follow a Gaussian (or normal) distribution and has significant implications for risk management and portfolio optimization (Gopikrishnan et al. (1999)).

Quantum Economics and the Connection between Quantum Phenomena and Economics

The connection between quantum phenomena and economics, as demonstrated in quantum economics—a subfield of econophysics—deserves special attention. This emerging discipline investigates the application of quantum mechanics principles to economic issues, potentially revolutionizing our view of economic systems by offering a more accurate depiction of agent interactions and market dynamics (Haven, Khrennikov (2013)).

For instance, the concept of quantum entanglement can be used to model the interdependence of economic variables, including exchange rates, interest rates, and stock prices. This approach enables a more in-depth understanding of how fluctuations in one variable affect others in the system, ultimately leading to more informed decision-making processes.

Another example is the use of quantum game theory, which extends traditional game theory by incorporating quantum mechanics principles. Researchers have applied this methodology to analyze strategic interactions in financial markets, such as high-frequency trading and algorithmic trading, providing insights into market stability and vulnerability to manipulation (Paul, Chatterjee (2018)).

Econophysics has become an essential field within economic science due to its ability to model complex systems, reveal hidden patterns, and forecast market behavior. By integrating principles from physics, specifically statistical mechanics and quantum phenomena, econophysics has the potential to radically transform our understanding of economic systems and inform more effective policy decisions.

2.10 The importance of wealth inequality

Wealth inequality, the uneven distribution of assets and resources within a society, is a pressing issue that has garnered increasing attention from economists, policymakers, and social scientists (Alvaredo et al. (2018)). The widening gap between the rich and the poor not only impacts individual economic well-being but also has broader ramifications for economic growth, social mobility, and political stability. In this article, we present a comprehensive and analytical approach to understanding the importance of wealth inequality and its consequences, drawing on real-life examples and mathematical models to emphasize the need for rigorous research that informs targeted policy interventions.

Economic Growth:

Wealth inequality can significantly affect economic growth through various channels. When resources are disproportionately concentrated among the wealthiest individuals, lower and middle classes may have limited access to education, healthcare, and capital, ultimately restricting their potential for economic mobility and productivity (Cingano (2014)).

Mathematically, the relationship between wealth inequality and economic growth can be expressed using the Gini coefficient (G), which ranges from 0 (perfect equality) to 1 (maximum inequality). A negative correlation between G and economic growth rate (g) suggests that greater inequality hampers growth:

$$g = f(G) \tag{1}$$

In the 1980s, Latin American countries faced a challenging period marked by high levels of wealth inequality, with Gini coefficients frequently surpassing 0.5, indicating substantial disparities in resource distribution (Deininger, Squire (1998)). This period, often referred to as the "Lost Decade," was characterized by economic stagnation, high inflation, rising unemployment, and increasing external debt, which exacerbated existing inequalities and contributed to social unrest (Edwards (1995); Sachs (1989)).

One contributing factor to the high wealth inequality during this period was the historical legacy of land concentration and the exclusion of large segments of the population from the formal economy (Morley (1995)). Additionally, the implementation of structural adjustment programs, which were advocated by international financial institutions like the International Monetary Fund (IMF) and the World Bank, played a role in perpetuating wealth disparities (Williamson, 1990). These programs often entailed austerity measures, trade liberalization, and deregulation, which disproportionately impacted lower-income households and led to a decline in social spending on public services such as education, healthcare, and housing (Cornia et al. (1987)).

China's remarkable economic growth over the past four decades has been accompanied by a substantial increase in wealth inequality, as demonstrated by a rising Gini coefficient (Xie, Zhou (2014)). While the country's GDP has experienced meteoric growth, the benefits have not been distributed equitably, resulting in a growing divide between the rich and the poor. This mounting inequality has sparked concerns about the sustainability of China's economic growth and the potential for social unrest (Wang, Zhang (2018)).

Several factors have contributed to the escalation of wealth inequality in China. First, rapid urbanization and industrialization have generated income disparities between urban and rural populations. Urban workers have benefited more from economic growth than their rural counterparts, who are often engaged in low-paying agricultural jobs with limited access to social services (Kanbur et al. (2017)). This has resulted in a considerable wealth gap between urban and rural areas.

Second, China's hukou (household registration) system has reinforced existing inequalities by restricting rural-to-urban migration and limiting access to social benefits for migrants (Chan (2009)). This system has effectively created a class of internal migrants who work in urban areas but are unable to access the same services and benefits as urban residents, exacerbating wealth disparities.

Third, disparities in educational opportunities have further widened the wealth gap. Access to quality education remains uneven across the country, with students in rural areas often facing inadequate resources and limited opportunities for higher education (Yue et al. (2019)). These educational disparities can have long-lasting consequences, as they limit social mobility and perpetuate income inequality across generations.

Finally, the rapid growth of China's real estate market has contributed to wealth inequality. As housing prices have soared, wealthier individuals have reaped significant profits, while low-income families have faced difficulties in affording housing or have been priced out of the market altogether (Wu (2015)). This has exacerbated wealth disparities and raised concerns about social stability.

In summary, the significant increase in wealth inequality in China can be attributed to multiple factors, including rapid urbanization, the hukou system, educational disparities, and the booming real estate market. Addressing these issues is essential to ensuring the sustainability of China's economic growth and mitigating the potential for social unrest.

Social Mobility:

Wealth inequality can also have profound effects on social mobility, or the ability of individuals to move up or down the socioeconomic ladder. High levels of wealth inequality can result in reduced opportunities for upward mobility, perpetuating intergenerational cycles of poverty and exacerbating income disparities (Chetty et al. (2014)).

A simple model that captures the impact of wealth inequality on social mobility is the intergenerational earnings elasticity (IGE), which measures the persistence of income across generations:

$$IGE = \beta * G \tag{2}$$

where β is the degree of correlation between parental and offspring income. A higher IGE implies lower social mobility.

In the United States, intergenerational earnings elasticity (IGE) has been estimated at around 0.5, signifying that approximately 50% of parental income differences carry over to the next generation (Mazumder (2005)). This high level of income persistence reveals a concerning "opportunity gap" between children from high- and low-income families, with long-lasting consequences for economic success (Chetty et al. (2014)).

The opportunity gap manifests itself in various aspects of life, including access to quality education, healthcare, and social networks. Children from low-income families are more likely to attend underfunded and underperforming schools, hindering their chances of acquiring the necessary skills and knowledge for future success (Reardon (2011)). This educational disparity can then translate into limited access to higher education, reducing the likelihood of obtaining high-paying jobs and perpetuating the cycle of poverty (Haveman, Smeeding (2009)).

Additionally, low-income families often face challenges in accessing quality healthcare, which can negatively impact children's physical and cognitive development (Braveman et al. (2008)). These disparities in health outcomes can further exacerbate the opportunity gap by limiting low-income children's potential for educational achievement and labor market success (Currie (2009)).

Lastly, social networks play a significant role in determining future economic opportunities. Children from high-income families are more likely to be exposed to influential social connections, enabling them to access better job opportunities and resources (Lin (2000)). In contrast, children from low-income families may lack access to such networks, limiting their chances of upward mobility (Granovetter (2005)).

Scandinavian countries, such as Denmark and Sweden, are known for their relatively low levels of wealth inequality and high social mobility. This outcome can be attributed to their comprehensive social welfare systems, which include universal healthcare, education, and robust social safety nets (Björklund, Jäntti (2009)). These policies foster an environment that promotes equal opportunities, social cohesion, and a more inclusive society.

One of the key components of the Scandinavian model is its focus on education. Both Denmark and Sweden offer free, high-quality education at all levels, from pre-school to university. This commitment to education ensures equal access to knowledge and skills development, regardless of socioeconomic background, and has been linked to higher levels of social mobility (Breen (2005)). Additionally, the provision of extensive vocational training and apprenticeship programs helps to further reduce income disparities and improve employment opportunities for young people (OECD (2010)).

Another critical aspect of the Scandinavian welfare system is the provision of universal healthcare. Citizens in Denmark and Sweden have access to comprehensive, publicly-funded healthcare services, which contributes to lower levels of health disparities and improved health outcomes overall (Mackenbach et al. (2008)). This access to quality healthcare is essential for ensuring that individuals can reach their full potential and participate fully in the workforce.

Furthermore, Scandinavian countries provide generous social safety nets, such as unemployment benefits, parental leave, and pension schemes. These policies help to cushion the impact of economic shocks and promote income security for all citizens, regardless of their socioeconomic status (Kangas, Palme (2000)). This comprehensive social protection system acts as an equalizer, preventing extreme disparities in wealth and fostering social cohesion.

Political Stability:

Wealth inequality can also have a profound influence on political stability by exacerbating social tensions, fueling populist movements, and undermining democratic institutions. Unequal access to resources and opportunities can erode social cohesion and trust, leading to unrest and political polarization (Inglehart, Norris (2016)).

The Arab Spring uprisings in 2011 were a series of anti-government protests and demonstrations that swept across the Middle East and North Africa. These uprisings were partly triggered by high levels of wealth inequality, as marginalized populations protested against economic exclusion and limited access to resources (Campante, Chor (2012)). The events underscore the potential for wealth inequality to contribute to political instability, particularly in regions with weak governance structures.

Several factors contributed to the high levels of wealth inequality in the Arab region during this period. One key factor was the widespread corruption and crony capitalism that led to the concentration of wealth among a small elite, while the majority of the population struggled with poverty and unemployment (You (2014)). In addition, the lack of political representation and restricted civil liberties exacerbated feelings of frustration and marginalization among the lower-income segments of the population (Beissinger et al. (2012)).

The Arab Spring uprisings exposed the interplay between wealth inequality and political instability. In countries like Tunisia and Egypt, where the uprisings began, the discontent over wealth disparities combined with dissatisfaction over the lack of political freedom and perceived government corruption (Gelvin (2012)). The subsequent protests and demonstrations led to the toppling of long-standing regimes in these countries, highlighting the powerful impact of wealth inequality on political unrest.

The rise of populist movements in Europe and the United States in recent years can be partly attributed to increasing wealth inequality. Disaffected citizens who feel left behind by globalization and economic shifts have turned to populist leaders who promise to address their grievances (Rodrik (2018b)). This trend has led to a rise in political polarization, threatening democratic norms and institutions.

Populist movements often emerge in response to a perceived disconnect between the needs and aspirations of ordinary citizens and the policies pursued by mainstream political parties. In the context of rising wealth inequality, this disconnect becomes more pronounced, as many citizens feel that political elites are unresponsive to their concerns and prioritize the interests of the wealthy (Colantone, Stanig (2018)). This sentiment can drive support for populist parties and leaders who offer a stark alternative to the status quo and promise to restore a more equitable distribution of resources and opportunities.

One factor that has contributed to the growth of populist movements is the decline in manufacturing jobs due to globalization and automation. These economic shifts have disproportionately impacted lower-skilled workers, leading to job losses, stagnating wages, and increased economic insecurity (Autor et al. (2016)). As a result, many individuals in these sectors feel increasingly marginalized and left behind by the mainstream political discourse, turning to populist leaders who promise to reverse these trends.

Another important aspect of the relationship between wealth inequality and the rise of populist movements is the erosion of social trust. As wealth becomes more concentrated among a small elite, citizens may perceive that the political and economic systems are rigged against them, undermining trust in democratic institutions and increasing support for anti-establishment figures (Dalton (2014)). This decline in trust can further fuel political polarization and weaken the foundations of democratic societies.

Policy Interventions:

Given the far-reaching implications of wealth inequality, it is essential for policymakers to develop targeted interventions that promote inclusive growth and mitigate the negative consequences of this phenomenon. Some potential policy interventions include:

- **Progressive taxation:** Implementing a progressive tax system that imposes higher tax rates on the wealthiest individuals can help redistribute resources and reduce wealth inequality (Saez, Zucman (2019)).
- **Social welfare programs:** Expanding access to education, healthcare, and social safety nets can improve social mobility and create more equitable opportunities for all members of society (Heckman, Masterov (2007)).
- **Financial market regulation:** Implementing policies that prevent excessive risk-taking and speculative behavior in financial markets can help mitigate the concentration of wealth among a few individuals and promote economic stability (Rajan (2010)).
- **Labor market policies:** Ensuring fair wages, strong labor protections, and access to skills training can help reduce income disparities and promote social mobility (Autor (2019)).
- **Inheritance tax:** Implementing or increasing inheritance taxes can help prevent the intergenerational transmission of wealth inequality and promote a more equitable distribution of resources (Piketty (2014)).

2.11 The Interplay Between Consumer Choice and Wealth Inequality: Understanding the Complex Dynamics Shaping Economic Outcomes

Consumer choice and wealth inequality are two key factors that can significantly impact economic outcomes and societal well-being. The relationship between these factors is multifaceted, with consumer preferences and purchasing power influencing market dynamics and wealth distribution. In this article, we provide an in-depth analysis of the interplay between consumer choice and wealth inequality, using real-life examples and case studies to illustrate the complex dynamics shaping economic outcomes.

The Impact of Consumer Choices on Wealth Disparity: Consumer choices significantly influence market demand and, as a result, the distribution of wealth within an economy. When high-income consumers exhibit a strong preference for luxury goods or services, wealth tends to concentrate among the producers of these items, further exacerbating wealth inequality (Stiglitz (2012)). This phenomenon, known as the "luxury effect," can contribute to persistent disparities in wealth distribution and economic opportunity.

Economists argue that this luxury effect can drive up prices for high-end goods and services, enabling producers and suppliers to accumulate more wealth (Frank (2014)). As a result, wealth becomes more concentrated at the top of the income distribution, widening the gap between the rich and the poor. This trend can also create barriers to entry for small businesses and entrepreneurs, as they may struggle to compete with established luxury brands.

Furthermore, the luxury effect can have trickle-down consequences for the broader economy. As high-income consumers increasingly allocate their spending to luxury goods and services, it can lead to reduced demand for products and services produced by middle- and low-income workers, limiting their economic opportunities and exacerbating income inequality (Milanovic (2016)).

Luxury Cars and Their Contribution to Wealth Disparity: The preference of high-income consumers for luxury vehicles from premium automobile manufacturers, such as BMW, Mercedes-Benz, or Tesla, leads to wealth concentration among these companies and their shareholders, thus exacerbating wealth inequality. For instance, luxury vehicle sales in the United States have been on the rise, with brands like BMW and Mercedes-Benz capturing a significant market share (Cain (2019)).

The structure of the automobile industry, which features high barriers to entry and considerable economies of scale, enables luxury carmakers to generate substantial profits. As a result, the wealth produced by these companies tends to concentrate among their top executives, shareholders, and high-ranking employees. This dynamic contributes to an expanding wealth gap both within the industry and throughout the broader economy (Piketty (2014)).

The Impact of Luxury Real Estate on Wealth Disparities: Affluent individuals often gravitate toward luxury real estate in exclusive neighborhoods, which in turn drives up property values in these areas and concentrates wealth among a select group of property owners. A study by (Albouy et al. (2018)) found that high-income households in the United States tend to cluster in neighborhoods with elevated housing prices and superior amenities, resulting in a higher concentration of wealth in these locations.

As property values in exclusive neighborhoods escalate, the wealth gap between luxury property owners and the rest of the population also widens. This effect is further intensified by the perception of real estate as a secure and profitable investment, prompting wealthy individuals to allocate a considerable portion of their assets to property holdings (Saez, Zucman (2016)).

Moreover, the concentration of luxury real estate in exclusive neighborhoods can contribute to social segregation and diminished access to quality education, healthcare, and other vital services for low-income households, thereby perpetuating cycles of poverty and inequality (Chetty et al. (2014)).

Purchasing Power Disparities and Wealth Inequality:

Variations in purchasing power among different income levels can contribute to wealth inequality. Lower-income consumers may face limited purchasing power, constraining their ability to access specific goods and services. This situation can lead to a concentration of wealth among producers targeting higher-income consumers, reinforcing existing wealth disparities (Milanovic (2016)).

Healthcare Access Disparities and Wealth Inequality:

Low-income individuals often face limited access to quality healthcare due to high service costs and inadequate insurance coverage. This disparity in purchasing power results in wealth concentration among healthcare providers catering to wealthier patients, further intensifying wealth inequality. A study by (Woolhandler, Himmelstein (2016)) found that the U.S. healthcare system, marked by high costs and unequal access to care, has been a significant factor driving wealth inequality in the country.

As wealthier individuals can afford more costly medical treatments and access superior healthcare services, the wealth generated by healthcare providers is frequently concentrated among those serving this population. Moreover, the ability of higher-income patients to pay for advanced and expensive treatments may incentivize healthcare providers to focus on offering specialized and high-priced services, potentially neglecting the needs of lower-income patients (Papanicolas et al. (2018)).

Educational Access Disparities and Wealth Inequality:

Financial constraints can limit low-income individuals' access to quality education, impacting their long-term earning potential and perpetuating poverty cycles. Conversely, wealthier families can afford to send their children to prestigious private schools, reinforcing existing wealth disparities and contributing to greater income inequality. Reardon (2011) discovered that the achievement gap between children from high- and low-income families in the United States has been growing steadily over recent decades.

This educational divide can have long-term implications for wealth inequality, as individuals with limited access to quality education may face reduced economic mobility and opportunities for well-paying jobs. Furthermore, the concentration of resources and high-quality teachers in elite private schools and well-funded public schools can perpetuate wealth disparities, as these institutions offer students better educational opportunities and a solid foundation for future success (Chetty et al. (2011)).

Market Dynamics and Wealth Concentration:

Market dynamics, shaped by consumer choices and purchasing power, can exacerbate wealth inequality. When a small group of producers dominates a specific market, it can lead to monopolistic or oligopolistic structures, concentrating wealth and market power among a select few.

Technology Sector's Impact on Wealth Inequality:

The technology sector, dominated by major companies such as Apple, Amazon, Google, and Facebook, has led to wealth concentration among their founders, shareholders, and top executives. This market dominance, driven by consumer preferences and purchasing power, has resulted in significant wealth inequality within the industry. Piketty et al. (2018) found that the emergence of "superstar firms" in the technology sector has contributed to a substantial increase in wealth and income inequality over recent decades.

These technology giants have accumulated significant market power, enabling them to generate substantial profits and concentrate wealth among their founders, shareholders, and top executives. As a result, individuals associated with these companies have experienced exponential wealth growth, contributing to a widening wealth gap both within the industry and across the broader economy (Autor et al. (2020)).

E-commerce and Wealth Inequality in the Retail Sector:

The retail sector has experienced significant disruption due to the rise of e-commerce giants such as Amazon, leading to a concentration of wealth among a small number of online retailers. This shift in the market, driven by consumer preferences for online shopping, has further contributed to wealth inequality. Furceri et al. (2019) found that the rapid expansion of e-commerce has resulted in a concentration of market power among a few dominant firms, which in turn has increased wealth inequality.

The decline of traditional retail businesses, struggling to compete with the lower prices and convenience offered by online retailers, has caused the wealth generated by the retail sector to become increasingly concentrated among a select number of e-commerce companies and their shareholders. This concentration exacerbates wealth inequality both within the sector and across the broader economy (Kenney, Zysman (2016)).

Policy Implications and Future Directions:

To address the issue of wealth inequality driven by consumer choice, policymakers can consider a variety of strategies, including:

- **Implementing progressive taxation:** A progressive tax system, which levies higher tax rates on wealthier individuals, can facilitate wealth redistribution and reduce inequality (Piketty et al. (2018)).
- **Expanding access to quality education and healthcare:** Ensuring equal access to quality education and healthcare services for individuals from all income levels can foster social mobility and diminish wealth disparities stemming from differences in purchasing power (Chetty et al. (2014)).
- **Promoting competition and regulating monopolies:** Enacting policies that encourage competition and prevent the formation of monopolies or oligopolies can mitigate the concentration of wealth and market power among a select few (Furceri et al. (2019)).
- **Supporting affordable housing initiatives:** Implementing policies that promote affordable housing and prevent the excessive concentration of wealth in high-end real estate markets can help address wealth disparities driven by differences in consumer preferences and purchasing power (Albouy et al. (2018)).
- **Encouraging financial inclusion:** Expanding access to affordable credit and financial services for low-income individuals can help reduce wealth inequality by providing opportunities for asset accumulation and economic mobility (Demirgc-Kunt et al. (2018)).

The relationship between consumer choice and wealth inequality is multifaceted, with consumer preferences, purchasing power, and market dynamics all playing a role in shaping economic outcomes. By providing an extensive analysis of this complex interplay, this article aims to inform future research and policy interventions aimed at promoting more equitable economic outcomes. Understanding the nuances of consumer choice and wealth inequality is essential for the development of targeted policies that address the root causes of wealth disparities and promote inclusive growth and social cohesion.

3 Problem Definition

In the realm of economics, a fundamental and enduring question concerns the emergence of macroeconomic phenomena from the intricate interplay of numerous individual agents, each with their own unique behaviors and decision-making processes. This research delves into the exploration of realistic and novel approaches to understanding how macroeconomic patterns and dynamics materialize from the complex interactions of diverse agents in various scenarios. By integrating advanced computational methods, particularly agent-based modeling, this study seeks to unravel the layers of economic systems, challenging traditional models and offering fresh perspectives on consumer behavior, market dynamics, and the overall structure of economic networks. The core inquiry revolves around how these individual interactions coalesce into larger economic trends and outcomes, and how this understanding can reshape our approach to economic theories and policies. In the following sections I introduce the main questions that are about to be answered. Then problems in more in depth detail can be viewed in the relative paper section.

3.1 An agent-based model of consumer choice: Some preliminary results

The central investigative premise of the paper revolves around unraveling the layers and dynamics of consumer choice within the framework of economic theories, keenly probing into the quintessential question: How does the incorporation of agent heterogeneity and sequential decision-making within an Agent-Based Model (ABM) augment our understanding of consumer choice in contrast to the conventional 'representative agent' model adhered to by the Standard Economic Model (SEM).

Anchoring the critical exploration in the contrast and comparison between ABM and SEM, the paper intricately weaves through the corridors of classical demand theory and the representative agent framework, which traditionally subsumes an implicit assumption of agent homogeneity and thereby, potentially glosses over the nuanced dynamics and interactions within a consumer base that is inherently diverse and heterogeneous.

This core question cascades into various sub-questions and exploratory channels:

- How does the inherent heterogeneity in preferences and incomes among agents, modelled within the ABM, influence the evolution of expenditure and demand across different socioeconomic groups.
- To what extent does the ABM, with its sequential nature and bottom-up approach, provide a more comprehensive and realistic depiction of consumer choice dynamics, especially in scenarios involving asset pricing and market bubbles.
- Can the ABM effectively mirror and possibly predict market phenomena (such as bandwagon effects and market psychology) that are often witnessed but not adequately explained by the SEM.
- How does the ABM negotiate the theoretical terrain marked by the Walrasian equilibrium model, and what implications does it present for understanding market equilibrium in the absence of a Walrasian Auctioneer.
- What are the practical implications and potential applications of adopting ABM in understanding and potentially mitigating economic downturns and crises.

The problem definition primarily grapples with the paradigmatic shift from a homogeneous representation of consumer behavior (as depicted by SEM) to a more complex, nuanced, and heterogeneously-charged depiction facilitated by ABMs. Through a nuanced journey of comparing these models, exploring their implications in historical economic contexts, and diving deep into the theoretical mechanics of each, the paper aims to extract insights that could potentially reshape how economists and researchers approach, understand, and model consumer choice and market dynamics.

In essence, the paper critically evaluates the validity, reliability, and practical applicability of ABMs in accurately modeling and predicting economic phenomena by delving into its mechanical difference and theoretical divergence from the SEM, thus providing a panoramic view of its strengths, limitations, and prospective horizons. This endeavor is not merely academic but pivots towards a tangible impact on economic policymaking, crisis aversion, and in shaping future research methodologies within the realm of consumer choice theory and beyond.

3.2 An agent-based data-driven model of consumer demand

The core objective of this study revolves around understanding the intricate dynamics of consumer choice through the lens of agent-based modeling and simulation. This modeling approach stands in contrast with classical theories of consumer behavior. The paper delves deep into several layers of this overarching issue, leading us to the fundamental research question:

> How does the agent-based modeling and simulation framework provide insights into consumer choice, especially in light of commodities classified as necessities or luxuries, and against the backdrop of classical consumer behavior theories, given agents' heterogeneity in income and preferences?

To unpack the complexities and nuances of this central question, the paper further dissects it into several sub-questions:

1. How do varying *states of the world*, introducing elements of risk, influence consumer behaviors in the agent-based model characterized by distinct income and preference parameters?
2. In comparison to the standard economic model (SEM), how does the agent-based approach, built on weaker foundational assumptions, shed light on the dynamics of demand, especially when faced with price uncertainties and other market volatilities?
3. Considering the recent relevance and application of agent-based models in asset pricing, how can this methodology augment our understanding of major financial events, such as the financial crisis of 2008?
4. How does the agent-based model juxtapose with, and possibly challenge, dominant economic paradigms like the Walrasian equilibrium, especially when market dynamics include decentralized exchange mechanisms, strategic behaviors, and asymmetric information?
5. Utilizing advancements like the random forest algorithm from the realm of artificial intelligence, how can the demand function be effectively estimated for homogenous agent groups within this framework?

By addressing these questions, the paper seeks to demystify the potential of the agent-based modeling approach as a robust tool for understanding, predicting, and interpreting consumer choices and market dynamics in a contemporary economic landscape.

3.3 A bounded rational agent-based model of consumer choice

In the vast realm of economic research, understanding the complexities of consumer choice and its impact on demand has been a foundational pursuit. With the evolution of methodologies and tools, there's an impetus to redefine and refine our exploration methods. This paper is no exception and endeavors to address a multifaceted, but coherent, research question:

How can the marriage of agent-based modeling, stochastic utility optimization, and machine learning techniques provide a more granular, dynamic, and nuanced understanding of multiparametric consumer demand functions, especially in the context of varying consumer types and scenarios?

Nested within this overarching inquiry are several sub-questions that illuminate the different facets of our research:

1. How can agent-based modeling, given its inherent property of emergent behaviors, provide insights into the nonlinear, complex interactions between various economic agents and commodities, particularly when considering heterogeneous consumer preferences and behaviors?

2. Given the historical assumption of rational consumers, how does introducing stochastic elements through utility optimization refine our understanding of consumer choices, particularly in scenarios that involve unpredictable or uncertain preferences?

3. What is the potential of machine learning, specifically Gaussian Process Interpolation, in effectively interpolating and modeling the vast array of data outcomes derived from the aforementioned stochastic utility optimization, especially when considering its ability to capture intricate relationships in demand data?

4. How do our derived multiparametric demand functions illuminate established economic phenomena, such as the Veblen effect, and do they provide new insights or challenge conventional understanding of consumer behavior and market dynamics?

5. In what ways can the integration of agent-based modeling, stochastic optimization, and machine learning present a comprehensive framework that not only enhances our understanding of consumer choice dynamics but also provides tools for predicting future market behaviors?

By navigating through these queries, this paper seeks to carve out a niche in economic research, intertwining established theories with contemporary methodologies, and thereby contributing to the broader discourse on consumer behavior, demand theory, and the implications of technology in economics.

3.4 An agent-based study on the dynamic distribution and firms concentration in a closed economy

In light of the recent global economic downturn, traditional models have proven inadequate to provide an understanding of intricate market dynamics, leading to a pressing need for alternative and more refined models. This paper tackles the challenging question:

1. How does the interaction of universal constants with stochastic fluctuations in unit prices, within a closed (supply) economy consisting of non-interacting firms, influence the dynamics of business cycles and the distribution of capital among firms?

   This central problem can be broken down into several sub-questions:

2. Given a closed economy with non-interacting firms, how do individual firm capitals evolve over time, given that their dynamics are influenced by a combination of capital productivity, expenditure coefficient, and the stochastic fluctuation of unit prices?

3. How does the multiplicative random walk with drift, represented by the interplay of capital productivity coefficient and expenditure coefficient against the backdrop of fluctuating unit prices, influence the trajectories of firm capitals?

4. Do certain economic parameters or combinations thereof lead to market concentrations, and under what conditions might monopolies emerge within the given economic framework?

5. What role does the relationship between the universal constants and the stochastic process, which governs unit price, play in determining the phases of business cycles experienced by firms in the economy?

6. Given the dynamics of the model, how can metrics like the Modified Hirschman-Herfindahl Index (MHHI) be employed to measure market concentration and its implications on firm behavior and overall market outcomes?

By addressing these queries, this research seeks to develop an agent-based model that captures the intricate and multifaceted dynamics of firm behavior in a closed economy, while also bridging the gap between micro-level actions and macro-level outcomes.

3.5 Simulating Market Concentration and Capital Distribution: Insights into Firm Dynamics and Economic Inequality

In the intricate realm of economic dynamics, understanding the allocation of capital, its distribution among firms, and the consequential rise of banking capital stands paramount. The intricate weave of economic variables defines the heart of these dynamics, establishing a sophisticated network that is both captivating and challenging to interpret. Thus, the core research problem addressed by this paper can be defined as:

1. How does the stochastic fluctuation of firm profits, driven by a myriad of economic variables, influence the allocation of capital, market concentration, and the overarching dynamics of large-scale economic systems? Moreover, in such a complex system, how can shifts in economic parameters be modeled to discern their impact on the capital distribution and concentration within the market?

   To further dissect this primary research problem, several sub-questions arise:

2. How can agent-based modeling be leveraged to simulate the interactions between firms and financial institutions, thereby shedding light on the multifaceted economic dynamics?
3. In a simulated economy, what roles do variables like firm capital, bank capital, interest rates, and market indices play in influencing the overall market behavior and individual firm performance?
4. Given the inherent unpredictability of real-world markets, how can stochastic elements be seamlessly integrated into the simulation, ensuring a balanced representation of risk and randomness?
5. How does the distribution of wealth evolve over time, particularly in scenarios where wealth is initially evenly distributed among participants? Can the model shed light on the phenomenon where a minor segment of agents accumulates substantial wealth, leaving a majority with a minimal share?
6. How can indices like the Herfindahl-Hirschman Index (HHI) and its modified counterpart (HHIM) be employed within the model to quantify market concentration and infer levels of competition?

The intention behind exploring these questions is to create a robust and versatile tool for stakeholders across the spectrum - from policymakers and economists to corporate strategists. This tool, grounded in rigorous mathematical modeling and simulations, aims to provide insights into market dynamics, wealth distribution, and potential strategic responses to a variety of economic scenarios. The ultimate aspiration is not merely to mirror the intricacies of actual markets but to provide a structured platform for investigation, comprehension, and response formulation in the face of ever-evolving economic landscapes.

3.6 Undestrandig money distribution in closed economic systems: A graph theoritical approach with quantum random walks

In the evolving field of economic systems modeling, there is an increasing need to understand how wealth and resources are distributed among agents. Classical models and simulations offer certain insights, but they often lack the nuance and depth to capture the complex and probabilistic nature of economic exchanges. This has led researchers to search for more sophisticated models that can encapsulate the inherent uncertainties and non-linear complexities of economic transactions.

1. **Main Research Question:**
   How does the distribution of money in closed economic systems, characterized by conserved total wealth and fixed agent numbers, behave when modeled as quantum random walks on graph networks?

   This overarching question can be broken down into the following sub-questions:
2. *Graphical Representation:* How can we effectively represent economic agents and their interactions in closed economic systems using graph theory? What is the significance of vertices (representing economic agents) and edges (signifying potential transactions) in this context?

3. *Quantum Walks in Economics:* How do quantum random walks, which allow superpositions of states and inherently embrace the probabilistic nature of quantum mechanics, serve as tools to model money exchanges between economic agents?

4. *Effect of Network Topology:* How do different attributes of the graph, such as vertex connectivity, vertex degree, and global clustering coefficient, influence the distribution of wealth, as measured by the Gini coefficient?

5. *Comparison with Classical Models:* How does this quantum approach differ from classical random walks or other traditional models in terms of outcomes, predictability, and insights into economic dynamics?

6. *Applications and Implications:* What potential insights or patterns emerge from modeling economic exchanges as quantum random walks, and how can these findings be utilized to understand real-world economic scenarios better?

This research aims to bridge the gap between traditional economic models and the probabilistic, uncertain nature of real-world economic transactions. By integrating principles from quantum mechanics and graph theory, the study seeks to offer a more nuanced and comprehensive understanding of money distribution in closed economic systems.

4 Description of Tools and Methodologies

In addressing the multifaceted aspects of consumer choice, market concentration, and wealth distribution, particularly within varied economic structures, a robust set of tools and methodologies is imperative. This chapter elaborates on the selection and application of several quantitative techniques, ranging from probabilistic simulations to machine learning models and economic indices.

4.1 Monte Carlo Simulations

One of the standout attributes of MCS is its remarkable versatility. It's adaptable to almost any mathematical model or framework, and it's this flexibility that has made it particularly appealing to researchers and analysts. Additionally, its proficiency in handling intricate scenarios is notable. Monte Carlo simulations can competently manage models with numerous inputs and variables, which is essential in the realm of complex economic systems. Another noteworthy capability is the method's aptitude to incorporate diverse distribution types for its inputs. Whether we're looking at normal, binomial, Poisson, or any other distributions, MCS has it covered. This is of paramount significance in the world of economics, where the distribution of various elements—be it income, demand, or supply—is often far from uniform.

Shifting the lens to computational economics, MCS has emerged as a potent tool to simulate and dissect multifarious economic systems, especially where conventional analytical techniques might not be sufficient. A prime use-case is in the domain of financial risk analysis. Imagine simulating different economic jolts on a particular model to discern potential results; MCS is tailor-made for such endeavors. Then there's the optimization conundrum. When faced with a vast solution landscape, especially where alternate routes are computationally taxing, MCS offers a valuable ally.

Moreover, forecasting, a cornerstone of economic modeling, finds a dependable companion in MCS. It facilitates the creation of diverse future models to interpret possible economic paths.

But where MCS truly shines is in its synergy with Agent-Based Modeling (ABM). The latter is a computational methodology where entities, termed as agents, endowed with specific traits, engage within a predefined setting. This interaction leads to emergent outcomes at the system level. In many instances, initializing conditions for agents or the milieu they operate in draws from statistical distributions. Here, MCS steps in, producing these conditions and thereby ensuring a comprehensive and representative assortment of commencement points. A solitary execution of an agent-based model showcases one manifestation of a system, given a particular set of conditions. Yet, real-world systems are inherently fluctuating. Through its iterative random sampling, MCS facilitates multiple runs of an ABM. This capability ensures each model starts differently or undergoes varied random events. Such a multifaceted approach fosters a profound understanding of potential system behaviors.

A defining feature of ABM is the system-level patterns that arise from individual interactions among agents. Marrying ABM with MCS allows for the observation of how these emergent behaviors shift across diverse simulation iterations. This blend gifts us with a probabilistic comprehension of such emergences. Moreover, in numerous agent-based models, decision-making by agents is often influenced by probabilities, such as the chances of transitioning between brands or embracing new technology. Here, MCS offers an instrumental mechanism to bring these probabilistic decisions to life within the simulations.

In the context of this study, there's a seamless integration of Monte Carlo simulations with the agent-based modeling approach. By melding the two, the study ensures a comprehensive capture of the outcomes of diverse agent decisions, especially those steered by inherent stochastic processes. This repeated modeling, made feasible by MCS, facilitates the exploration of a range of emergent patterns. Whether it's dissecting wealth distribution dynamics or understanding market behaviors, the interplay between MCS and ABM offers invaluable insights into plausible real-world actions, especially within interconnected and isolated economic frameworks.

4.2 Machine Learning: Gaussian Process and Random Forest

In the evolving realm of computational economics, two notable machine learning models, Gaussian Process (GP) and Random Forest, have found substantial traction for their nuanced capabilities.

A Gaussian Process is intriguingly viewed as a distribution over functions. At its essence, it's a collection of random variables, and any specific subset of these variables adheres to a joint Gaussian distribution. Every function within this model symbolizes a potential association between input and output spaces. The kernel, sometimes called the covariance function, plays a pivotal role, shaping the behavior and attributes of the GP. It outlines the intricate relationships between various points in the input domain. Remarkably, GPs are non-parametric. Such a design lets them embody a myriad of functional forms, uninhibited by a predefined format, making them agile and adaptable to diverse datasets. Among the myriad advantages they offer, their ability to quantify uncertainty is unparalleled. This not only presents confidence intervals around predictions but also proves invaluable in computational economics, where exact functional relationships, say between demand and supply, often remain elusive. GPs can deftly approximate these nebulous relationships, shedding light on intricate economic dynamics. Forecasting, an indispensable component of economics, gets a boost from GPs, particularly since they can delineate the span of possible future metrics, arming policymakers and businesses with enhanced predictive power. Moreover, within the purview of

Agent-Based Modeling (ABM), GPs act as crucial catalysts. Agents can harness the power of GPs to continually learn and adapt to their environment, refining decisions based on empirical observations. Furthermore, GPs can significantly enhance model calibration, particularly when specific parameters within an ABM remain indeterminate, by gleaning the most fitting parameters using observed real-world datasets.

Pivoting to Random Forest, this ensemble machine learning technique is a conglomerate of numerous decision trees framed during the training phase. When presented with new data, it either expounds the mode of the classes for classification tasks or averages out predictions for regression tasks. The ensemble character of Random Forest is its bulwark against overfitting, making it sturdier than individual decision trees. One of its most lauded features is its knack for discerning feature importance. It can incisively detect pivotal variables that significantly sway the outcome. Versatility is another feather in its cap, as it seamlessly processes both categorical and numerical data. Random Forests' prowess doesn't end here. They can adeptly predict economic outcomes, sculpted by diverse factors. For instance, they can forecast market demand, weighing in variables like pricing strategies, advertising expenditures, or even actions by competitors. The feature importance metric of Random Forests can be a goldmine for researchers, pinpointing the linchpin economic drivers in a system. This understanding can guide more targeted policy decisions or interventions. Within the context of ABM, agents equipped with a trained Random Forest model can anticipate the ramifications of their actions or even gauge probable actions of their peers based on historical patterns. Furthermore, after orchestrating ABM simulations, Random Forest stands as a sentinel, validating the model by juxtaposing simulated outcomes against the real-world dataset, ensuring that the simulations mirror real-world intricacies with precision.

In summation, both Gaussian Process and Random Forest bring a rich tapestry of capabilities to the fore in computational economics, making them indispensable tools for researchers and analysts in the field. Their intricate designs, coupled with their capacity to handle diverse datasets and scenarios, ensure they remain at the forefront of predictive and analytical modeling.

4.3 Markov Processes and Stochastic Processes

The realm of computational economics is replete with methodologies that dive deep into uncertainties intrinsic to economic systems. Among these, stochastic processes and Markov processes emerge as powerful tools, each illuminating different facets of economic dynamics.

Stochastic processes are often perceived as sequences of random variables that advance through time, melding both predetermined conditions and innate randomness. These processes elegantly capture the temporal evolution of systems, tracing their dynamics and potential trajectories. They are remarkable for their versatility, evident in their adaptability to discrete or continuous time or space states and their ability to incorporate diverse probability distributions. This adaptability is invaluable in computational economics. For instance, the unpredictability and volatilities of stock prices, commodities, or even assets find their mathematical representation in stochastic differential equations. These equations provide insights into the underlying uncertainties that are part and parcel of market dynamics. Beyond asset prices, stochastic processes also offer a framework to model and analyze unexpected macroeconomic perturbations, such as abrupt changes in employment rates. By doing so, they grant economists the ability to scrutinize the cascading impacts of these shocks throughout the broader economic fabric.

Diving deeper into the subset of stochastic processes, Markov processes stand out, characterized by the eponymous Markov property. This property ensures that the subsequent state of the process hinges solely on its present state, rendering past states irrelevant. This 'memorylessness' is pivotal, signaling that the future trajectory of a system doesn't rely on its historical path. Markov processes, especially in contexts with finite states, often utilize state transition matrices to denote probabilities of inter-state transitions, streamlining analyses and enhancing the clarity of predictions.

The applicability of Markov processes in computational economics is vast. Consumer behavior, for example, is susceptible to constant evolution. In market dynamics, the propensity of a consumer migrating from one brand preference to another finds its analog in the Markov chains, delineating the ebb and flow of brand loyalties. Similarly, the lending sector can harness Markov processes to gauge the evolution of a borrower's credit standing – the transition probabilities can indicate potential shifts from commendable credit scores to possible delinquencies or even outright defaults.

Yet, the potential of both stochastic and Markov processes is most prominently realized in the domain of Agent-Based Modeling (ABM) in economics. ABM, with its emphasis on individual entities (agents) and their interactions, finds a perfect partner in these processes. Agents can be ascribed various states, and their subsequent behaviors, interactions, and decisions can be orchestrated by the transition probabilities intrinsic to Markov processes. Envision a financial ABM where agents oscillate between roles such as 'savers,' 'investors,' or 'defaulters.' Markov processes can steer these transitions, influenced by both external environmental variables and inherent propensities.

As these agents engage, interact, and transition, they invariably impact the broader system. Utilizing Markov processes within an ABM framework can amplify our comprehension of these intricacies, shedding light on how localized behaviors might coalesce to spawn overarching system-level patterns. Furthermore, the introduction of Markovian assumptions can occasionally trim down the complexity of agent-based models, particularly when historical data doesn't significantly dictate future outcomes. Such simplifications can be a boon, enhancing computational efficiency and expediting simulations.

In conclusion, both stochastic and Markov processes offer a profound depth of understanding in computational economics. Their ability to model uncertainties, transitions, and inherent randomness makes them invaluable. When synergized with Agent-Based Modeling, they provide an encompassing lens, enabling researchers to decipher intricate economic dynamics, agent behaviors, and emergent phenomena. Their significance in economic modeling cannot be overstated, particularly in an era where the blend of computation and economics is steering policy decisions, market strategies, and academic research.

4.4 Random Walks:

Random walks, at their core, encapsulate paths shaped by a sequence of random steps. Typically conceived within spatial frameworks, these paths might manifest on structured grids, like lattices, or sprawl within continuous realms. Crucially, each step's direction emerges from probabilistic determinants. An intrinsic characteristic of random walks is their memorylessness, closely mirroring the traits of Markov processes. In this context, the subsequent step in a random walk relies solely on its present position, indifferent to the journey that led there.

Delving into its diverse types, the simple random walk is a prominent archetype, unfolding on a lattice. Here, each step the walker takes offers equal chances of veering in any conceivable direction. In contrast, the biased random walk has a predilection for specific directions, while the continuous random walk meanders unrestrictedly through both space and time.

The pertinence of random walks in computational economics is profound. Financial economics frequently invokes them, particularly when addressing stock prices. This stems from the Efficient Market Hypothesis, which suggests that stock prices navigate a random walk trajectory, given that consecutive price changes remain independent and uniformly distributed. Beyond the world of stocks, even the oscillations of exchange rates sometimes find their explanations in random walks, especially when their short-lived fluctuations appear inscrutable. An intriguing application also emerges in consumer behavior, where in the face of overwhelming choices and the absence of a definitive strategy, consumers' decisional paths might resonate with the unpredictable trajectories of a random walk.

However, the potential of random walks truly shines when integrated into Agent-Based Modeling (ABM). Within this milieu, agents can emulate random walk behaviors, especially when they grapple with incomplete information or an environment riddled with uncertainties. This modeling approach is especially pertinent in fragmented economic systems. Furthermore, agents often oscillate between the compulsion to explore their surroundings, reminiscent of a random walk, and the urge to harness known resources. This exploration-exploitation conundrum finds its exploration roots in the random walk paradigm. More intriguingly, when multiple agents within an ABM adhere to random walk principles, the aggregate system behavior, influenced by localized interactions, can unveil emergent patterns, even if individual paths appear haphazard. This phenomenon becomes even more evident in models that probe the spread of information, where agents disseminate knowledge or hearsay as they randomly intersect with others.

Reflecting on our research context, random walks proved indispensable. They simulated individual agents' paths as they navigated markets, especially when these markets lacked interconnected structures. In such terrains, the dearth of holistic, accessible information means agents predominantly lean on localized decision frameworks. While their choices retain a semblance of rationality, they often align closely with the unpredictability of random walks. Such modeling illuminated intricate facets of market dynamics, offering insights into wealth dispersion and consumer decisions, particularly in settings devoid of overarching structure.

In summation, random walks emerge as a potent tool, capturing the inherent uncertainties, serendipities, and individualistic trajectories prevalent in diverse systems. By weaving them into computational economics and agent-based modeling, we can simulate and decipher intricate systems from a granular perspective, acknowledging and embracing the often capricious paths entities carve. Their significance in economic modeling, especially in agent-based contexts, underscores their ability to resonate with real-world unpredictability, making them an invaluable asset in the researcher's toolkit.

4.5 Economic Indices in Computational Economics and Agent-Based Modeling: Gini Coefficient and Herfindahl Index

The exploration of economic indices, especially in computational economics and agent-based modeling, provides a robust framework to analyze intricate systems and phenomena. Two such pivotal metrics that have gained widespread prominence are the Gini coefficient and the Herfindahl Index.

The Gini coefficient offers a comprehensive measure of inequality, particularly in the distribution of attributes such as wealth or income. This coefficient scales between 0 and 1, where the former denotes an idyllic scenario of equality, and the latter epitomizes the pinnacle of inequality. Often, this metric finds its graphical representation in the Lorenz curve, which charts the cumulative proportion of wealth against the population. In the realm of computational economics, the Gini coefficient emerges as a powerful tool, especially for model evaluations. Whether it's assessing the repercussions of policy modifications or diving into a comparative analysis of income disparities across regions, this coefficient provides unparalleled insights. Within the agent-based modeling context, the Gini coefficient further aids in delineating emergent wealth distributions. As agents operate based on localized rules, tracking the Gini coefficient iteratively during simulations can unravel how minute interactions snowball into overarching patterns of wealth distribution. Moreover, when the goal is to mimic real-world conditions, the coefficient acts as a benchmark, steering the calibration of the model until it mirrors observed inequalities.

Transitioning to the Herfindahl Index, or more comprehensively, the Herfindahl-Hirschman Index (HHI), one dives into the nuances of market share concentration within an industry. Often hailed as the touchstone for antitrust authorities and economists, the HI quantifies market structures. The index is derived from the summation of squared market shares of individual firms, and its values oscillate between near 0 (reflecting a fragmented market teeming with small entities) to 1 (a monopoly). Computational economists frequently harness the HI, especially when delineating the contours of monopolies and competition. For instance, in a speculative exercise involving potential mergers, the HI can forecast the ensuing market concentration, subsequently spotlighting any looming anti-competitive concerns. Within agent-based models representing market dynamics, the HI morphs into a dynamic metric. Here, as agents — which might symbolize firms or producers — evolve, merge, or even bow out, the HI unfurls the shifting tapestry of market concentration. Moreover, when different regulatory landscapes are simulated, the HI becomes an instrumental measure, enabling analysts to gauge the impacts of policy changes on market structures.

Reflecting upon our in-depth exploration of consumer choices, market concentration, and wealth dispersal, it becomes evident that both the Gini coefficient and the Herfindahl Index are invaluable. These metrics crystallize the intricacies of agent interactions into discernible, quantifiable narratives. By monitoring their evolution within an agent-based model, we unearth the undercurrents steering wealth inequalities and market power shifts. In essence, these indices, while rooted in traditional economic theories, find renewed relevance in modern computational economics, bridging micro-level interactions with macro-level patterns.

4.6 Networks, Small-World Properties, and Agent-Based Modeling in Economics

In the intricate world of computational economics, understanding networks and their characteristics can unravel the fabric of complex systems. At its core, a network or graph is essentially a compilation of nodes, representing entities, connected by edges which symbolize the relationships or interactions between these entities.

One of the fascinating quirks of certain networks is the small-world property. These networks are intriguing due to their high clustering coefficient, which means nodes tend to form closely-knit groups. Yet, despite this close-knittedness, the average path length—or the average number of steps it takes to connect one node to another—is quite low. This phenomenon gives rise to the concept of 'six degrees of separation,' a popular notion that every individual on Earth is approximately six introductions away from any other person.

While the small-world property is an exciting network characteristic, there are various ways to generate or model networks. The Erdős–Rényi (ER) model, for instance, offers a straightforward approach, building a network by randomly linking nodes. In such networks, a pair of nodes connects with a certain probability 'p'. The simplicity of the ER model results in a binomial degree distribution in large networks. However, one of its limitations is the lack of significant clustering, which is often observed in real-world networks.

Beyond the ER model, there are other compelling network types. Scale-free networks, for example, boast a power-law degree distribution, indicating the presence of certain highly connected nodes amidst many low-connectivity ones. On the other end of the spectrum, regular lattices display nodes connected in an orderly pattern, resembling a grid. They naturally possess high clustering coefficients, yet they also have extensive path lengths. Then there's the Watts-Strogatz model, which introduces an intriguing blend by starting with a regular lattice and then introducing random rewiring, giving rise to small-world properties.

Networks hold immense significance in computational economics. They can elegantly model trade dynamics between countries, provide insights into interbank lending patterns and potential systemic risks, and even shed light on the intricate flows within supply chains. When one infuses these network theories into agent-based modeling (ABM), the results are profound. In ABM, agents can be linked via a network, determining their interactions. Such a framework is particularly apt for market models, as not all agents might interact universally.

Furthermore, the very structure of the network can become a pivotal factor influencing emergent behaviors within the model. In a small-world network, for instance, the rapid dissemination of information or innovations becomes a hallmark. By modeling economic systems as intricate networks within ABM, one can probe deeper into the propagation of shocks, identify nodes that are vital for maintaining the health of the system, and even decipher how alterations in the network structure can transform the overarching system behavior.

By juxtaposing different market structures—like monopolies and oligopolies—and the inherent network of agent interactions, ABMs can simulate and interpret the genesis of these economic phenomena.

In essence, the fusion of network theory, especially the nuances of small-world properties, with agent-based modeling offers a robust analytical toolkit for computational economics. This combination demystifies not just the actions of individual agents but also how their cumulative actions orchestrate broader economic phenomena. Such an understanding is invaluable, setting a strong foundation for deciphering the multidimensional dynamics inherent in economic systems.

5 Literature Review

Agent-based modeling (ABM) has emerged as a powerful tool for studying complex economic systems, particularly in the context of consumer choice and demand, as well as wealth inequality. This literature review examines the use of ABM to explore consumer choices between two goods, focusing on the incorporation of new economic thinking, historical context, and the role of heterogeneity and bounded rationality in shaping individual decision-making processes. The review also discusses the application of ABM in understanding market stability and the impact of global economic events, such as the COVID-19 pandemic, on consumer behavior and wealth disparities. As the global economy becomes more interconnected and dynamic, ABMs can provide valuable insights into the mechanisms driving consumer choices, wealth inequality, and their implications for the broader economic landscape (Farmer, Foley (2009)).

5.1 Agent-Based Modeling: Consumer Behavior and Wealth Inequality

Agent-based models (ABMs) have emerged as an alternative to traditional economic models that rely on assumptions of perfect competition, rationality, and optimization. ABMs offer a more flexible and realistic approach to modeling complex economic systems by accounting for the heterogeneity of market participants, their adaptive behavior, and the role of wealth inequality in shaping individual decisions (Macal, North (2005)). By simulating the interactions of many agents with diverse attributes and behaviors, ABMs can capture emergent phenomena that may not be easily explained by aggregate models.

The concept of bounded rationality, introduced by (Simon (1955)), challenges the assumption that individuals have perfect information and make optimal decisions. Bounded rationality acknowledges the limitations and constraints faced by individual decision-makers in real-world situations. ABMs incorporate this concept to provide a more nuanced understanding of consumer behavior, wealth inequality, and their impact on the economy (Tesfatsion (2006b)).

Moreover, the incorporation of behavioral economics principles, such as loss aversion and social preferences, can further enhance the realism of agent-based models in capturing consumer decision-making processes and wealth disparities (Thaler (2015b)). Loss aversion, a concept introduced by Kahneman, Tversky (1979a), suggests that individuals are more sensitive to losses than gains, which can lead to suboptimal decision-making. Social preferences, on the other hand, emphasize the importance of fairness, reciprocity, and other social factors in shaping individual decisions (Fehr, Schmidt (1999)). By integrating these principles, ABMs can more accurately model the complexity of human behavior in economic settings.

In recent years, agent-based models have been used to study various economic phenomena, such as financial market dynamics, labor market behavior (Neugart, Richiardi (2018)), and macroeconomic dynamics. These studies illustrate the versatility and applicability of ABMs in exploring different aspects of the economy. Moreover, ABMs have been instrumental in understanding the impact of policy interventions on wealth inequality, market stability, and economic growth (Gintis, Helbing (2015)).

Despite their advantages, ABMs are not without limitations. One challenge associated with ABMs is the calibration and validation of model parameters (Windrum et al. (2007)). Since ABMs involve numerous parameters and complex interactions among agents, it can be difficult to determine the appropriate parameter values to accurately represent real-world situations. Additionally, computational constraints can limit the scalability and scope of agent-based models, making it challenging to simulate large-scale economic systems.

5.2 Incorporating Artificial Intelligence, Machine Learning, and Wealth Inequality

The incorporation of artificial intelligence (AI) and machine learning (ML) techniques has significantly expanded the potential of agent-based models (ABMs) in understanding complex economic systems and the role of wealth inequality. By leveraging AI algorithms, researchers can better analyze and predict agent behaviors and interactions, contributing to a more comprehensive understanding of market dynamics, consumer behavior, and wealth **?**. In this section, we will discuss the integration of AI and ML techniques in ABMs and their implications for studying wealth inequality.

One of the primary AI algorithms that has been integrated into ABMs is the random forest algorithm (Breiman (2001)). Random forests, an ensemble learning method, combine multiple decision trees to improve predictive performance and reduce overfitting. They have been utilized in ABMs to estimate demand functions for groups of homogeneous agents and to analyze the impact of policy interventions on wealth inequality (Pangallo et al. (2019)). By incorporating random forests, researchers can enhance the predictive accuracy of ABMs and gain valuable insights into the underlying mechanisms driving wealth disparities in economic systems.

In addition to random forests, other ML techniques such as neural networks and reinforcement learning have been employed to improve the representation of agent behavior in ABMs. Neural networks, a class of ML models inspired by the structure and function of biological neural networks, can be used to uncover hidden patterns in large datasets and simulate adaptive behaviors more effectively (Soltoggio et al. (2018)). They have been applied in various economic contexts, including modeling consumer behavior, predicting market dynamics, and simulating the impacts of fiscal and monetary policies on wealth distribution (Castelnuovo, Nistico (2010)).

Reinforcement learning, another ML technique, enables agents in ABMs to learn from their experiences and adapt their behavior over time (Zhang et al. (2021)). This approach has been particularly useful in modeling adaptive behavior in economic settings, such as labor market participation, financial investment, and consumption decisions. By incorporating reinforcement learning, ABMs can better capture the dynamics of learning and adaptation, which are crucial for understanding the emergence and persistence of wealth inequality in economic systems (Lippe et al. (2019)).

The integration of AI and ML techniques in ABMs has also facilitated the analysis of large-scale economic systems and the study of wealth inequality at various levels of aggregation (Zhang, Vorobeychik (2019)). For instance, researchers have employed AI-enhanced ABMs to examine the effects of tax policies, redistribution mechanisms, and other policy interventions on wealth distribution at the national and regional levels. These studies have provided valuable insights into the potential consequences of policy decisions on wealth inequality and have informed the design of more effective and equitable policy interventions.

Despite the advancements brought by the integration of AI and ML techniques in ABMs, there are several challenges that need to be addressed. One major challenge is the interpretability and transparency of AI and ML algorithms (Akinosho et al. (2020)). As these models become more complex, it may become increasingly difficult for researchers to understand the underlying mechanisms driving the model's predictions and behaviors. This can limit the applicability of AI-enhanced ABMs in informing policy decisions and guiding economic theories (Inderwildi et al.

(2020)). Another challenge is the potential for biases and unfairness in AI and ML algorithms, which may inadvertently perpetuate existing inequalities or introduce new ones (Barocas et al., 2017). To address these challenges, researchers must develop rigorous validation and verification procedures, as well as ethical guidelines for the use of AI and ML in ABMs, to ensure that the models are both accurate and fair.

5.3 Decentralized Markets, Resource Allocation, and Wealth Inequality

Agent-based modeling (ABM) has emerged as a valuable tool for studying decentralized markets, resource allocation, and the impact of these factors on wealth inequality (Albin and Foley, 1992). In contrast to traditional economic models that rely on assumptions of perfect competition and rationality, ABMs provide a more flexible and realistic approach to understanding the complexities of real-world markets, where these assumptions often do not hold (Axtell, Farmer (2022)). By exploring decentralized exchange mechanisms and simulating the interactions of heterogeneous agents, researchers can gain valuable insights into market dynamics and the consequences of wealth disparities in decentralized markets.

One important contribution of ABMs in studying decentralized markets is the analysis of the emergence and evolution of market institutions, such as auctions, bargaining processes, and matching mechanisms. By simulating the interactions among market participants and the formation of these institutions, researchers can investigate the factors that drive the development and persistence of various market arrangements, as well as their implications for resource allocation, market efficiency, and wealth inequality (Grewal, Dharwadkar (2002)). This research can help inform the design and regulation of market institutions, and contribute to the development of innovative market mechanisms that promote equitable and efficient resource distribution while addressing wealth disparities (George et al. (2012)).

Another area where ABMs have proven valuable is in the study of market frictions and their impact on wealth inequality. Market frictions, such as asymmetric information, transaction costs, and search costs, can lead to inefficient resource allocation and exacerbate wealth disparities (Kingsley et al. (2015)). By incorporating market frictions into their models, researchers can better understand the mechanisms through which these frictions contribute to wealth inequality and explore potential policy interventions to mitigate their effects.

Moreover, ABMs have been employed to investigate the role of social networks, trust, and social capital in decentralized markets, and how these factors influence resource allocation and wealth distribution . Social networks can shape economic outcomes by affecting information flows, risk-sharing, and the enforcement of contracts. By simulating the formation and evolution of social networks and their effects on market interactions, ABMs can provide insights into the ways in which social structures contribute to wealth inequality and inform the design of policies that promote social cohesion and economic equity.

Furthermore, ABMs have been used to study the impact of market power and strategic behavior on resource allocation and wealth inequality (Bakos (1991)). In decentralized markets, firms and individuals may engage in strategic behavior to exploit their market power, leading to inefficiencies and exacerbating wealth disparities (North, Macal (2007)).

5.4 Policy Implications and Future Directions

The insights gained from agent-based modeling (ABM) have significant implications for evidence-based policy decisions, as they can help shape a more resilient, equitable, and sustainable economy (Hynes et al. (2020)). By examining the impact of various policy interventions, such as progressive taxation, wealth redistribution, and regulatory measures, ABMs can provide valuable information for policymakers aiming to reduce wealth inequality and promote economic stability (D'Orazio (2019)). In this section, we discuss some of the policy implications and future research directions in the field of agent-based modeling.

One area where ABMs can inform policy decisions is the design and implementation of taxation and wealth redistribution mechanisms. By simulating the effects of different tax structures and redistribution schemes on wealth inequality, ABMs can help policymakers identify the most effective strategies for promoting economic equity and social welfare (Ghorbani et al. (2014)). For instance, ABMs have been used to analyze the impact of progressive income taxes, wealth taxes, inheritance taxes, and capital gains taxes on wealth distribution, allowing researchers to compare the effectiveness of different policy instruments in addressing wealth disparities (Kuziemko et al. (2015)).

Another policy domain where ABMs can provide valuable insights is the regulation of market institutions and financial markets. By modeling the interactions between market participants and the effects of various regulatory measures, ABMs can help policymakers assess the potential consequences of their decisions on market efficiency, financial stability, and wealth inequality (Vermeulen, Pyka (2016)). For example, ABMs have been employed to study the impact of capital requirements, leverage restrictions, and macroprudential policies on the stability and resilience of the financial system, as well as their implications for wealth distribution (Popoyan et al. (2017)).

In addition to informing the design of specific policy interventions, ABMs can also contribute to a broader understanding of the factors that drive wealth inequality and the potential feedback loops between inequality, economic growth, and social welfare. By simulating the complex interactions between individual behavior, market dynamics, and policy interventions, ABMs can help researchers identify the underlying mechanisms that contribute to wealth disparities and explore the potential consequences of different policy scenarios for long-term economic development and social cohesion (Coburn (2000)).

Future research in agent-based modeling could explore the role of social networks, cultural factors, and cognitive biases in shaping individual decision-making processes, consumer behavior, and wealth disparities. This multidisciplinary approach can lead to a more comprehensive understanding of the complex interactions between individual behavior, market dynamics, and wealth inequality (Collins et al. (2011)). For example, researchers could investigate how social norms, values, and beliefs influence consumption patterns, savings behavior, and investment decisions, and how these factors, in turn, affect wealth distribution and economic outcomes (Guiso et al. (2006)).

Moreover, the continued development and integration of artificial intelligence (AI) and machine learning (ML) techniques in agent-based modeling can further enhance the accuracy and predictive capabilities of these models. As computational power and data availability continue to increase, ABMs will play an increasingly important role in advancing our understanding of consumer choices, wealth inequality, and their implications for economic outcomes in an increasingly interconnected and dynamic global economy (Chappin et al. (2020)).

5.5 General and further application

Agent-based modeling has proven to be a valuable approach to understanding consumer behavior and demand in complex economic systems. By incorporating more realistic assumptions about human decision-making processes, such as bounded rationality, and leveraging advanced computational techniques, such as artificial intelligence and machine learning, researchers can develop richer representations of market dynamics and responses to various policy interventions or economic shocks (Dixon et al. (2020)).

The literature on ABM and its application to consumer choice and demand has significant implications for both academic research and real-world policy interventions. As global economic events, such as the COVID-19 pandemic, continue to evolve, the insights gained from agent-based modeling research will prove invaluable in informing evidence-based policy decisions and shaping a more resilient and sustainable economy (Bak-Coleman et al. (2021)).

By employing agent-based modeling techniques to study consumer choice and its impact on demand, researchers can develop more comprehensive and accurate models that better reflect the complexities of consumer behavior and contribute to a deeper understanding of the economy. This literature review highlights the importance of exploring alternative models that account for the complexities of real-world markets and emphasizes the potential of agent-based modeling in advancing our understanding of consumer choices and their implications for economic.

As the world continues to face an increasingly interconnected and dynamic economic landscape, the need for more sophisticated tools and methodologies to study consumer choice and its impact on demand becomes paramount. Agent-based modeling has emerged as a powerful instrument to fill this need, enabling researchers to model complex economic systems with a greater degree of accuracy and flexibility (Ringler et al. (2016)).

Recent advances in computational power and the widespread availability of big data have further expanded the potential of agent-based modeling in the study of consumer choice. For example, the integration of social network analysis in ABMs allows researchers to better understand the role of social influence and information diffusion in shaping consumer preferences and behaviors. Additionally, the incorporation of behavioral economics principles into agent-based models, such as loss aversion, endowment effects, and mental accounting, can offer a more holistic and realistic representation of consumer decision-making processes (Jeffrey, Putman (2013)).

Furthermore, agent-based modeling provides a valuable tool for exploring the impact of regulatory interventions and policy changes on consumer behavior and market outcomes. For example, ABMs have been used to assess the effectiveness of carbon taxes and emissions trading schemes on consumer choices and the overall emissions reduction (Kothe et al. (2021)). These insights can inform the design and implementation of more effective and targeted policy interventions aimed at achieving specific economic, social, or environmental goals.

In the context of market stability, agent-based models have been used to study the emergence of market crashes and financial contagion. By simulating the interactions among heterogeneous agents, researchers can gain insights into the feedback loops and nonlinear dynamics that contribute to market volatility and systemic risk (Paulin et al. (2019)). This knowledge can help inform the development of macroprudential policies aimed at mitigating financial instability and promoting economic resilience (Giese et al. (2013)).

The literature on ABM and consumer choice has also examined the role of marketing and advertising in shaping consumer preferences and behavior. By incorporating advertising strategies and competitive dynamics into their models, researchers can better understand the impact of marketing campaigns on market shares, brand loyalty, and consumer welfare (Huang et al.

(2012)). This information can prove useful for businesses and policymakers alike in the design and evaluation of marketing and advertising regulations, as well as the development of more effective and consumer-centric marketing strategies (Harcar, Kaynak (2008)).

Econophysics offers a different perspective on economic systems, treating them as complex systems with many interacting agents, similar to the behavior of particles in physical systems (Schinckus (2013a)). The application of econophysics to agent-based models allows researchers to explore the emergent properties of economic systems, including wealth distribution and inequality, through the lens of statistical mechanics and complex systems theory (Sornette (2014)).

Incorporating econophysics concepts into agent-based modeling can help reveal the underlying mechanisms driving wealth inequality, such as the role of network structures, social stratification, and the distribution of economic opportunities (Axtell, Farmer (2022)). By simulating the interactions among heterogeneous agents with different preferences, constraints, and access to resources, agent-based models can capture the dynamics of wealth accumulation and redistribution more accurately than traditional economic models (Dosi, Roventini (2019)).

One notable application of agent-based modeling and econophysics to wealth inequality is the study of Pareto distributions, which often characterize the distribution of wealth in real-world economies. By simulating the interactions of agents following simple exchange rules, agent-based models can reproduce the emergence of such distributions and provide insights into the factors that contribute to wealth concentration among a small fraction of the population (Chakraborti et al. (2011)).

By combining agent-based modeling with insights from econophysics, researchers can explore the complex dynamics of wealth inequality, its underlying causes, and the potential impact of various policy interventions aimed at reducing disparities in wealth distribution. These insights can inform policy debates on issues such as taxation, social welfare, and labor market regulations, and contribute to the design of more effective and equitable economic policies (Bianchi, Squazzoni (2015)).

5.6 Econophysics: An Interdisciplinary Approach in the Study of Wealth Distribution

Econophysics is an interdisciplinary field that combines ideas and methods from physics, particularly statistical mechanics, with economics to study complex systems, such as wealth distribution and financial markets (Rickles (2011)). This approach has led to the development of new models and techniques, providing valuable insights into the mechanisms underlying wealth inequality and the evolution of wealth distribution.

5.6.1 Origins and Development

The foundations of econophysics can be traced back to the application of statistical mechanics principles in the study of social phenomena. One of the early pioneers of this interdisciplinary approach was physicist Ludwig Boltzmann, who attempted to understand the emergence of macroscopic patterns and collective behavior in economic systems using the principles of statistical mechanics. These initial efforts laid the groundwork for what would later become econophysics (Richmond et al. (2013)).

In the 1960s and 1970s, researchers such as Benoit Mandelbrot, Eugene Stanley, and Thomas Schelling began to apply statistical mechanics and complex systems theory to economic problems, laying the groundwork for the modern field of econophysics (Dash (2014)). This nascent field aimed to provide a more quantitative and rigorous approach to understanding economic phenomena, which was in stark contrast to the largely qualitative and deductive methods of traditional economics.

The term "econophysics" was first coined by H. Eugene Stanley and his colleagues in the early 1990s. The field has grown rapidly since then, attracting researchers from diverse disciplines, including physics, mathematics, computer science, and economics. The interdisciplinary nature of econophysics has led to the development of novel models, methods, and techniques for studying complex economic systems, bridging the gap between the natural and social sciences (Chen, Li (2012)).

The emergence of econophysics as a distinct field was facilitated by several factors. First, advances in computational power and data availability enabled researchers to perform large-scale simulations and analyze vast amounts of economic and financial data. Second, the increasing complexity of economic systems and the limitations of traditional economic models in explaining real-world phenomena, such as financial crises and wealth inequality, motivated researchers to explore alternative approaches (Farmer et al. (2012)). Finally, the growing acceptance of interdisciplinary research in academia and the recognition that economic systems exhibit many features common to complex systems in physics and other natural sciences contributed to the development of econophysics as a field (Schinckus (2013b)).

Since its inception, econophysics has produced a plethora of research contributions, including the development of power-law distributions for income and wealth, the application of complex network theory to economic systems, and the study of financial market dynamics (Jovanovic et al. (2019)). As the field continues to grow and mature, it is likely that econophysics will continue to provide valuable insights into the underlying mechanisms driving economic phenomena and contribute to the development of more accurate and realistic models of economic systems.

5.6.2 Statistical Mechanics and Wealth Distribution

One of the most significant contributions of econophysics to the study of wealth distribution has been the development of statistical mechanics-based models that explain the emergence of power-law distributions in income and wealth (Venkatasubramanian et al. (2015)). This approach has its roots in the principles of statistical mechanics, which describes the collective behavior of large ensembles of particles. Econophysicists have drawn inspiration from the kinetic exchange mechanism in particle systems, where particles can exchange energy through collisions, to model the dynamics of wealth redistribution and the emergence of wealth inequalities (Chakrabarti et al. (2013)).

One such model is the Boltzmann-Gibbs distribution, which is derived from the principle of maximum entropy and has been applied to describe the distribution of wealth in certain economic systems (Dragulescu, Yakovenko (2000)). This model posits that wealth is distributed according to an exponential function, which is consistent with the empirical observations of income distribution in some economies.

Another important econophysics model is the Pareto-Levy distribution, which is characterized by a power-law tail and can be used to model the distribution of wealth among the richest individuals in society (Iglesias et al. (2021)). This model has been successful in reproducing empirical wealth and income distribution patterns observed in real-world economies and provides insights into the underlying mechanisms driving wealth inequality.

Similarly, Gibrat's law of proportional growth has been used to model the distribution of firm sizes and income. According to this model, the growth rate of a firm or individual's income is proportional to its current size. This leads to a log-normal distribution of firm sizes and income, which has been observed empirically in various economic systems (Reichstein, Jensen (2005)).

Econophysics models have also investigated the role of savings and market fluctuations in driving wealth accumulation and inequality. For instance, the kinetic exchange models with savings have shown that the introduction of a savings parameter can lead to a more equitable wealth distribution. Moreover, the impact of market fluctuations on wealth accumulation has been studied using models that incorporate stochastic elements, such as the multiplicative stochastic process (Diaz et al. (2003)).

These econophysics models have provided valuable insights into the mechanisms driving wealth inequality, such as the role of savings, the impact of market fluctuations, and the influence of economic policies. By offering a probabilistic framework for understanding the dynamics of wealth redistribution, these models have advanced our understanding of wealth distribution and contributed to the development of more realistic and accurate models of economic systems. As the field of econophysics continues to evolve, it is likely that these models will be further refined and extended, providing even more valuable insights into the complex dynamics of wealth inequality (Chakrabarti et al. (2013)).

5.6.3 Complex Networks and Economic Systems

Econophysics has significantly contributed to the study of economic systems from a complex network perspective. Complex networks are characterized by their intricate patterns of connections among various elements, which can provide insights into the structure and dynamics of the systems they represent (Cimini et al. (2015)). By analyzing the structure and dynamics of economic networks, researchers can gain valuable insights into the stability and resilience of financial systems, the propagation of risk, and the impact of network topology on wealth distribution (Cimini et al. (2015)).

Interbank networks, for example, have been the subject of extensive research in econophysics. These networks consist of financial institutions connected through lending and borrowing relationships (Klinger et al. (2014)). By studying the topology and dynamics of interbank networks, researchers have been able to identify channels of contagion during financial crises and pinpoint potential vulnerabilities in financial systems. This research has informed the development of macroprudential policies and regulatory measures aimed at enhancing the stability and resilience of the global financial system.

Econophysics approaches have also been applied to the study of international trade networks, which consist of countries connected through trade relationships. By analyzing the structure and dynamics of these networks, researchers have been able to uncover the impact of globalization on wealth inequality and economic growth (Smith, White (1992)). Furthermore, this research has highlighted the importance of considering the global network structure when analyzing the impact of trade policies on wealth distribution and economic development.

Another area of interest in econophysics is the study of social networks and their impact on economic outcomes. Researchers have investigated the role of social networks in shaping consumer behavior, labor market dynamics, and the diffusion of information and innovations. By understanding the structure and dynamics of social networks, policymakers can develop more effective interventions to address issues such as unemployment, inequality, and technology adoption (DiMaggio, Garip (2012)).

Finally, econophysics has also contributed to the study of stock market networks, where the nodes represent individual stocks and the links represent correlations in stock price movements. By analyzing the structure and dynamics of these networks, researchers have been able to identify early warning signals of financial crises and develop strategies for portfolio optimization and risk management (Zhu, Liu (2021)).

In summary, the application of complex network analysis in econophysics has provided valuable insights into the structure and dynamics of various economic systems, including interbank networks, international trade networks, social networks, and stock market networks. This research has informed the design and implementation of policies aimed at promoting financial stability, addressing wealth inequality, and fostering economic growth. As the field of econophysics continues to evolve, it is likely that the use of complex network analysis will continue to play a crucial role in advancing our understanding of economic systems and their impact on wealth distribution.

5.6.4 Financial Market Dynamics

Econophysics has significantly contributed to the understanding of financial market dynamics, particularly in the areas of price formation, market microstructure, and the role of human behavior in financial markets (Mantegna and Stanley, 1995; Lux and Marchesi, 1999; Plerou et al., 1999). By analyzing high-frequency financial data, researchers have uncovered the presence of scaling laws, long-range correlations, and other statistical regularities in financial markets, which has led to the development of more realistic models of financial market dynamics (Lux, Alfarano (2016)).

One of the most notable achievements in this area is the empirical identification of stylized facts, such as the fat-tailed distribution of returns, volatility clustering, and the absence of autocorrelation in returns (Takahashi et al. (2019)). These stylized facts have become the cornerstone for the development of advanced models, such as agent-based models and continuous-time stochastic models, which capture the complex dynamics of financial markets more accurately than traditional models based on the efficient market hypothesis and Gaussian assumptions (Wang et al. (2018)).

Additionally, econophysicists have delved into the study of market microstructure, which examines the process of price formation and the impact of trading mechanisms, liquidity, and information asymmetries on market dynamics. This research has led to a better understanding of how order flow, limit order book dynamics, and market participants' strategies influence price movements and market stability. Consequently, this knowledge has informed the design of more efficient trading mechanisms and regulatory measures aimed at enhancing market stability and preventing market manipulation (Balp, Strampelli (2018)).

The role of human behavior in financial markets has also been a subject of interest in econophysics. By incorporating insights from behavioral finance and cognitive psychology, econophysicists have developed models that capture the impact of cognitive biases, herd behavior, and other behavioral factors on market dynamics (Baddeley (2010)). This research has not only improved our understanding of the origins of financial market anomalies, such as bubbles and crashes, but has also informed the development of investment strategies and policy interventions aimed at mitigating the impact of these phenomena on financial stability and wealth inequality.

In conclusion, econophysics has made substantial contributions to the understanding of financial market dynamics by identifying statistical regularities, developing advanced models that capture market complexity, studying market microstructure, and exploring the role of human behavior in financial markets. This interdisciplinary approach has provided valuable insights into the functioning of financial markets and has informed the development of evidence-based policies and investment strategies. As the field of econophysics continues to evolve, it is likely that its contributions to the study of financial market dynamics will remain significant and continue to shape our understanding of the complex interplay between economic agents, market structures, and wealth distribution.

6 An introductory agent based modelling for consumer choice

6.1 An agent-based model of consumer choice: Some preliminary results

The paper presents a study of an agent-based model (ABM) to understand consumer choice, emphasizing the advantages of reflecting the heterogeneity of agents over the traditional 'representative agent' model. This heterogeneity is the crux of the paper's argument, challenging the representative agent framework of the classical demand theory.

One of the key advantages of the ABM approach is its ability to derive dynamic relations due to its sequential nature. This feature allows for a unique exploration of the evolution of expenditure within a homogeneous group and the relationship between demand, preferences, and income across heterogeneous agents (Krusell, Smith (2006)). Each agent in the model is described comprehensively using five components - four preference parameters and income, which ensures a detailed characterization for more nuanced insights into consumer behavior.

Interestingly, the paper draws an analogy between the ABM approach and physical theories. It suggests that the ABM is akin to inferring constitutive models from experiments, adopting a bottom-up approach that allows for individual agent attributes and behaviors to manifest distinctly (Gatti et al. (2011)).

From a comparative standpoint, the paper posits that the understanding derived from ABMs surpasses that from the standard economic model (SEM). The classical theory of consumer behavior or SEM relies heavily on a representative agent model. This means all attributes and behavioral characteristics are aggregated to represent a single agent, making homogeneity an inherent assumption (Stango et al. (2017)).

Highlighting practical implications, the paper showcases that ABMs have been instrumental in understanding asset pricing dynamics, specifically replicating phenomena like bandwagon effects, technical trading, and market psychology. The reference to the work of Arthur et al. (1996) is particularly illuminating as it sheds light on how a theory of asset pricing with heterogeneous agents, characterized by endogenous expectations, can lead to temporary market bubbles and crashes. Furthermore, the relevance of ABMs in deciphering the dynamics leading to and during the 2008 financial crisis underscores their significance in modern economic analysis.

Hamill, Gilbert (2016) exploration of the applications of ABMs post the 2008 global economic downturn is noteworthy. Their work paints ABMs as crucial tools in understanding various economic processes, emphasizing the advantages of this approach in encapsulating heterogeneity, bridging the micro-macro divide, and accommodating non-optimizing behavior.

Regarding the specifics of the model presented in the paper, agents must choose between two commodities, and they are characterized by both preference and income heterogeneity. Through a sequential algorithm, agents use a discrete analogue of Gossen's second law to find equilibrium. This approach contrasts with classical demand theory, which offers a single optimal consumption bundle solution, whereas the ABM presents a path of optimal bundles (McKenzie (1989)).

Lastly, the discussion around the Walrasian equilibrium model provides valuable insights. This model, rooted in the Walrasian Auctioneer pricing mechanism, remains a fundamental paradigm in economic thinking regarding equilibrium attainment. However, attempting to remove this mechanism presents myriad challenges, from handling asymmetric information and strategic interaction to understanding learning and social norms. Albin, Foley (1992) exploration of decentralized exchange mechanisms is particularly intriguing as it paints a picture of a world without an auctioneer, focusing on real costs of communication and bounds of rationality.

In conclusion, the paper effectively showcases the strengths and potential of the agent-based model, particularly its capability to capture and represent heterogeneity and dynamic interactions among agents. It makes a compelling argument for researchers and economists to consider ABM as a versatile and encompassing approach, especially when studying systems with significant heterogeneity and dynamic interactions.

6.2 Model Framework

In the context of our study, we visualize an economy encompassing n distinct agents denoted by indices $j = 1, 2, \ldots, n$ and L unique commodities represented by $l = 1, 2, \ldots, L$. Every agent, j, possesses a strictly quasiconcave utility function $u_j : \mathbb{R}^L_{\geq 0} \longrightarrow \mathbb{R}$. The domain, $\mathbb{R}^L_{\geq 0}$, signifies the nonnegative orthant, which is expressed as $R^L_{\geq 0} = \{(x_{j1}, x_{j2}, \ldots, x_{jL}) \in \mathbb{R}^L : x_{jl} \geq 0\}$. Each agent's attributes are encapsulated in a vector comprising $2L + 1$ components: $2L$ elements reflecting preference parameters (two for each commodity) and the last element denoting agent j's earnings.

Two fundamental aspects constitute this model:

1. **Agent Proximity Measure**: A metric assesses the variance in preferences and incomes amongst agents in set $N := \{1, 2, \ldots, n\}$. This metric evaluates the 'closeness' between any two agents within N. Multiple methodologies can define this metric, such as relying solely on the $2L$ preference parameters from agent j's utility function or incorporating both the preference parameters and agent j's income, denoted m_j. Regardless of the approach, a larger metric value indicates greater heterogeneity between agents. In this context, we adopt the Euclidean metric $d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|$ from the normed vector space $\left(\mathbb{R}^{2L+1}, \|\cdot\|\right)$. Alternatively, heterogeneity between agents can be assessed through the angle $\theta = \cos^{-1}\left(\frac{\langle \mathbf{x}, \mathbf{y} \rangle}{\|\mathbf{x}\|\|\mathbf{y}\|}\right)$ between vectors of points $\mathbf{x}$ and $\mathbf{y}$.

2. **Computational Algorithm**: The model incorporates an iterative algorithm, culminating in a final consumption bundle, $\mathbf{x}_T$. While there are various potential approaches, we select an optimization method where each step maximizes agent j's utility. Another possible strategy involves computing all feasible combinations $(x_{j1}, x_{j2}, \ldots, x_{jL})$ within agent j's budget set given by

$$B_j(\mathbf{p}, m_j) = \{(x_{j1}, x_{j2}, \ldots, x_{jL}) \in \mathbb{R}^L_{\geq 0} : p_1 x_{j1} + p_2 x_{j2} + \ldots + p_L x_{jL} \leq m_j\} \tag{3}$$

where $\mathbf{p} = [p_1, p_2, \ldots, p_L]^t$ represents the pricing structure. Given the assumption of a strictly monotonic preference relation, the utility function u_j increases in at least one axis, implying the constraint in correspondence (2.1) equates to equality. Consequently, agent j's budget hyperplane is described as

$$H_j(x_{j1}, x_{j2}, \ldots, x_{jL}) = p_1 x_{j1} + p_2 x_{j2} + \ldots + p_L x_{jL} - m_j = 0 \tag{4}$$

An alternative algorithm might adopt a Markov chain to identify $\mathbf{x}_T$.

Thus, the model yields a multi-dimensional demand function, illustrated as

$$\begin{aligned}&\mathscr{D}(p_1,\dots,p_L,m_1,\dots,m_n,a_{11},\dots,a_{1L},\dots,a_{n1},\dots,a_{nL},b_{11},\dots,b_{1L},\dots,b_{n1},\dots,b_{nL})\\&\quad=(x_1,x_2,\dots,x_L)\end{aligned} \tag{5}$$

This function, $\mathscr{D}$, can be derived either as a panel data model, capturing unobserved individual or group variances, or as a traditional linear regression when specific attributes are observable for every agent.

6.3 Simulations

There exists a finite set of agents $N = \{1, 2, \dots, n\}$ and $L = 2$ commodities in the simulated economy. Each agent $j \in N$ is characterized by his income and preference parameters contained in the five-dimensional vector

$$\mathbf{a} := \begin{bmatrix} m_j \\ a_{jA} \\ b_{jA} \\ a_{jB} \\ b_{jB} \end{bmatrix}_{5\times 1} \tag{6}$$

where m_j denotes agent j's income and

$$\mathbf{b} := \begin{bmatrix} a_{jA} \\ b_{jA} \\ a_{jB} \\ b_{jB} \end{bmatrix}_{4\times 1} \tag{7}$$

is the vector of preference parameters related to commodities A and B. Hence, we may write the vector (3.1) in partitioned form as

$$\mathbf{a} = \begin{bmatrix} m_j \\ \hline \mathbf{b} \end{bmatrix}_{5\times 1} \tag{8}$$

We use $\mathbf{b}^*$ to denote a fixed value of $\mathbf{b}$. The utility function of agent j is given by the exponential sum

$$u_j(x_{jA}, x_{jB}; \mathbf{b}) = a_{jA} b_{jA}^{1-x_{jA}} + a_{jB} b_{jB}^{1-x_{jB}}, a_{jA} < 0, a_{jB} < 0, b_{jA} > 1, b_{jB} > 1 \tag{9}$$

where x_{jA} and x_{jB} are the quantities of commodities x_A and x_B with prices p_A and p_B, respectively. The sign conditions on the preference parameters a_{jA}, b_{jA}, a_{jB}, and b_{jB} are sufficient in order for the marginal utility of commodity x_A and commodity x_B to be strictly positive and strictly decreasing. The simulated economy can thus be defined as a collection of n agents with heterogeneous preferences over $L = 2$ commodities. In addition, agents are heterogeneous with respect to their income. The degree of heterogeneity is captured by the Euclidean metric induced by the normed vector space $(\mathbb{R}^5, \|\cdot\|)$, where the set $\mathbb{R}^5 =: \Theta$ is the parametric space of the simulated economy and the function $\|\cdot\| : \Theta \longrightarrow \mathbb{R}$ is the Euclidean norm. Denoting by $\mathcal{S}$ the sequential computer algorithm used for generating the demand function as well as all additional relationships and by $(e^j_{x_A}, e^j_{x_B})$ an exogenously given supply of commodities x_A and x_B, the simulated or artificial economy is a quintet

$$\langle \{(e^j_{x_A}, e^j_{x_B})\}_{j=1}^n, \{u_j : \mathbb{R}_{L\geq 0} \longrightarrow \mathbb{R}\}_{j=1}^n, \|\cdot\|, \Theta, \mathcal{S}\rangle \tag{10}$$

For each simulation run $k \in \{1, 2, \ldots, K\}$ we fix the characteristics of agent g from a group of agents $G \subsetneq N$. In addition, we fix the price vector

$$\mathbf{p} := \begin{bmatrix} p_A \\ p_B \end{bmatrix}_{2\times 1} \tag{11}$$

For all simulation runs, $p_A, a_{jA}, b_{jA}, a_{jB}, b_{jB}$, and m_j are the same which means that the price of commodity x_A as well as all agents are the same throughout all simulations. Each simulation is characterized by its value of p_B and the aggregate demand, $D_{x_B} = \sum_{g\in G} x_{gB}$. Therefore, we get the demand correspondence $p_B \longrightarrow D_{x_B}$. This is done for a group of agents from the set of all agents. This group has the property that its members are homogeneous which implies that the Euclidean distance between any two points with position vectors given by (3.1) is 'small'. For each homogeneous group of agents $G \subsetneq N$ we have the map

$$p_{Bk} \longrightarrow \sum_{g\in G} x_{gB} = D_{x_{Bk}} \tag{12}$$

We consider 100 separate agents each of which is perturbed into 30 agents who are 'close by' and form a homogeneous group, G. The distance between these groups is 'large'. Hence, while agents are close by within each group they are 'far apart', i.e., heterogeneous, between groups.

Agents are assumed to be unboundedly rational. The behavioral rule that characterizes each agent is utility maximization. Note that this type of behavior is not incompatible with an agent-based model. Indeed, these models can accommodate the behavioral rules of classical demand theory and many more. This is their advantage. Furthermore, they can accommodate heterogeneity as is the case here and thus skip the representative agent framework. In this sense, agent-based models are a superset of the standard economic model. For a given agent we start with some initial bundle, (x^*_{jA}, x^*_{jB}), where the star superscript denotes a fixed number of units from commodities x_A and x_B demanded by agent j and examine two scenarios. (i) $(x^*_{jA}+1, x^*_{jB})$. (ii) $(x^*_{jA}, x^*_{jB}+1)$. Hence, each scenario involves a unit increase in the demand of one of the two commodities. The following numbers are computed

$$u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*) \tag{13}$$

$$u_j(x^*_{jA}+1, x^*_{jB}; \mathbf{b}^*) \tag{14}$$

$$u_j(x^*_{jA}, x^*_{jB}+1; \mathbf{b}^*) \tag{15}$$

$$\frac{u_j(x^*_{jA}+1, x^*_{jB}; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_A} \tag{16}$$

$$\frac{u_j(x^*_{jA}, x^*_{jB}+1; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_B} \tag{17}$$

If

$$\frac{u_j(x^*_{jA}+1, x^*_{jB}; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_A} > \frac{u_j(x^*_{jA}, x^*_{jB}+1; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_B} \tag{18}$$

then agent j will choose the consumption bundle $(x^*_{jA}+1, x^*_{jB})$. If

$$\frac{u_j(x^*_{jA}+1, x^*_{jB}; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_A} < \frac{u_j(x^*_{jA}, x^*_{jB}+1; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_B} \tag{19}$$

then agent j will choose the consumption bundle $(x^*_{jA}, x^*_{jB}+1)$. Equality

$$\frac{u_j(x^*_{jA}+1, x^*_{jB}; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_A} = \frac{u_j(x^*_{jA}, x^*_{jB}+1; \mathbf{b}^*) - u_j(x^*_{jA}, x^*_{jB}; \mathbf{b}^*)}{p_B} \tag{20}$$

is the discrete analogue of Gossen's second law. Assume inequality (3.14) holds, i.e., assume that the new consumption bundle chosen by agent j is $(x^*_{jA}, x^*_{jB}+1)$. Agent j's new income will be $m'_j = m_j - p_B$. Similarly, the consumption bundle $(x^*_{jA}, x^*_{jB}+1)$ is compared with the consumption bundles $(x^*_{jA}+1, x^*_{jB}+1)$ and $(x^*_{jA}, x^*_{jB}+2)$. The strict convexity of the indifference curves

$$u_j(x_{jA}, x_{jB}; \mathbf{b}) = a_{jA} b_{jA}^{x_{jA}-1} + a_{jB} b_{jB}^{x_{jB}-1} = \overline{u}, \tag{21}$$

where $\overline{u}$ denotes a fixed number, implies that the terminal consumption bundle $\mathbf{x}_T = \begin{bmatrix} x_{jA,T} \\ x_{jB,T} \end{bmatrix}_{2\times 1}$ will have both entries strictly positive. We may denote this as $\mathbf{x}_T \gg \mathbf{0}$. The algorithm $\mathcal{S}$ ends when agent j exhausts his income. Hence, Walras' law holds.

An advantage of the simulation algorithm outlined above is that the commodity bundle which results from each step is optimal. In particular, the result of the algorithm is a sequence of optimal commodity bundles each of which constitutes a utility maximizer. This contrasts with the non agent-based modeling approach where the solution is 'static', i.e., a single point in Euclidean 2-space, or in Euclidean L-space more generally, as opposed to a complete serial path. Hence, in classical demand theory it is not feasible to observe demand build up gradually. This can be observed here without imposing an intertemporal structure. This is the result of the use of the algorithm. A second advantage of the proposed simulation algorithm is that by considering a unit increase in either x_{jA} or x_{jB} in each step we are able to reduce the computational complexity that would arise were we to examine all possible commodity bundles that maximize an agent's utility and at the same time exhaust his income. The number of such bundles is greater with a higher income and lower prices.

Each homogeneous group of agents is generated by perturbing an agent with respect to his income and preferences and producing a group G of 30 close by agents. In all, 100 groups are produced in this way each of which consists of 30 'neighboring' agents. We average over the preference parameters and income of the agents that belong to each of the 100 different groups. The average produced will be close to the values of the preference parameters and income of each agent separately because all them are close by in terms of the Euclidean metric $d(\mathbf{x}, \mathbf{y}) = \|\mathbf{x} - \mathbf{y}\|$. The expenditure for commodity x_B in each of the 100 groups $\{G(1), G(2), \ldots, G(100)\}$ is equal to the product of the price p_B times the quantity demanded denoted $D_{x_B,G(i)}$, $i = 1, 2, \ldots, 100$, where each group is represented by a 'mean agent' close to whom there are another 29 agents making up a group of 30 homogeneous agents in total for each $i = 1, 2, \ldots, 100$. The following model is fitted

$$D_{x_B,G(i)} = \frac{m_i^{\beta_1}}{p_B} \exp\left(\beta_2 a_{A,i} + \beta_3 b_{A,i} + \beta_4 a_{B,i} + \beta_5 b_{B,i}\right) \tag{22}$$

Taking logarithms on both sides of equation (3.17) and adding an error term ϵ_i yields the equation

$$\log\left(D_{x_B,G(i)}\right) = \beta_1 \log(m_i) + \beta_2 a_{A,i} + \beta_3 b_{A,i} + \beta_4 a_{B,i} + \beta_5 b_{B,i} - \log(p_B) + \epsilon_i \tag{23}$$

Equation (3.18) can be estimated by linear least squares. In particular, we generate new variables $\log\left(D_{x_B,G(i)}\right)$, $\log(m_i)$, and $\log(p_B)$ and then regress $\log(m_i)$, and $\log(p_B)$ along with the remaining variables to $\log\left(D_{x_B,G(i)}\right)$. The estimated values of the coefficients along with the observed values of the t statistic on the explanatory variables and the p-values for the t tests are shown in Table 1

Independent variables	Estimates	Standard error	t-Statistic	p-value
m_i	1.228	0.017	73.560	0.0000
$a_{A,i}$	0.255	0.089	2.885	0.005
$b_{A,i}$	-0.125	0.023	-5.421	0.0000
$a_{B,i}$	0.097	0.091	1.057	0.293
$b_{B,i}$	0.178	0.022	8.143	0.0000

Table 1 Summary of estimation results for the model (3.17)

Hence, the estimated model is the following

$$D_{x_B,G(i)} = \frac{m_i^{1.228}}{p_B} \exp\left(0.255a_{A,i} - 0.125b_{A,i} + 0.097a_{B,i} + 0.178b_{B,i}\right) \tag{24}$$

This implies a relationship between price versus the quantity demanded of the general form $p_B = f(D_{x_B,G(i)}) := \frac{C}{D_{x_B,G(i)}}$ for each i. This is graphed in Figure 1 for varying levels of income that affect the value of the constant C. Demand, $D_{x_B,G(i)}$, is depicted on the horizontal axis while the price of commodity x_B is depicted on the vertical axis. A higher income, m_i, shifts the demand curve to the right. Without loss of generality, the assumption made is that commodity x_B is normal or otherwise the demand curve would shift to the left for $\Delta m_i > 0$.

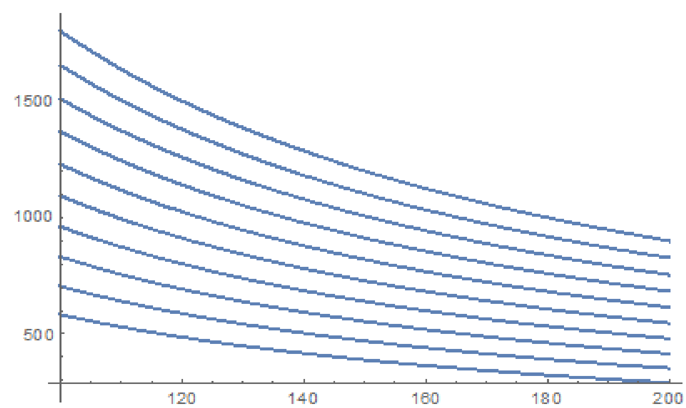


Figure 1: Demand curves for varying levels of income.

Equation (3.17) can be used for the purpose of analyzing the relationship between any two variables that appear in it while holding the remaining variables or parameters fixed, i.e., ceteris paribus. It is worth emphasizing that this particular equation is purely a feature of the agent-based approach adopted here in modeling demand. In the framework of classical demand theory one need only solve the static utility maximization problem (UMP) in order to obtain the optimum consumption bundle which is a point in commodity space. Assume now that only the price of commodity x_B, p_B, and income, m_i, change. Equation (3.17) implies a functional relationship of the form $m_i = C \sqrt[\beta_1]{p_B D_{x_B,G(i)}}$, where C is some constant. Demand $D_{x_B,G(i)}$ is also treated as constant each time. Figure 2 shows some level curves of this function. A higher level curve is associated with an increased level of demand, $D_{x_B,G(i)}$.

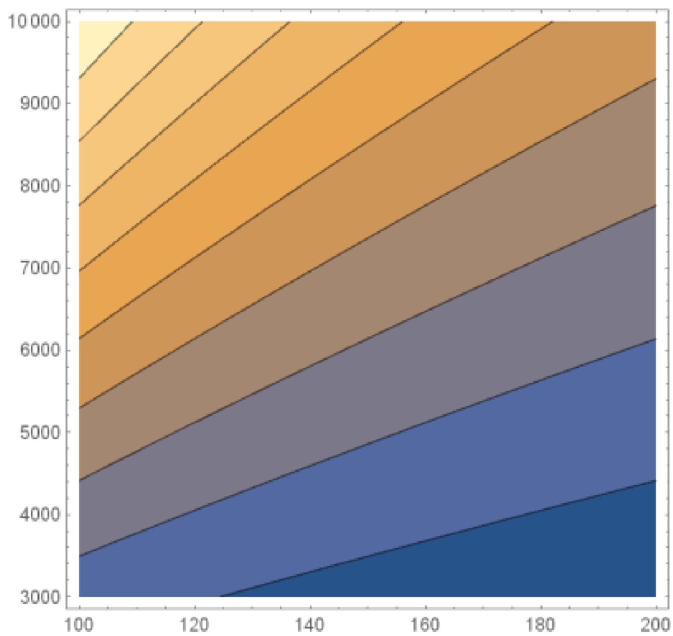


Figure 2: Income-price level curves with fixed demand.

This result is also intuitive. In particular, an increase in demand for commodity x_B is associated with an increase in income given that every agent exhausts his income and the commodity is assumed to be normal. Consider now a change in preferences while income remains unchanged. Equation (3.17) implies a functional form for price p_B that is similar to that analyzed in the case of Figure 1. The difference now is that the constant C in the function $p_B = f(D_{x_B,G(i)}) := \frac{C}{D_{x_B,G(i)}}$ is affected by a change in either one of the preference parameters as opposed to group i's income. Figure 3 shows some representative demand curves. The price of commodity x_B is depicted on the horizontal axis whereas the quantity demanded is depicted on the vertical axis. Again, the result is intuitive. A larger value for a certain preference parameter related to commodity x_B implies an increase in demand and thus a vertical shift upwards of the demand curve.

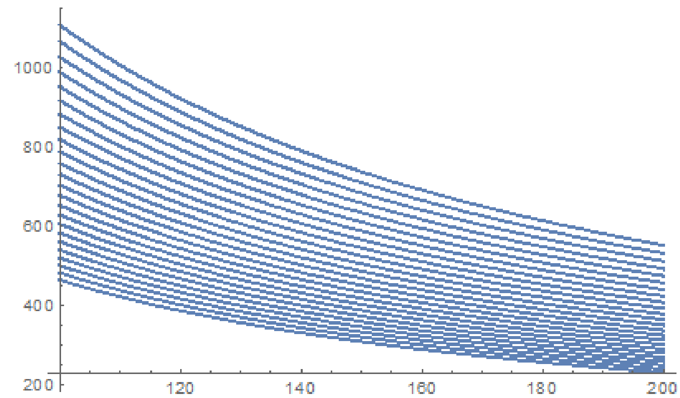


Figure 3: Demand curves with a varying preference parameter.

A fourth scenario related to equation (3.17) concerns the relationship between the model's parameters, namely, income and the preference parameters $a_{A,i}$, $b_{A,i}$, $a_{B,i}$, and $b_{B,i}$. Assume that demand and the price of commodity x_B are fixed and all but one of the preference parameters are also fixed. Income varies and the same is true for any single preference parameter contained in the vector $\mathbf{b}$ in equation (3.2). In particular, assume that $a_{B,i}$ varies. Then, equation (3.17) implies a functional relationship of the form $a_{B,i} = C_0 + C_1 \ln(m_i)$. Some typical level curves are shown in Figure 4. The function graphed is increasing and concave which implies that as the agents' income increases the preference parameter $a_{B,i}$ increases at a decreasing rate. From equation (3.17) it is easy to see that the same type of behavior with respect to the agents' income is true not only for $a_{B,i}$ but also for the other three preference parameters $a_{A,i}$, $b_{A,i}$, and $b_{B,i}$ since all four enter in (3.17) in the same way, i.e., exponentially. An increase in expenditure, $p_B \cdot D_{x_B,G(i)}$, shifts the level curves to the right. Indeed, since an agent in some group $G(i)$ is assumed to exhaust his income, a higher expenditure is associated with a higher income and a shift to the right in Figure 4.

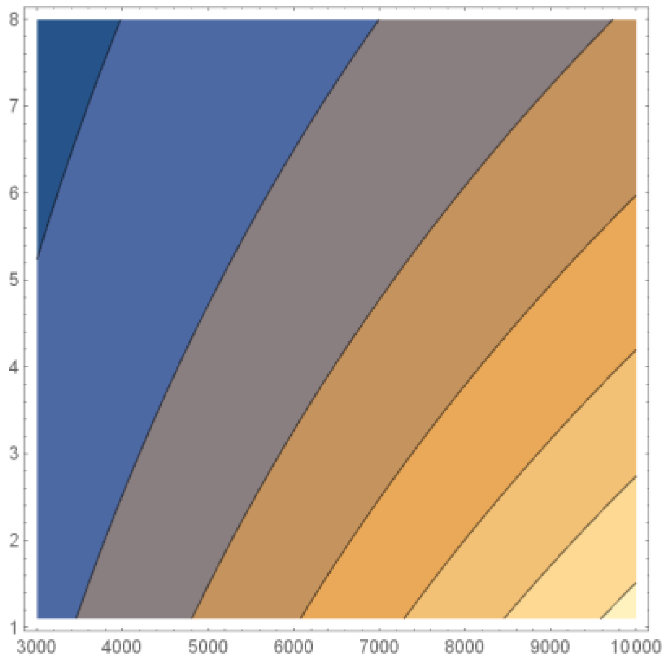


Figure 4: Preference-income level curves.

A feature of the sequential algorithmic approach adopted here is the ability to derive relations of a dynamic sort. In particular, the steps of the algorithm can be thought of as playing the same role as time does in the framework of a dynamic model. In each step of the algorithm we can thus observe the evolution of agents' expenditure until their income is exhausted at the terminal bundle. The following figure shows the number of steps required in order to reach the terminal bundle $\mathbf{x}_T$ for some group $G(i)$ of homogeneous agents when the price p_B is lower compared to the price of the same commodity, i.e., commodity x_B in Figure 7. In particular, p_B is assumed to be higher in the latter case. Then, the number of steps required to reach $\mathbf{x}_T$ is smaller. In both figures, the horizontal axis depicts the number of steps while the vertical axis depicts the agents' expenditure in each step.

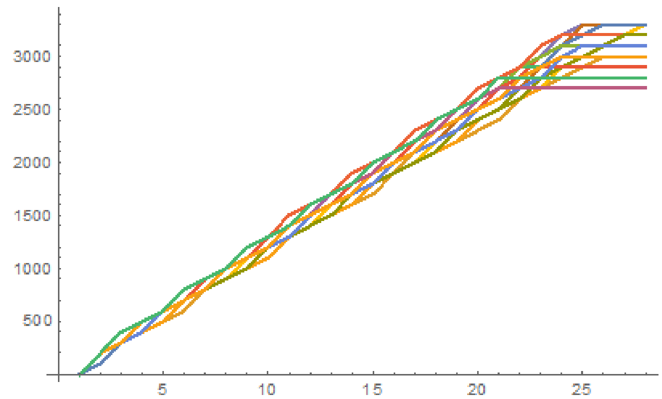


Figure 5: Expenditure evolution within a group $G(i)$ when the price p_B is low.

The evolution of the average expenditure for the agents that comprise some group $G(i)$ is shown in Figure 6 below. Note that this averaging makes sense because agents belong to the same group. Hence, they are 'close by' and so taking the average over their expenditures produces a value at each step in the simulation algorithm that is close to the expenditure of each isolated agent in the group.

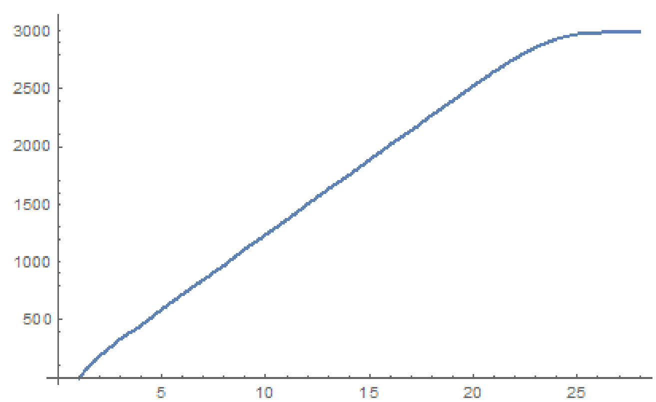


Figure 6: Average expenditure evolution within group $G(i)$ when the price p_B is low.

When the price p_B is higher, the terminal bundle is attained faster. This result agrees with intuition. Indeed, when the price of the commodity is higher then the agents' income is exhausted faster as shown in Figure 7 below.

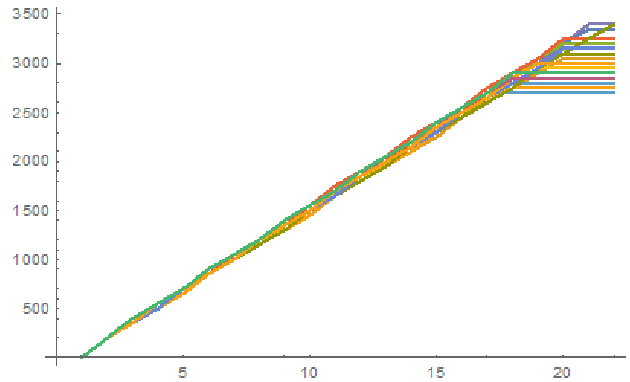


Figure 7: Expenditure evolution within a group $G(i)$ when the price p_B is high.

Similarly to Figure 6, the average expenditure evolution when the price p_B is higher is shown in Figure 8. The results related to Figures 5 through 8 are ABM−specific, i.e., they are obtainable only in the framework of agent-based modeling. This is obvious if one considers the analogous problem in classical demand theory, namely, the utility maximization problem (UMP)

$$\begin{aligned} \max_{x_A, x_B} \quad & u(x_A, x_B) \\ \text{s.t.} \quad & p_A x_A + p_B x_B \leq m \\ & x_A \geq 0, x_B \geq 0 \end{aligned} \tag{25}$$

The solution to the UMP (3.20) is simply a point in the feasible set

$$\mathcal{D} := \{(x_A, x_B) \in \mathbb{R}^2 : p_A x_A + p_B x_B \leq m, x_A \geq 0, x_B \geq 0\} \tag{26}$$

which is the image of the budget set correspondence. Hence, an optimal trajectory of commodity bundles is undefined in this case. The same is true for an an agent's expenditure path and the average expenditure path of a group of agents. In fact, a group of agents is undefined since there is only the representative agent.

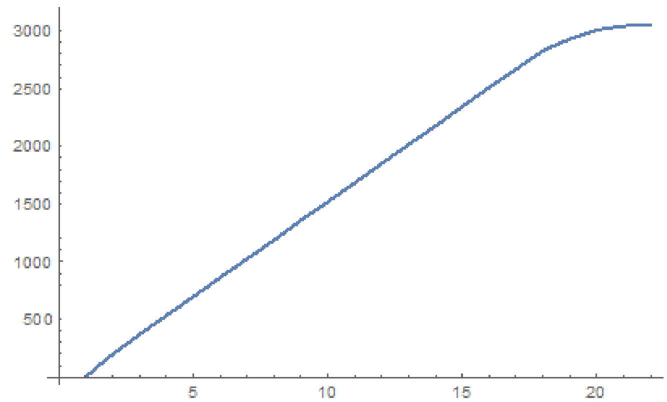


Figure 8: Average expenditure evolution within a group $G(i)$ when the price p_B is high.

6.4 Conclusions

The study delves into the realms of consumer choice and classical demand theory, leveraging agent-based modeling and simulation as its core framework. Contrary to the traditional representative agent paradigm in classical demand theory, this research characterizes agents in heterogeneous groups, differing based on their unique preferences and income levels. This heterogeneity is visualized within a distinct Euclidean space, wherein the distance between its points illustrates the variance between agents.

In the synthesized artificial economy:

- There exist 3,000 agents, segmented into 100 groups.
- Every group encompasses 30 akin agents, derived by perturbing a 'representative agent'.
- Agents within a specific group resonate similarity, but exhibit stark contrasts inter-group.
- Agents, when confronted with choices related to commodity bundles, opt for the best, depleting their income upon reaching the terminal bundle, thereby upholding the rationality principle intrinsic to classical demand theory.

The computational technique embraced by the study, owing to its sequential nature, bestows enriched insights into agent behavior. It proffers a trajectory of optimal consumption decisions, as opposed to a singular optimal juncture as discerned in classical demand theory. This agent-centric methodology singularly unveils the expenditure patterns of agent groups over temporal spans, facilitating a profound exploration into diverse heterogeneity determinants and their interrelations.

7 An agent-based data-driven model of consumer demand

7.1 An agent-based data-driven model of consumer demand

The paper titled "Agent-Based Data-Driven Model of Consumer Demand" delves into the intricate world of consumer demand through the lens of an agent-based model (ABM). This model's appeal lies in its bottom-up approach, allowing individualized modeling of agents, each defined by specific attributes and behaviors. A central tenet of this research is the heterogeneity of consumers, with varying preferences and incomes, leading to diverse classifications of agents into distinct groups.

The dynamic nature of commodity prices, contingent upon different 'states of the world,' introduces an element of risk into the model. When juxtaposed with the classical theory, the agent-based model unfolds as a more robust framework. It offers a deeper and more generalized comprehension of consumer behavior as opposed to the traditional Standard Economic Model (SEM). This superiority emerges from the ABM's ability to operate on less stringent assumptions, effectively spotlighting the repercussions of agent diversity.

The versatility of agent-based models shines as they find applications in diverse fields, from asset pricing to understanding pivotal financial occurrences like the 2008 crisis. A significant attribute of the ABM is its capacity to produce aggregate-level phenomena that sprout from individual agent interactions. These emergent behaviors pave the way for path-dependent outcomes, breeding complexities that are often elusive when analyzed at the individual level.

The paper also critically examines the established Walrasian equilibrium model, advocating for a shift from its centralized pricing mechanism. By removing the standard Walrasian Auctioneer, the research champions more decentralized procurement processes, which resonates with real-world market dynamics.

The fusion of modern computational techniques with economic study is another hallmark of this research. The integration of the random forest algorithm, an advanced artificial intelligence tool, underscores the paper's commitment to harness cutting-edge tech solutions. By leveraging this algorithm, the research astutely estimates demand functions for agent clusters, underpinning the symbiotic relationship between economics and technological advancements.

In conclusion, this manuscript embarks on an exploration of consumer behavior by leveraging the granularity and adaptability inherent in agent-based modeling. It stands as a beacon in contemporary economic literature, emphasizing the seamless blend of classical theories with modern computational methodologies.

7.2 Basic Model Overview

Let's consider an economy comprising of n individuals, where each individual is indexed by $j = 1, 2, \ldots, N$ and L goods indexed by $l = 1, 2, \ldots, L$. The utility function for agent j is given by $u_j : \mathbb{R}^L_{\geq 0} \longrightarrow \mathbb{R}$. This utility function is defined over a domain of nonnegative real numbers, represented by $\mathbb{R}^L_{\geq 0}$. Every agent has a profile consisting of $2L + 1$ elements, which includes $2L$ preference parameters (two for each good) and an income value.

There are two primary components in our model:

1. A metric that measures the difference in preferences and income among the agents. This metric can be constructed in various manners. It could be based solely on the preference parameters or both the preference parameters and the agent's income, represented as m_j. We use the Euclidean

metric for determining this:

$$d(\mathbf{x},\mathbf{y}) = \|\mathbf{x}-\mathbf{y}\| = \left[\sum_{m=1}^{2L+1}(x_m - y_m)^2\right]^{1/2} \tag{27}$$

Another way to measure difference is by calculating the angle θ between vectors of agents $\mathbf{x}$ and $\mathbf{y}$.

2. The second crucial component is a machine learning algorithm, which in our context is denoted by $\mathcal{A}$. We employ the random forest algorithm, a type of ensemble method, to approximate the demand function for the L goods based on a modified agent profile. The underlying principle of random forests is the bootstrap method. A bootstrap sample, represented as $\mathcal{X}_n^*$, is a randomized version of the original dataset $\mathcal{X}_n$. In the context of estimating a parameter θ, its standard error can be bootstrapped, resulting in a more accurate approximation.

Agent j has a budget set given by:

$$B_j(\mathbf{p}, m_j) = \{(x_{j1}, x_{j2}, \ldots, x_{jL}) \in \mathbb{R}^L_{\geq 0} : p_1x_{j1} + p_2x_{j2} + \ldots + p_Lx_{jL} \leq m_j\} \tag{28}$$

With the presumption of strict monotonicity in preferences, the utility function for each agent increases in at least one dimension, ensuring the budget constraint aligns with the agent's preferences. Specifically, agent j's budget hyperplane is given by:

$$H_j(x_{j1}, x_{j2}, \ldots, x_{jL}) = p_1x_{j1} + p_2x_{j2} + \ldots + p_Lx_{jL} - m_j = 0 \tag{29}$$

7.3 Simulations

There exists a finite set of agents $N = \{1, 2, \ldots, n\}$ and $L = 6$ commodities in the artificial economy. Each agent $j \in N$ is charácetrized by his income and preference parameters contained in the 13×1 column vector

$$\mathbf{a}_j = \begin{bmatrix} m_j \\ a_{j1} \\ \vdots \\ a_{j6} \\ b_{j1} \\ \vdots \\ b_{j6} \end{bmatrix}_{13\times 1} \tag{30}$$

where m_j is agent j's income and

$$\mathbf{b}_j = \begin{bmatrix} a_{j1} \\ \vdots \\ a_{j6} \\ b_{j1} \\ \vdots \\ b_{j6} \end{bmatrix}_{12\times 1} \tag{31}$$

is the vector of preference parameters related to the closed set $\mathcal{C} \triangleq \{x_1, x_2, \ldots, x_6\}$ whose elements are the commodities in the artificial economy. Hence, we may write the vector (3.1) in partitioned form as

$$\mathbf{a}_j = \left[\frac{m_j}{\mathbf{b}_j}\right]_{13\times 1} \tag{32}$$

The utility function of agent j is *additively separable*, given by the multivariable exponential sum

$$u_j(\mathbf{x}_j; \mathbf{b}_j) = \sum_l a_{jl} b_{jl}^{1-x_{jl}}, a_{jl} < 0, b_{jl} > 1 \tag{33}$$

where x_{jl} are the quantities of commodities $x_1, x_2, \ldots, x_6$ with prices $p_1, p_2, \ldots, p_6$, respectively, from which agent $j = 1, 2, \ldots, n$ obtains utility. The vector $\mathbf{x}_j$ is the commodity bundle (or commodity vector)

$$\mathbf{x}_j = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}_{6\times 1} \tag{34}$$

which can be viewed as a point in $\mathbb{R}^6$, the commodity space. The sign conditions on the preference parameters a_{jl} and b_{jl} in equation (3.4) are sufficient in order for the marginal utilities of commodities $x_1, x_2, \ldots, x_6$ to be strictly positive and strictly decreasing. The commodity set $\mathcal{C}$ is partitioned into two disjoint subsets $\mathcal{C}_1 \subsetneqq \mathcal{C}$, $\mathcal{C}_2 \subsetneqq \mathcal{C}$ with $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$ and $\mathcal{C}_1 \cup \mathcal{C}_2 = \mathcal{C}$. One of the two sets, say $\mathcal{C}_1$, contains only necessity commodities while $\mathcal{C}_2$ contains only luxury commodities. For each commodity in $\mathcal{C}$ we consider 100 different *states of the world* with respect to its price. Hence, the model developed here is conducive to the presence of risky alternatives. In this framework, the set of risky alternatives, say $\mathcal{R}$, coincides with the agents' consumption set, i.e., $\mathcal{R} = \mathcal{C}$. Then, one has a simple lottery $\mathscr{L} = (\rho_1, \rho_2, \ldots, \rho_L)$ with $\rho_l \geq 0$, $l = 1, 2, \ldots, L$ being the probabilities of the state-dependent prices $p_1, p_2, \ldots, p_L$, respectively, and $\sum_{l=1}^{L} p_l = 1$. In any case, each commodity price $p_l \in \mathbb{R}_{>0} := (0, \infty)$ is bounded, i.e., there exist constants $M_{1,l}$ and $M_{2,l}$ such that for all $l \in \{1, 2, \ldots, L\}$, $M_{1,l} \leq p_l \leq M_{2,l}$.

We consider 5 separate agents each of which is perturbed into 10 agents who are close by in terms of their Euclidean distance and form a homogeneous group, say, G. For all combinations of agents and sates of the world we have a total of $5 \times 10 \times 100 = 5,000$ cases. Hence, we estimate five thousand 6-dimensional demand vectors. The demand function of each commodity has the general form

$$\mathscr{D}_l(p_1, \ldots, p_L, m_1, \ldots, m_L, \mathbf{b}_1, \ldots, \mathbf{b}_n) = (x_1, x_2, \ldots, x_L) \tag{35}$$

for $l \in \{1, 2, \ldots, L\}$, where the vectors $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n$ are given by (3.2) for $j = 1, 2, \ldots, n$. Based on these generated data we interpolate a machine learning algorithm such as a random forest which is an ensemble of decision tree algorithms and Gaussian process and produce six demand functions, one for each commodity contained in the closed set $\mathcal{C} = \{x_1, x_2, \ldots, x_6\}$.

Agents are assumed to be rational. Their behavior is characterized by utility maximization expressed in the indifference equation

$$\frac{\frac{u_j(x_{jg}^* + \Delta x_{jg}, x_{jh}^*; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jg}}}{p_g} = \frac{\frac{u_j(x_{jg}^*, x_{jh}^* + \Delta x_{jh}; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jh}}}{p_h} \tag{36}$$

for any two commodities $g, h \in \{1, 2, \ldots, L\}$ with $g \neq h$. Equation (3.7) is the discrete analogue of Gossen's second law. We use $\mathbf{b}_j^*$ to denote a fixed value of $\mathbf{b}_j$. If

$$\frac{\frac{u_j(x_{jg}^*+\Delta x_{jg}, x_{jh}^*; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jg}}}{p_g} > \frac{\frac{u_j(x_{jg}^*, x_{jh}^*+\Delta x_{jh}; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jh}}}{p_h} \tag{37}$$

then agent j will choose the consumption bundle $(x_{jg}^* + \Delta x_{jg}, x_{jh}^*)$. If

$$\frac{\frac{u_j(x_{jg}^*+\Delta x_{jg}, x_{jh}^*; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jg}}}{p_g} < \frac{\frac{u_j(x_{jg}^*, x_{jh}^*+\Delta x_{jh}; \mathbf{b}_j^*) - u_j(x_{jg}^*, x_{jh}^*; \mathbf{b}_j^*)}{\Delta x_{jh}}}{p_h} \tag{38}$$

then agent j will choose the consumption bundle $(x_{jg}^*, x_{jh}^* + \Delta x_{jh})$. Inequalities (3.7)-(3.9) are used in order to generate 5,000 data points which are then interpolated by the random forest algorithm to produce the demand schedules. In particular, the graphs of the demand functions for the six commodities in the set $\mathcal{C} = \{x_1, x_2, \ldots, x_6\}$ for the third group of agents are shown in Figures 9 and 2. The quantity demanded is depicted on the vertical axis. The price of the commodity is depicted on the horizontal axis. The values of the parameters are kept constant and only the price of the commodity varies.

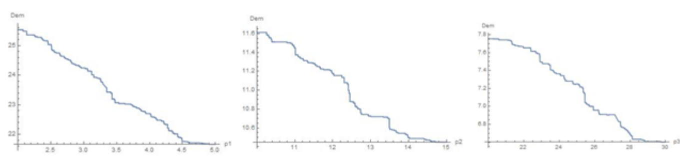


Figure 9: Demand functions for the third group of agents for the necessity commodities.

Commenting on the results in Figure 9, we see that the demand curve for the necessity commodities has a much more 'regular' shape compared to the same curve for the luxury commodities. The change in quantity, depicted on the vertical axis, is more or less nonzero as opposed to the case of luxury commodities for which the quantity demanded seems to be unaffected by changes in price for an initial range of values. This seems to contradict the idea that luxury goods are much more price-elastic compared to necessity goods. However, this result is not 'global' in the sense that it does not hold in the entire price range. Indeed, as the price increases further beyond this initial interval, the price elasticity of demand assumes large values with the demand curve being close to vertical before it flattens out again.

This variation in price elasticity reflects on factors such as the rate of growth in an economy and exchange rate dynamics (Gordon et al., 2013; Fouquet, 2014; Corsetti et al., 2006). In this respect, one may discern between short-run and long-run price elasticity of demand. A similar pattern is evidenced for the remaining groups of agents in Figures 8-11 in the Appendix. Figure 3 plots a three-dimensional graph with the price of the commodity on the x-axis, income on the y-axis, and quantity on the z-axis. The cross-sections of the demand function with the price of the commodity held fixed and the level of income held fixed are also displayed. The quantity-income curves correspond to the Engel curves. In this framework, the consumption of a particular commodity is depicted on the horizontal axis while the level of income is depicted on the vertical axis.

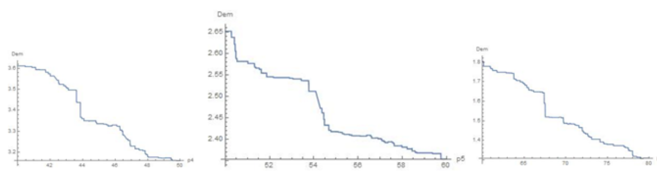


Figure 10: Demand schedules for the third group of agents for the luxury commodities.

The graphs of the demand functions for the first, second, fourth, and fifth group of consumers are given in Appendix A. Figure 3 plots the aggregate demand of the three necessity commodities as a function of the price of each such commodity and group income for the third group of agents. Holding income fixed and letting demand vary yields the demand schedules in Figures 1 and 2.

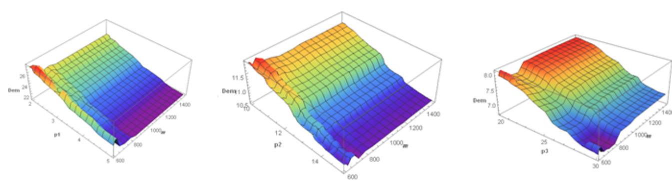


Figure 11: Demand schedules for the third group of agents for the three necessity commodities. Price is depicted on the x-axis. Income is depicted on the y-axis. Demand is depicted on the z-axis.

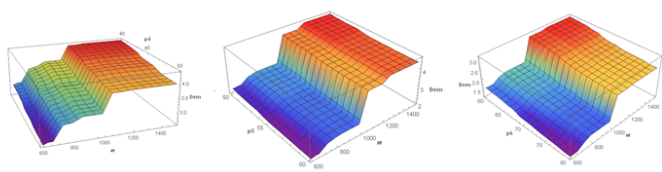


Figure 12: Demand schedules for the third group of agents for the three luxury commodities. Price is depicted on the x-axis. Income is depicted on the y-axis. Demand is depicted on the z-axis.

Holding the price of each commodity fixed and letting income vary yields the Engel curve for each commodity. A positive slope for the Engel curve implies that the commodity is normal and vice versa.

Random forest interpolation allows us to explore relationships between different economic variables keeping the rest of the variables constant. One such relationship is the Engel curve that relates an agent's income to the demand for a commodity. For a normal commodity, one expects that this relationship is increasing. For the demand functions we interpolate, this actually happens to all commodities and groups of agents, as depicted in the graphs which follow.

Due to the nature of the random forest algorithm, these curves are piecewise linear or in some cases piecewise constant which practically means that these functions are almost *concave and increasing*, i.e., on the one hand, an increase in income implies an increase in demand but on the other hand it means that this appears with sharp jumps, i.e., changes, and also it means that the percentage increase in demand is smaller than that of income.

These assertions are demonstrated in the following examples.

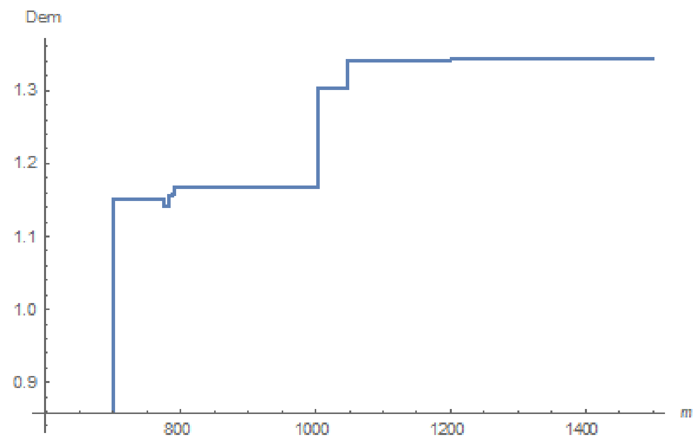


Figure 13: Engel curve for the first group of agents and the fourth (luxury) product.

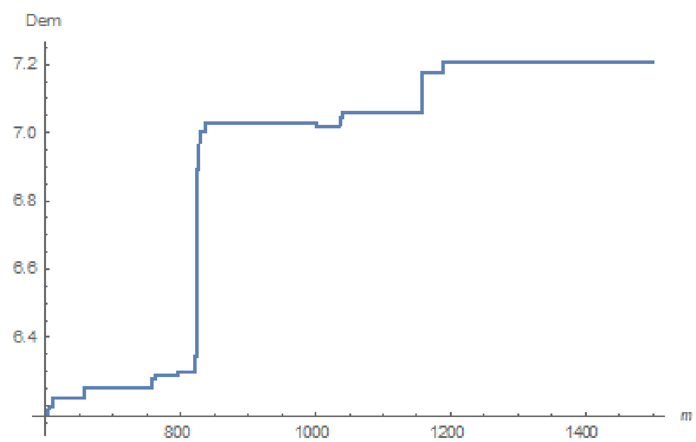


Figure 14: Engel curve for the second group of agents and the third (necessity) commodity.

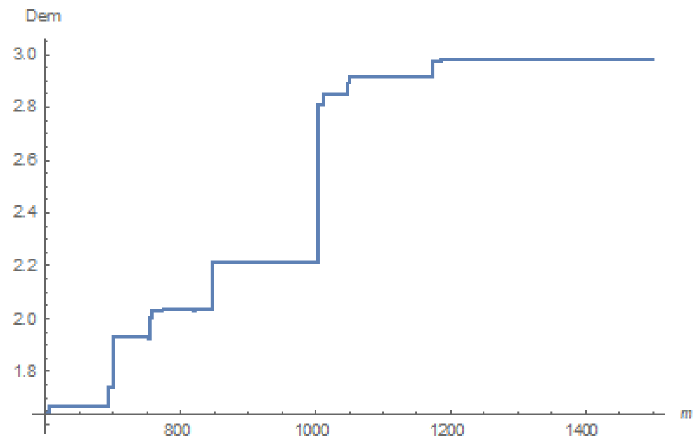


Figure 15: Engel curve for the fourth group of agents and the sixth (luxury) commodity.

The limitations of the method we apply are: (1) The random forest interpolation does not extend out of sample. (2) The derived functions are not smooth.

7.4 Remarks on the data generating methodology

The primary goal of the present study has been to model basic relations between variables that are prevalent in demand theory from the point of view of *data-driven modeling*. In order to generate the data required for the application of the random forest algorithm, or any other machine learning algorithm for that matter, we solved a nontrivial constrained maximization problem. In particular, the utility drawn *separately* from each commodity indexed by $l \in \{1, 2, \ldots, L\}$ is given by the sum $\sum_{k=0}^{x_l} a_l b_l^{1-k}$ where x_l denotes the amount, x, consumed from commodity l. For example, if $x_l = 3$, then the level utility in this case is equal to the sum $a_l b_l + a_l + a_l \frac{1}{b_l} + a_l \frac{1}{b_l^2}$. Similarly, if $x_l = 4$ the level of utility is equal to the sum $a_l b_l + a_l + a_l \frac{1}{b_l} + a_l \frac{1}{b_l^2} + a_l \frac{1}{b_l^3}$. The utility drawn from all L commodities is equal to the double sum $\sum_{l=1}^{L} \sum_{k=0}^{x_l} a_l b_l^{1-k}$ where the number of terms in the inner sum is not fixed but varies according to the amount of commodity $l \in \{1, 2, \ldots, L\}$ that is consumed. The maximization problem for the data generating process then becomes

$$\begin{aligned} \max_{x_1, x_2, \ldots, x_L} \quad & \sum_{l=1}^{L} \sum_{k=0}^{x_l} a_l b_l^{1-k} \\ \text{s.t.} \quad & p_1 x_1 + p_2 x_2 + \ldots p_L x_L \leq m \\ & x_1 \geq 0, x_2 \geq 0, \ldots, x_L \geq 0 \end{aligned} \tag{39}$$

Problem (4.1) is rather nontrivial for three main reasons
1. The inner sum in the objective function can vary according to the amount of commodity x_l that is consumed.

2. Solutions need to be integer.
3. Solutions need to be nonnegative.
Under these conditions, classical optimization algorithms like steepest descend cannot be applied since they require differentiability of the objective function which in turn requires that the control variables be continuous rather than discrete-valued. To overcome these restrictions, we applied the methodology described in the previous section.

7.5 The rest of the graphs

We give here some of the results of the simulations for the demand functions for the necessity and luxury commodities for the first, second, fourth and fifth groups of agents.

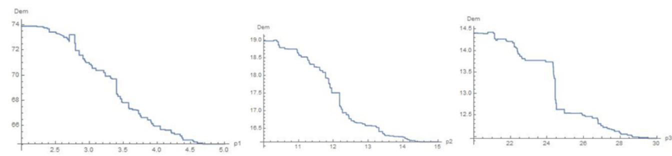


Figure 16: Demand functions for the first group of agents for the necessity commodities.

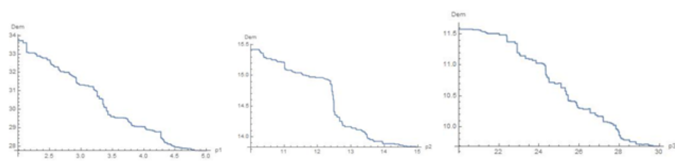


Figure 17: Demand functions for the second group of agents for the necessity commodities.

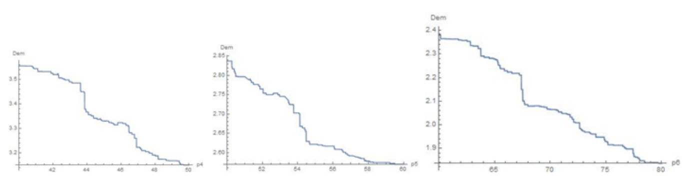


Figure 18: Demand functions for the fourth group of agents for the luxury commodities.

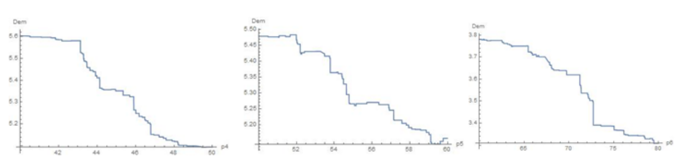


Figure 19: Demand functions for the fifth group of agents for the luxury commodities.

7.6 Conclusion

We employed an *agent-based data-driven* approach to deduce the demand functions for a diverse group of agents and a market with varied commodities. These agents, distinct in preferences and income, have their demand functions determined using commodity prices, their own preferences, and income, and these functions were further refined with a random forest interpolation.

The derived demand functions, which are multi-parametric, were visualized using various parameters like agent type, commodity price, and income brackets. This allowed for the creation of Engel curves and facilitated insights into the simulated economy and market dynamics. A key finding is the successful interpolation of our intricate data using the random forest algorithm, although its limitations were also highlighted.

While the data in our study were synthesized, the proposed *data-driven* methodology is versatile and can be adapted to any relevant dataset, making it a robust model for forecasting demand based on agent characteristics and commodity prices.

8 A bounded rational agent-based model of consumer choice

8.1 A bounded rational agent-based model of consumer choice

The paper introduces a novel methodology that seeks to revolutionize the modeling of consumer behavior by seamlessly intertwining Agent-Based Modeling (ABM), random utility models, and machine learning techniques. Delving deep into this model's mechanics, it incorporates the principles of bounded rationality and integrates Markov processes. Furthermore, Gaussian Process Interpolation, a machine learning approach, plays a pivotal role in aggregating individual agent data, offering a refined lens to examine the complex consumer marketplace. Set within a simulated market framework housing 1,000 consumers and a pair of products, the model hinges on a stochastic process to account for the capricious nature of consumer preferences.

Emerging from this intricate modeling is a spectrum of intriguing phenomena that at times challenge the bedrock of classical economic demand theory. Among the key observations is the counterintuitive rise in a product's demand as its price augments, potentially echoing phenomena such as the Veblen effect. Additionally, a noteworthy discovery reveals that the demand for a certain product may climax when the consumer preferences for the two available products are almost at parity.

The paper firmly roots itself amidst a vast backdrop of prior research on consumer choice, all approached through the agent-based modeling lens. The potency of ABMs lies in their ability to allow macro-level patterns to burgeon from granular interactions between individual agents. Such interactions beget layers of complexity characterized by path-dependence, unpredictability, and challenges in deriving analytical inferences solely based on system components. The authors, no strangers to this realm, have a suite of prior publications that meander through the corridors of consumer demand, choices, and the dynamics of firms using ABMs. Classical theories on consumer choice, which historically revolved around the tenet of consumers as paragons of rationality, have been nudged and nudged over time. The concept of bounded rationality makes a case for consumers as entities swayed by heuristics and biases, leading to decisions that might be sub-optimal but satisfactory.

A salient feature of this paper is its methodological prowess. Gaussian Process Interpolation stands out as a beacon of innovation. The authors navigate the intricacies of understanding demand by initially addressing a stochastic utility optimization challenge tailored for diverse consumer personas and scenarios. This data then undergoes a transformation through Gaussian Process Interpolation, culminating in the derivation of multiparametric demand functions.

These explorations and findings don't merely linger in the academic stratosphere; they resonate with profound real-world implications. Instances such as the potential Veblen effect highlighted in the paper, where price increments might shift consumer perceptions, suggest that the economic fabric might be more nuanced than traditionally believed. The confluence of ABM, random utility models, and machine learning in this study paves the way for fresh insights into consumer behaviors. The incorporation of Gaussian Process Interpolation, especially, offers an avant-garde approach to derive multiparametric demand functions. Ultimately, this paper unfurls as a tapestry of insights, emphasizing the profound impact of micro-level consumer behaviors on shaping the grand tapestry of economic patterns.

In summation, this paper stands as a testament to the potential of contemporary modeling methodologies when merged with time-tested economic theories. It promises a holistic, enriched understanding of consumer behavior's intricate nonlinear dynamics, casting a spotlight on the often-underrated complexities of the economic domain.

8.2 Overall Framework

Converting economic problems to mathematical ones typically involves using mathematical structures, operations, and constructs to represent economic agents, processes, and principles. This mathematical representation's accuracy relies on several foundational assumptions:

1. We assume that choices are made from a discrete set, and an appropriate index facilitates the selection within this set. Instead of a singular calculation using constrained optimization, this choice model operates by repetitively employing a basic selection mechanism, making it sequential.

2. The research also incorporates the idea of "bounded rationality". Traditional assumptions posit that rational consumers possess full knowledge of available products, their own preferences, and budget limitations. The "bounded rationality" principle asserts that consumers and firms, with their limited ability to process data and make decisions, often resort to heuristics or approximations.

3. Our methodology employs Agent-Based Modeling (ABM). In this approach, agents are depicted as individual entities, each with unique attributes and decision-making capabilities. The agent-based models use a bottom-up perspective, initiating from individual agents and aggregating interactions to capture the broader system behavior.

4. We integrate the theory of consumer behavior with Markov processes and ABM, aiming for a more comprehensive depiction of consumer behavior over time, especially by introducing stochastic components.

Through this multi-faceted approach, consumer agents are modeled as individual decision-making entities, each equipped with distinct preferences and constraints. By examining these agents' interactions, we can explore consumer behavior at a macro level, deciphering patterns and understanding their roots in individual decisions.

8.3 Constructing the Model

We model an economy comprising 1000 consumers. Each is considered as having bounded rationality, represented by a 12-element vector detailing their preferences, income, and the prices of two accessible products. By leveraging machine learning techniques on aggregated consumer data, we discern multiparametric demand functions, offering a macroscopic perspective based on individual behaviors.

Given the assumptions in subsection, the study employs an agent-based model to derive demand functions. Each consumer, characterized by preferences and disposable income, interacts within an environment where products are defined by their prices. Uncertainty in preferences is encapsulated by a random utility model. For our purposes, we employ an exponential utility model with dual parameters, with utility sets defined by two functions, chosen at each decision point with probabilities $(p, 1-p)$.

The utility function is expressed as:

$$u(x) = \begin{cases} \sum_{i=1}^{x_1} x_3 x_4^{1-i} & \text{if } x_1 > 0 \\ 0 & \text{otherwise} \end{cases} + \begin{cases} \sum_{j=1}^{x_2} x_5 x_6^{1-j} & \text{if } x_2 > 0 \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

This stochastic demand model functions as follows:

With a budget M, prices p_1 and p_2, and the utility function parameters, the consumer iteratively evaluates whether to purchase more of the first or second product based on respective utilities.

For each consumer possessing a product bundle denoted by vector x, the choice set S encompasses n vectors. These represent the possible selections available to the consumer.

The consumer opts for the product offering the maximum utility-price ratio:

$$\text{Good } k = \text{argmax}_{i\in\{1,2\}} \frac{\Delta u_i}{p_i} \tag{41}$$

The described process is iteratively applied until the consumer has utilized their entire budget.

The micro-level captures a consumer with specific observable traits, and a Monte Carlo simulation, following the process previously detailed, can simulate the consumer's decisions. These simulations can be applied across diverse consumer groups.

In cases of extensive data and non-linear relationships, machine learning methods are apt for creating these demand functions. In this paper, we implement a Gaussian Process Interpolation model which anticipates the demand for each product, taking into consideration the prices, budget, and utility function parameters.

8.4 Simulations

In our model economy we have a total of 1000 consumers and 2 products. Every consumer is characterized by a 12 component state vector

$$(\mathrm{p}_1, \mathrm{p}_2, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4, \mathrm{prob})^{\mathrm{T}}, \tag{42}$$

where p_1 and p_2 are the prices of the 2 products, m is the money of the consumer, $\mathrm{a}_1, \mathrm{b}_1$ and $\mathrm{a}_3, \mathrm{b}_3$ determine the two utility functions of the first product, whereas $\mathrm{a}_2, \mathrm{b}_2$ and $\mathrm{a}_4, \mathrm{b}_4$ determine the two utility functions of the second product. The component prob gives the probability p $\mathrm{a}_1, \mathrm{b}_1$ and $\mathrm{a}_2, \mathrm{b}_2$ to make transition to $\mathrm{a}_3, \mathrm{b}_3$ and $\mathrm{a}_4, \mathrm{b}_4$ respectively with a Markov chain process. By following the three Step programme which follows we derive multiparametric demand functions. The first two steps are materialized with a Monte Carlo simulation.

Step 1

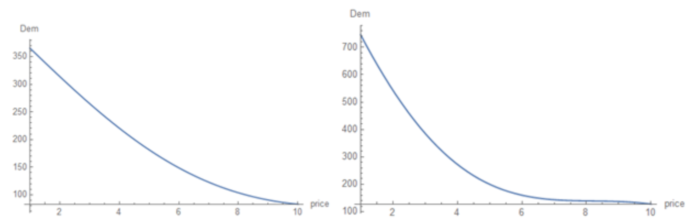
If the current bundle of products is $(\mathrm{q}_1, \mathrm{q}_2)$ the possibilities $\mathrm{A}(\mathrm{q}_1 + 1, \mathrm{q}_2)$ and $\mathrm{B}(\mathrm{q}_1, \mathrm{q}_2 + 1)$ are evaluated by first choosing a utility among u_1, u_2 via the probability selection measure (p,1-p) and calculating the ratio of marginal utility/price for the two possibilities A, B. The one with the highest ratio is selected. The trajectory of optimal consumption terminates when the income of the consumer is exhausted and the final bundle $(\overline{\mathrm{q}_1}, \overline{\mathrm{q}_2})$ defines the demand of the two products.

Step 2 An ensemble of 1000 consumers and prices is constructed. An ensemble of 1000 vectors is constructed each of which contains the attributes of the consumer and prices of the 2 goods with the use of a beta distribution so that we obtain heterogenous random consumers. For every vector which is the state vector of the consumer appended with a pair of prices of the two goods.

Step 1 is applied and a demand for the two products is calculated. This process gives rise to a data set which corresponds consumer attributes and prices of goods to demand of goods.

Step 3 For the created set of data calculated in Step 2 Gaussian Process Interpolation is applied to interpolate these data and create two demand functions, one for each product.

We give now some of the results of the simulations.



Product 1

Product 2

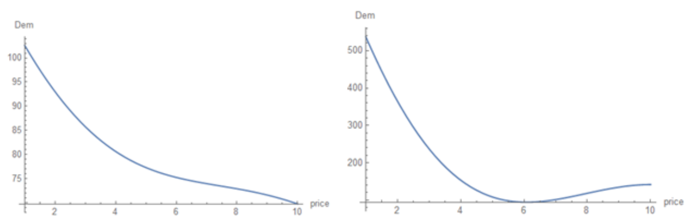
Figure 20: Demand curves for products 1 and 2 for a consumer $\mathscr{A}$.

In figure Product 1 the demand function for a consumer $\mathscr{A}$ of the product 1 as a function of its price is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_2, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4, \mathrm{prob})^{\mathrm{T}} &= \\ (10, 1899, 0.292285, 1.05871, 0.100436, 1.12235, \\ 0.321698, 1.58774, 0.496347, 1.53882, 0.176273)^T. \end{aligned} \tag{43}$$

In figure Product 2 the demand function for a consumer $\mathscr{A}$ of the product 2 as a function of its price is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4, \mathrm{prob})^{\mathrm{T}} &= \\ (2, 1899, 0.292285, 1.05871, 0.100436, 1.12235, \\ 0.321698, 1.58774, 0.496347, 1.53882, 0.176273)^T. \end{aligned} \tag{44}$$



Product 1

Product 2

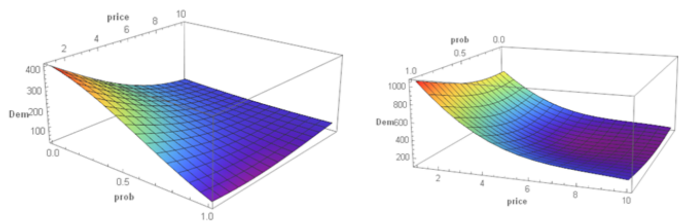
Figure 21: Demand curves for products 1 and 2 for a consumer $\mathscr{B}$.

In figure Product 1 the demand function for a consumer $\mathscr{B}$ of the product 1 as a function of its price is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_2, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4, \mathrm{prob})^{\mathrm{T}} &= \\ (7, 924, 0.205183, 1.28338, 0.410497, 1.24444, \\ 0.605611, 1.36808, 0.448373, 1.73304, 0.240323)^T. \end{aligned} \tag{45}$$

In figure Product 2 the demand function for a consumer $\mathscr{B}$ of the product 2 as a function of its price is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_1, m, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, prob})^{\mathrm{T}} &= \\ (2, 924, 0.205183, 1.28338, 0.410497, 1.24444, & \\ 0.605611, 1.36808, 0.448373, 1.73304, 0.240323)^T. & \end{aligned} \tag{46}$$



Product 1

Product 2

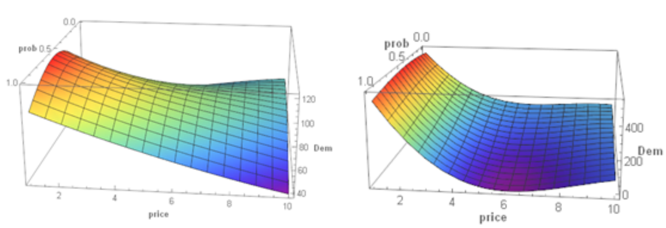
Figure 22: Demand curves for products 1 and 2 for a consumer $\mathscr{A}_{prob}$ as a function of price and probability.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{A}_{prob}$ as a function of its price and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_2, m, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4})^{\mathrm{T}} &= \\ (10, 1899, 0.292285, 1.05871, 0.100436, & \\ 1.12235, 0.321698, 1.58774, 0.496347, 1.53882)^T. & \end{aligned} \tag{47}$$

In figure Product 2 the demand function for a consumer $\mathscr{A}_{prob}$ of the product 2 as a function of its price and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_1, m, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4})^{\mathrm{T}} &= \\ (2, 1899, 0.292285, 1.05871, 0.100436, & \\ 1.12235, 0.321698, 1.58774, 0.496347, 1.53882)^T. & \end{aligned} \tag{48}$$



Product 1

Product 2

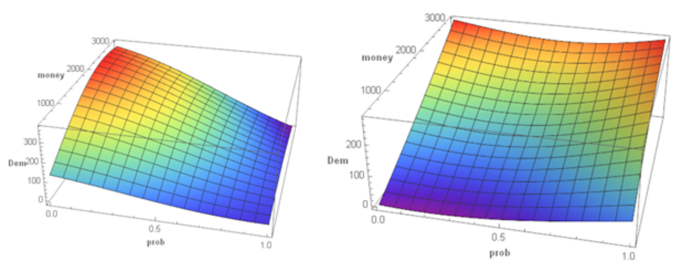
Figure 23: Demand curves for products 1 and 2 for a consumer $\mathscr{B}_{prob}$ as a function of price and probability.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{B}_{prob}$ as a function of its price and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_2, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4)^{\mathrm{T}} &= \\ (7, 924, 0.205183, 1.28338, 0.410497, & \\ 1.24444, 0.605611, 1.36808, 0.448373, 1.73304)^{T}. & \end{aligned} \tag{49}$$

In figure Product 2 the demand function for a consumer $\mathscr{B}_{prob}$ of the product 2 as a function of its price and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1, \mathrm{m}, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4)^{\mathrm{T}} &= \\ (2, 924, 0.205183, 1.28338, 0.410497, & \\ 1.24444, 0.605611, 1.36808, 0.448373, 1.73304)^{T}. & \end{aligned} \tag{50}$$



Product 1

Product 2

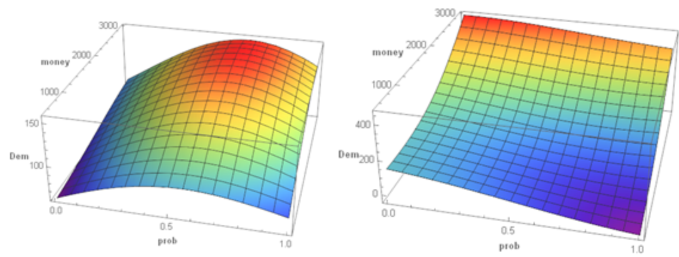
Figure 24: Demand curves for products 1 and 2 for a consumer $\mathscr{A}_{prob,money}$ as a function of probability and money.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{A}_{prob,money}$ as a function of the money of the consumer and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1, \mathrm{p}_2, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4)^{\mathrm{T}} &= \\ (2, 10, 0.292285, 1.05871, 0.100436, & \\ 1.12235, 0.321698, 1.58774, 0.496347, 1.53882)^{T}. & \end{aligned} \tag{51}$$

In figure Product 2 the demand function for a consumer $\mathscr{A}_{prob,money}$ of the product 2 as a function of the money of the consumer and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1, \mathrm{p}_2, \mathrm{a}_1, \mathrm{b}_1, \mathrm{a}_2, \mathrm{b}_2, \mathrm{a}_3, \mathrm{b}_3, \mathrm{a}_4, \mathrm{b}_4)^{\mathrm{T}} &= \\ (2, 1899, 0.292285, 1.05871, 0.100436, & \\ 1.12235, 0.321698, 1.58774, 0.496347, 1.53882)^{T}. & \end{aligned} \tag{52}$$



Product 1

Product 2

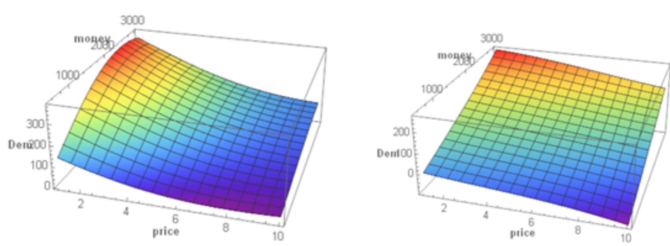
Figure 25: Demand curves for products 1 and 2 for a consumer $\mathscr{B}_{prob,money}$ as a function of money and probability.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{B}_{prob,money}$ as a function of the money of the consumer and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1,\mathrm{p}_2,\mathrm{a}_1,\mathrm{b}_1,\mathrm{a}_2,\mathrm{b}_2,\mathrm{a}_3,\mathrm{b}_3,\mathrm{a}_4,\mathrm{b}_4)^{\mathrm{T}} &= \\ (7, 924, 0.205183, 1.28338, 0.410497, \\ 1.24444, 0.605611, 1.36808, 0.448373, 1.73304)^T.\end{aligned} \tag{53}$$

In figure Product 2 the demand function for a consumer $\mathscr{B}_{prob,money}$ of the product 2 as a function of the money of the consumer and the probability of transition in the Markov chain process is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_1,\mathrm{p}_2,\mathrm{a}_1,\mathrm{b}_1,\mathrm{a}_2,\mathrm{b}_2,\mathrm{a}_3,\mathrm{b}_3,\mathrm{a}_4,\mathrm{b}_4)^{\mathrm{T}} &= \\ (2, 924, 0.205183, 1.28338, 0.410497, \\ 1.24444, 0.605611, 1.36808, 0.448373, 1.73304)^T.\end{aligned} \tag{54}$$



Product 1

Product 2

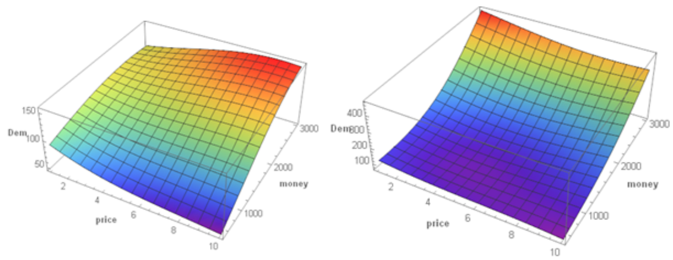
Figure 26: Demand curves for products 1 and 2 for a consumer $\mathscr{A}_{money}$ as a function of money.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{A}_{money}$ as a function of the money of the consumer is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p}_2,\mathrm{a}_1,\mathrm{b}_1,\mathrm{a}_2,\mathrm{b}_2,\mathrm{a}_3,\mathrm{b}_3,\mathrm{a}_4,\mathrm{b}_4,\mathrm{prob})^{\mathrm{T}} &= \\ 10, 0.292285, 1.05871, 0.100436, 1.12235, , \\ 0.321698, 1.58774, 0.496347, 1.53882, 0.176273)^T.\end{aligned} \tag{55}$$

In figure Product 2 the demand function for a consumer $\mathscr{A}_{money}$ of the product 2 as a function of the money of the consumer is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_1, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, prob})^{\mathrm{T}} &= \\ (2, 0.292285, 1.05871, 0.100436, 1.12235, & \\ 0.321698, 1.58774, 0.496347, 1.53882, 0.176273)^{T}. & \end{aligned} \tag{56}$$



Product 1

Product 2

Figure 27: Demand curves for products 1 and 2 for a consumer $\mathscr{B}_{money}$ as a function of money.

In figure Product 1 the demand function of the product 1 for a consumer $\mathscr{B}_{money}$ as a function of the money of the consumer is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_2, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, prob})^{\mathrm{T}} &= \\ 7, 0.205183, 1.28338, 0.410497, 1.24444, & \\ 0.605611, 1.36808, 0.448373, 1.73304, 0.240323)^{T}. & \end{aligned} \tag{57}$$

In figure Product 2 the demand function for a consumer $\mathscr{B}_{money}$ of the product 2 as a function of the money of the consumer is given. The rest of the parameters are given by

$$\begin{aligned}(\mathrm{p_1, a_1, b_1, a_2, b_2, a_3, b_3, a_4, b_4, prob})^{\mathrm{T}} &= \\ (2, 0.205183, 1.28338, 0.410497, 1.24444, & \\ 0.605611, 1.36808, 0.448373, 1.73304, 0.240323)^{T}. & \end{aligned} \tag{58}$$

8.5 Conclusions

In the study, the researchers ventured beyond classical demand theory to introduce a novel agent-based model for simulating consumer behavior in multi-product markets. This model, which accounts for 1,000 consumers, captures the heterogeneity in preferences and income, contrasting with the traditional representative agent approach.

Consumer choices in this model are serially optimal, but with "bounded rationality," differing from the strict rational assumptions of classical theory. Additionally, Markov processes were integrated to simulate consumer unpredictability over time.

The computational technique used provides dynamic insights into consumer behavior, capturing trajectories of optimal consumption and expenditure paths. Through Gaussian Process

Interpolation, the team developed multiparametric demand functions, revealing non-linear market dynamics.

This research combines machine learning with economic modeling, emphasizing the need to recognize individual consumer behavior. The findings suggest that understanding micro-level actions can help explain macro-level outcomes. Future studies could enhance this model to mirror the evolving complexities of real-world economies.

In essence, by blending classical economic theories with modern computational methods, the study offers a refined tool for decoding the intricacies of contemporary market economies.

9 An agent-based study on the dynamic distribution and firms concentration in a closed economy

In an era marked by continuous evolutions in economic theories and modeling, this study introduces a revolutionary approach, using an agent-based model, to comprehend the distribution and dynamics of firms in a closed supply economy. At its core, this research centers on an intriguing model where firms, each with its unique capital driven by a random walk process, coexist. Intriguingly, despite their independence, they're bound by shared constants like capital productivity and expenditure coefficient, which harmonize their economic actions.

One of the study's most groundbreaking observations is how even non-interacting firms can, due to stochastic fluctuations in product prices, fall into business cycles. The economy's heartbeat, it appears, is regulated by the intricate dance between universal constants and the unpredictable whims of the unit price. Through Monte Carlo simulations, the research reveals a potential drift away from homogeneity in capital distribution, sometimes even giving rise to monopolies.

The economic landscape becomes even more captivating when the lens is turned toward varied sectors. Here, the universal constants, which once seemed immutable, shift, varying from one sector to another. This variation unlocks new layers of complexity, allowing for a deeper exploration of the relation between capital distribution, capital productivity, and the unpredictable nature of unit prices.

The motivation for such an avant-garde approach to economic modeling is rooted in the 2008 global economic downturn, which underscored the pressing need for nuanced modeling methods. Agent-based models, as cited from a slew of prominent researchers including Hamill and Gilbert (2015) and Axtell and Farmer (2022), champion a break from traditional economic paradigms. These models gracefully navigate economic complexity without being tethered to notions like equilibrium or optimization.

A standout feature of agent-based modeling is its organic emergence at the aggregate level, sculpted by countless interactions amongst agents. This creates a riveting tapestry of complexity, replete with unpredictability and path-dependent phenomena. When delving into the dynamics of a market, it becomes clear that the interactions between agents and the resultant complexities are not just mere outcomes but integral to understanding the bigger picture.

As a nod to established economic theory, the research integrates the Hirschman-Herfindahl Index (HHI) – a trusted tool for gauging market concentration. Moreover, the study introduces an enhanced version, the Modified Hirschman-Herfindahl Index (MHHI), which lends an extra layer of precision by accounting for weight variations.

In sum, this paper is a testament to the potential of agent-based models, acting as a beacon for researchers aspiring to understand the intricacies of firm dynamics in closed economies.

9.1 Basic Framework

Consider a market consisting of N entities, indexed as $i = 1, 2, \dots, N$. The financial resource of the i^{th} entity at the subsequent time $t+1$ can be represented as:

$$R_{i,t+1} = R_{i,t} + (p_{i,t}\psi - e)R_{i,t}, \tag{59}$$

where ψ symbolizes the coefficient of capital efficiency, e stands for the coefficient of expenditure, and $p_{i,t}$ is the unit price set by the i^{th} entity at time t.

Equation 59 gives a concise version of a resource production function that deduces an entity's subsequent level of financial resource $R_{i,t+1}$ based on its present resource $R_{i,t}$, the price $p_{i,t}$ it assigns to its product, and its associated expenses. In essence, this equation determines the entity's next level of resource $R_{i,t+1}$ by summing its present resource $R_{i,t}$ with the net of its revenue (computed as the product's unit price times its output) and costs (given by $eR_{i,t}$).

We proceed with the assumption that $p_{i,t}$ experiences unpredictable variations and that entities function independently. Furthermore, it's assumed that:

$$\bar{p}\psi - e > 0, \tag{60}$$

where $\bar{p}$ signifies the average value of p.

From 59, we derive:

$$R_{i,t+1} = (1 + g_{i,t})R_{i,t}, \tag{61}$$

with $g_{i,t} = p_{i,t}\psi - e$. This represents the financial resource undergoing a consistent random walk with a directional tendency. The starting resources for entities are uniformly distributed between $[1000, 6000]$. The economy's behavior is dictated by the relation $g_{i,t} = p_{i,t}\psi - e$.

For a constant $g_{i,t}$ value, say g, the differential equation (61) would yield the trajectory:

$$R_{i,t+1} = R_{i,0}(1 + g)^t,$$

depicting how the i^{th} entity's financial resource evolves over time.

Upon computing the logarithm of (61), we get:

$$\log R_{i,t+1} = \log(1 + g_{i,t}) + \log R_{i,t}. \tag{62}$$

The term $g_{i,t} = p_{i,t}\psi - e$ defines the four dominant phases of the economic fluctuations among the market entities, and how the resources are distributed among them at the conclusion of a given time frame.

9.2 Simulations

We run Monte Carlo simulations for $N = 100$ firms with initial capitals homogeneously distributed in the interval $[1000, 5000]$. We depict (62) for the sum of the capitals of the firms of the economy. Every simulation is characterised by a vector $v = (v_1, v_2, v_3, v_4, v_5, v_6)$ with six components. The meaning of these components is as follows:

1. v_1 is the number of firms in the final distribution with capital greater than the 40% of the mean of the capitals of the firms.

2. v_2 is the modified Herfindahl-Hirschman Index.

3. v_3 is the logarithm of the mean of the capitals of the firms in the final distribution of the capitals of the firms.

4. v_4 is the standard deviation of the price of the unit which undergoes stochastic fluctuations.

5. $v_5 = \phi$ is the capital productivity coefficient.
6. $v_6 = c$ is the expenditure coefficient.

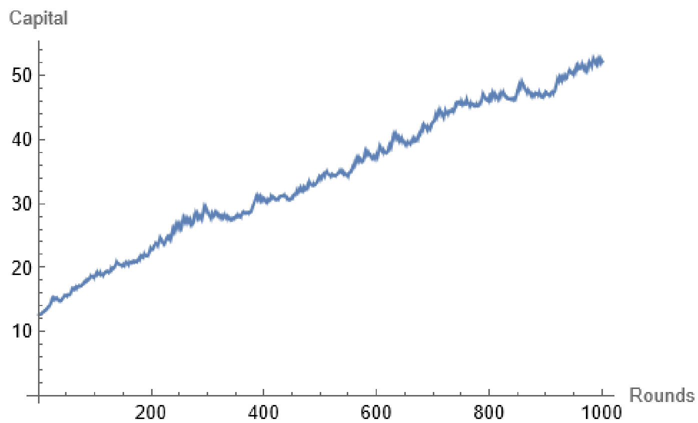


Figure 28: $(v_1, v_2, v_3, v_4, v_5, v_6) = (8, 0.702268, 47.4516, 0.895031, 0.4, 0.2)$.

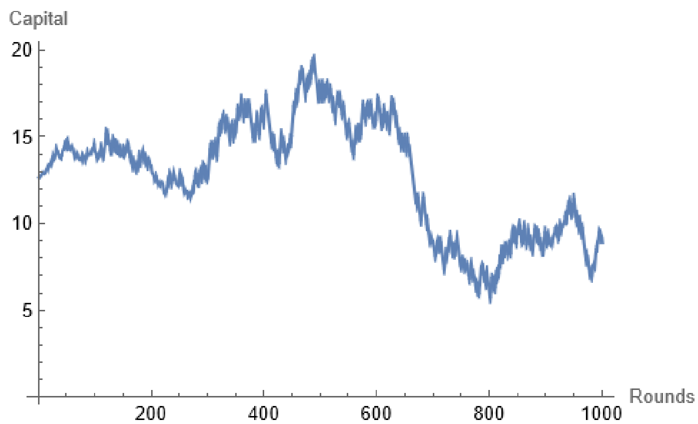


Figure 29: $(v_1, v_2, v_3, v_4, v_5, v_6) = (2, 0.622217, 4.30743, 0.887447, 0.4, 0.25)$.

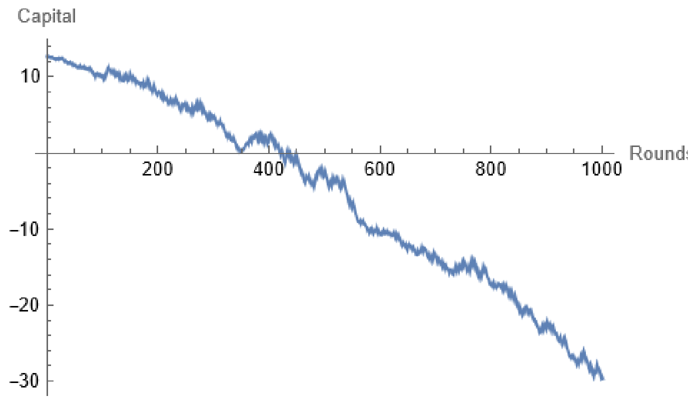


Figure 30: $(v_1, v_2, v_3, v_4, v_5, v_6) = (6, 0.779583, -34.2565, 0.792636, 0.4, 0.25)$.

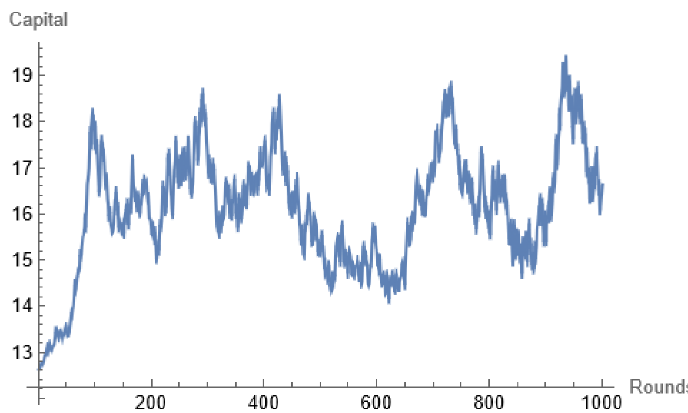


Figure 31: $(v_1, v_2, v_3, v_4, v_5, v_6) = (2, 0.982597, 12.0254, 0.766824, 0.4, 0.25)$.

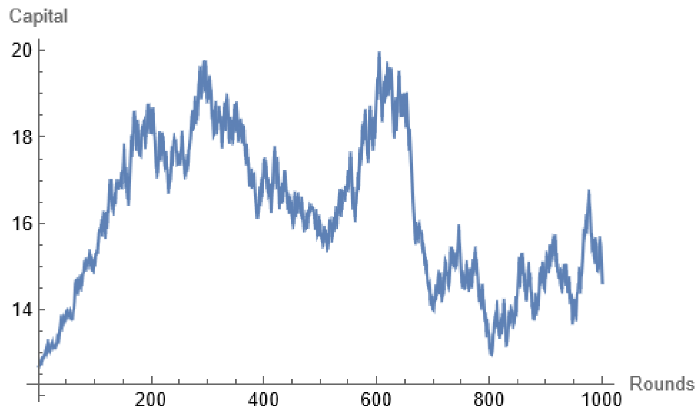


Figure 32: $(v_1, v_2, v_3, v_4, v_5, v_6) = (3, 0.652818, 10.0041, 0.712196, 0.4, 0.25)$.

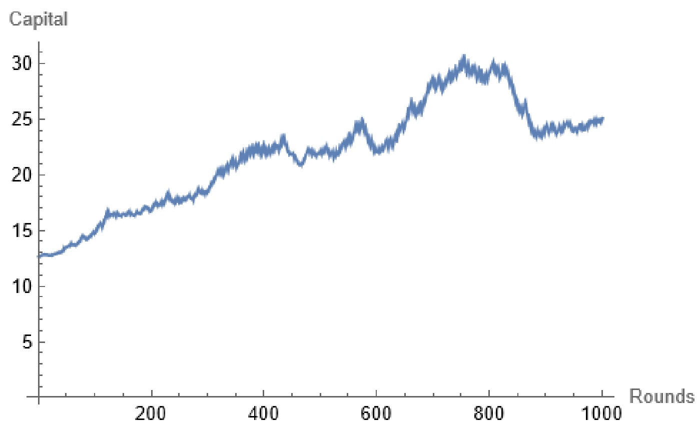


Figure 33: $(v_1, v_2, v_3, v_4, v_5, v_6) = (6, 0.634695, 20.4516, 0.602278, 0.4, 0.25)$.

9.3 Conclusions

Economic inequality, a topic of paramount importance in contemporary society, has been closely examined in our research. Through our study, we've identified an increasing trend in the uneven distribution of capital over time, echoing the alarming patterns witnessed in real-world scenarios. This accumulation and concentration of wealth, as observed, have profound implications on several facets of the economy, including market competition, the dynamics of firms, and the process of price formation. Such phenomena can, in turn, intensify the disparities in economic wealth among different segments of society.

While our research offers valuable insights into these dynamics, it is essential to approach our findings with a discerning eye. Our model, though comprehensive, provides a simplified representation of the multifaceted real-world economy. Therefore, any conclusions drawn from it should be judiciously interpreted, cross-referenced with empirical evidence, and situated within broader theoretical discussions to ensure a holistic understanding.

In wrapping up, our study adds a nuanced layer to the understanding of factors influencing market concentration, the distribution of capital, the behavior and dynamics of firms, and the mechanics of price formation. These elements play a pivotal role in molding the landscape of economic inequality. As we move forward, it's crucial for subsequent research to further untangle these intricate interrelationships, examining their broader repercussions in varied economic conditions and policy settings, thus equipping policymakers and economists with deeper, more actionable insights.

10 Simulation of Economic Market Dynamics: A Concentration and Capital Distribution Perspective

In an ever-evolving world of finance and economy, understanding the intricate relationships between variables remains a profound challenge. This study introduces a pioneering simulation model designed to decode these complexities, particularly focusing on market concentration, capital distribution, and the influential role of banking capital.

Drawing inspiration from established economic theories, this model marries foundational concepts like price formation and central banking with contemporary tools and indices. An innovative feature is its profit function, which intricately ties a firm's profitability to the ratio of its current to initial capital, effectively capturing real-time dynamics. A modified version of the Herfindahl-Hirschman Index further refines the analysis, shedding light on firm sizes and market concentration.

Executed iteratively, the model is a masterful reflection of the ebbs and flows seen in real-world markets, seamlessly adjusting to firm entries and exits. A noteworthy accomplishment is its juxtaposition with the Yard-Sale model of wealth distribution, reinforcing the robustness and versatility of our approach. The meticulous analysis offers a bird's-eye view of the cascading impact of economic parameters, painting a vivid picture of the ever-shifting terrains of capital allocation and market concentration.

For those navigating the mazes of global markets – policymakers, economists, and researchers – this paper serves as a beacon, offering unparalleled insights into the enigmatic world of market structures, wealth distribution patterns, and dynamic economic systems.

The intrigue of market dynamics has long captivated scholars and practitioners. Economically, understanding these dynamics – the intricate ballet of variables interweaving in a grand performance – is paramount. Simulations, in this regard, emerge as invaluable allies, demystifying these intricate dances by offering replicable, controlled environments for hypothesis testing and scenario exploration.

Our endeavor is anchored in the creation of an avant-garde mathematical model tailored for an in-depth exploration of capital distribution and banking capital evolution. More than just a tool, it acts as a magnifying glass, dissecting the impact of these dynamics on market concentration, offering a panoramic view of the market's heartbeat across various conditions.

At the model's core lies a meticulous blend of variables, a testament to the multifaceted nature of markets. Spanning across firm capital, bank capital, interest rates, and market indices, it walks the reader through a journey of 100 iterative stages, each echoing the previous in a domino-like fashion, true to real-world market dynamics.

While the simulation does not claim to be an all-encompassing mirror to the real world, it offers an adaptable framework, illuminating the impact of diverse economic policies, market conditions, and corporate strategies, akin to shining a torch in the dark alleys of the financial world.

In essence, this research does not lay out a one-size-fits-all answer. Rather, it presents a sophisticated tool, a compass, if you will, guiding stakeholders through the multifaceted world of economics. The insights harvested here can be instrumental in sculpting corporate blueprints, fine-tuning regulatory landscapes, and ultimately, forging more adaptive and resilient economic frameworks.

10.1 Basic framework

Our model economy is made up of a number of agents-firms whose activity is based on own capital but also on bank lending. We construct an agent-based model with discrete evolution in order to study certain aspects of market dynamics such as "market concentration" and inequality. Two basic notions we use in our model is the Profit function of the agents and the HHI and the HHIM indices. We elaborate now on these two basic notions.

We assume that the profit of the firms depends on various factors, undergoes stochastic fluctuations and is given by the following Profit function:

$$\text{Profit}(C,H,S) = \left[0.41 + 0.04 \cdot C^{\frac{1}{10}} + 0.3 \cdot H^{1.5}\right] \cdot 1.35 - \left[0.35 + \left(1 - S^3\right) \cdot R1^2\right] - \left[1.9 + R2^2\right] \cdot 0.08$$

- C: This represents the "capital" of a firm. In the context of this model, capital is a significant determinant of a firm's ability to make investments, generate revenue, and ultimately create profit. The exponent $\frac{1}{10}$ applied to the capital in the profit function might reflect diminishing returns to scale, a common assumption in economic production functions.

- H: This represents a measure of "market concentration". In this particular model, it could stand for the Herfindahl-Hirschman Index (HHI) or the Modified Herfindahl-Hirschman Index (HHIM). These indices provide a quantitative measure of market concentration and provide insight into the level of competition within the market.

- S: This represents the "size" of a specific firm relative to the market. In the model, a higher value of S implies a larger, more influential company.

- $R1$ and $R2$: These are realizations of random variables drawn from specific normal distributions, $N(0, 0.65)$ and $N(0, 0.5)$ respectively. They introduce a stochastic element into the profit function, capturing the inherent unpredictability and risk present in economic activities.

Profit(C, H, S) is calibrated so that the profit is positive most of the time; however, there is a small but not negligible probability the profit of the firm to become negative signalling bankruptcy of the firm and exit of the firm from the economy. We start with a number of firms and by following the steps which are stated in Subsection 3.2 we find the final "market concentration". We measure the "market concentration" by using the both the HHI and the HHIM indices. So a few remarks about these indices are deemed appropriate at this point.

Herfindahl-Hirschman Index (HHI):

$$\text{HHI} = \frac{\sum_{i=1}^{n} s_i^2}{(\sum_{i=1}^{n} s_i)^2}$$

The HHI is calculated by squaring the market share of each firm competing in the market and then summing the resulting numbers. The HHI takes into account the relative size and distribution of the firms in a market and approaches zero when a market is composed of a large number of firms of relatively equal size. The HHI increases both as the number of firms in the market decreases and as the disparity in size between those firms increases.

Modified Herfindahl-Hirschman Index (HHIM):

$$\text{HHIM} = \frac{\sum_{i=1}^{n} \frac{1}{n-i} \cdot s_{(i)}}{(\sum_{i=1}^{n} \frac{1}{n-i} \cdot s_{(i)})^2}$$

The HHIM modifies the original HHI by applying a weight to each firm's market share before squaring and summing them. The weight, 1/(n-i), where n is the total number of firms and i is the rank of a firm when the firms are sorted in increasing order of market shares, is applied to each firm's market share. The weight is smaller for larger firms and larger for smaller firms. This modification introduces a bias towards smaller firms, and the HHIM is sensitive to the distribution of the smaller firms in the market, even if the largest firms have a significant share of the market. This can be useful in markets where small firms play a critical role.

10.2 Simulation

Our methodology focuses on a simulation-based approach, leveraging various economic theories and concepts to study the intricate interactions between firms, banks, and market conditions. The goal of the simulation is to provide insights into how changes in economic parameters, such as total capital, market concentration, company size, and interest rates, affect capital distribution and market concentration.

10.2.1 Simulation Parameters and Variables

The simulation relies on several key parameters and variables:

- Company Capital (C): Represents the financial resources available to each firm. It is initially assigned randomly within a specified range for each company in the simulation.
- Market Concentration: Assessed using the Herfindahl-Hirschman Index (HHI) and the Modified Herfindahl-Hirschman Index (HHIM). The HHI is defined as the sum of the squares of the market shares of all firms, normalized by the square of total market share. The HHIM accounts for the market rank of each firm and is defined as the weighted sum of the squares of market shares, where the weights are the inverse of the rank, normalized by the square of the weighted total market share.
- Company Size (S): Represents the relative size of each company in the market. It is an essential factor in determining the company's influence on the market and its profitability.
- Interest Rates: They determine the cost of borrowing and are crucial for investment decisions.
- Profit: A function of the company's capital, market concentration, and company size, with stochastic elements representing economic uncertainty.

10.2.2 Simulation Model

The simulation is structured as follows:

1. Initialize the simulation by randomly assigning capital to each company within a specified range.

2. Calculate the total capital in the market (totcap) and the capital held by the bank (bankcap).

3. Determine the loan amount for each firm based on its share of total capital.

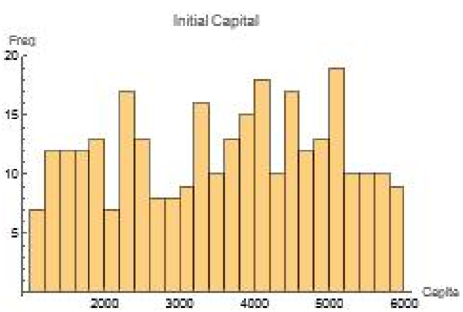
4. Compute the HHI and HHIM to assess the initial state of market concentration.

5. For each round of the simulation, calculate the profit for each company using the defined profit function. This process involves adjusting company capital, recalculating market concentration, and updating company size.

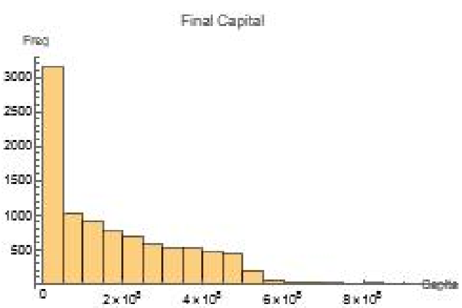
6. Iterate through the specified number of rounds, recording the state of the economy after each round.

This simulation model, combined with the detailed analysis of the resulting data, enables us to study the complex dynamics of capital distribution and market concentration in a controlled, reproducible environment. The insights derived from this model could prove instrumental in formulating more effective business strategies, informing regulatory policies, and fostering healthier, more resilient economies.

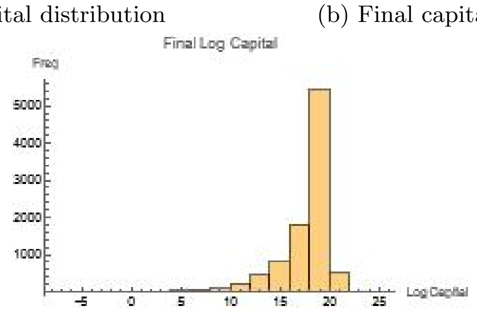
10.3 Results



(a) Initial capital distribution



(b) Final capital distribution



(c) Final (Log) capital distribution

Figure 34: Initial and final distribution of the capital in our simulated economy

The final distribution is considered by the simulation to be a mixed distribution of lognormal distributions with the following **PDF**:

$$\frac{0.304009e^{-0.85314(\log(x)-10.2261)^2}}{x}+\frac{0.047344e^{-0.0405695(\log(x)-6.80704)^2}}{x} \tag{63}$$

Economic systems are intricately complex and continuously evolving, with the distribution of wealth being one of the most fundamental elements to understand. Agent-based models act as a useful instrument for observing these changes, allowing us to explore the internal dynamics of an economy. One notable observation is the shift in capital distribution from a uniform to a geometric distribution.

In the beginning, all agents in the model possess an equal share of capital, reflecting an egalitarian society where wealth is evenly distributed among all participants. However, as time progresses, the capital distribution in the model shifts towards a "geometric type" distribution. In this situation, a small number of agents accumulate a substantial portion of the total wealth, while a large number of agents have a comparatively small share, often depicted as a long-tail curve graphically.

11 Understanding Money Distribution In Closed Economic Systems: A Graph Theoretical Approach With Quantum Random Walks

11.1 Understanding money distribution in closed economic systems: A graph theoretical approach with quantum random walks

In a closed economic framework with a constant sum of money and a set number of participants, Dragulescu, Yakovenko (2000) drew comparisons with statistical mechanics. They posited that the equilibrium money distribution in this confined environment should resemble the Boltzmann-Gibbs law from physics. This idea likens the average money per agent to an "effective temperature." This hypothesis was later corroborated through computer modeling. Other works that have explored the intersection of statistical physics and economics include Aoki (1998), Farmer (1999), and Mantegna, Stanley (2000b).

In this research, we envision the participants of the economic system as interconnected in a network, where financial exchanges occur between neighboring members. This interconnection is conceptualized as a graph, where each point symbolizes an economic player, and the connecting lines indicate potential monetary interactions. Notably, we employ Quantum Random Networks to depict the money transfer mechanisms among participants.

Historically, random walks have been employed to emulate system dynamics Knight (1921). Traditional models assume steps are independent and uniformly distributed, often leading to a Gaussian distribution over time Feller (1968). These models may fall short in representing the non-linear complexities and unpredictable nature of economic exchanges effectively. In contrast, quantum random walks, deeply embedded in quantum mechanics principles Aharonov et al. (1993), introduce a novel layer of unpredictability. Unlike their classical equivalents that advance in a deterministic direction at each interval, quantum random walks can exist in superposed states, representing several states concurrently Kempe (2003). This intricate behavior is reminiscent of the inherent unpredictability in economic transactions, suggesting a more appropriate model for such phenomena.

However, the absence of Gaussian traits in Quantum Random Walks presents challenges in their statistical analysis Venegas-Andraca (2012). These walks do not evolve into a consistent distribution. Therefore, while their incorporation in economics may shed light on uncharted aspects of system dynamics, their interpretation remains complex.

In our strategy, considering that the system doesn't evolve into a stable equilibrium, we use the Gini coefficient as a metric to assess monetary distribution among the network entities Dawkins (1980). Our primary focus is to understand the influence of the network's architecture on this distribution. A critical finding is that as the network's interconnectivity intensifies, the economic inequality, represented by the Gini coefficient, diminishes Yakovenko, Rosser (2009).

11.2 The economic network

We consider an economic network consisting of N economic agents, which may be individuals or corporations. The total number N of agents is assumed to remain fixed. Each agent i has some money, denoted by m_i, which changes with time, since the agent may exchange money with other agents. In this work, we do not assume exchanges between any pair of agents. On the contrary, any agent is allowed to have transactions only with those ones adjacent to him. In this setting, the economic network can be modelled with a directed graph $G = (V, E)$, where

- the set V of vertices is the set $\{1, 2, \ldots, N\}$ of agents;
- the set E of directed edges contains all pairs (i, j) such that, the agent i may lead money to the agent j.

In the case where any transaction is allowed between any pair of agents, then we have a so-called *complete* graph. However, in real economies, each corporation prefers to exchange money with a set of particular partners. Therefore, our assumption corresponds to a more realistic scenario.

Clearly, in each transaction the amount of money is conserved. We also assume that the economic system is closed, in the sense that there is no external flux of money. Finally, as usual, the agents are not allowed to produce new "paper" money. Consequently, the total amount of money, denoted by M, in the economic system is conserved. Clearly, $M = \sum_{i=1}^{N} m_i$.

11.3 Quantum Random Walks.

We now describe some basic framework for the Quantum Random Walk. First of all, the action in this type of walks takes place inside a Hilbert space $\mathscr{H}$, which is the tensor product of two other spaces, namely the coin space $\mathscr{H}_c$ and the position space $\mathscr{H}_p$. Hence, one has

$$\mathscr{H} = \mathscr{H}_c \otimes \mathscr{H}_p.$$

The position space $\mathscr{H}_p$ is the (infinite-dimensional) Hilbert space spanned by the position states $\{|i\rangle \ : \ i \in \mathbb{Z}\}$. We also assume that the previous basis is orthonormal. Therefore, any vector $|\psi\rangle$ in the space $\mathscr{H}_p$ has a unique decomposition $|\psi\rangle = \sum_{i\in\mathbb{Z}} c_i|i\rangle$, where c_i are the Fourier coefficients given by $c_i = \langle i|\psi\rangle$. The reader have already noticed that we adopt the Dirac bra-ket notation, which is very useful in quantum world. That is, the *ket's* $|\psi\rangle$ denote vectors in the Hilbert space (d-dimensional row vectors, if we work in $\mathscr{H} = \mathbb{C}^d$) and the *bra's* $\langle\psi|$ denote linear functionals belonging to the dual space $\mathscr{H}^*$ (the transposed d-dimensional row vector, if we are in the space $\mathbb{C}^d$. In more mathematical terminology, one could say that $\mathscr{H}_p$ is (isometrically isomorphic to) the space $\ell_2(\mathbb{Z})$.

The position space $\mathscr{H}_p$ is augmented by the coin space $\mathscr{H}_c$. In our case, the latter is the 2-dimensional space $\mathbb{C}^2$ generated by the basis vectors $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$. Following again the notation of quantum mechanics, we denote these vectors by $|\uparrow\rangle$, $|\downarrow\rangle$ respectively. So, these vectors represent the state of a spin-$\frac{1}{2}$ particle, where spin up is $|\uparrow\rangle$ and spin down is $|\downarrow\rangle$ and they act like the "flipping" of the coin.

Until now, we have only introduced the Hilbert space $\mathscr{H}$, which is the scene where the action takes place. Next, we need to describe the dynamics of the Quantum Walk, in other words, how the transition between different states is done. This transition is carried out through a unitary operator U that acts upon the whole space $\mathscr{H}$.

In order to define the operator U, we first need the Hadamard coin operator which acts on $\mathscr{H}_c$ and is written in the following matrix form

$$C = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\1 & -1\end{pmatrix}.$$

This operator acts on the basis operators $|\uparrow\rangle$ and $|\downarrow\rangle$ as follows: $C\,|\uparrow\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ and $C(|\downarrow\rangle) = \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle)$. If we make a measurement on one of these states, we see that it collapses either to $|\uparrow\rangle$ or to $|\downarrow\rangle$ with probability 1/2, thus the Hadamard coin is a balanced coin.

We also need the shift operator, denoted by S, which acts on $\mathscr{H}$ and its action is described by the relations

$$S\,(|\uparrow\rangle\otimes \mid i\rangle) = |\uparrow\rangle\otimes \mid i+1\rangle$$
$$S\,(|\downarrow\rangle\otimes \mid i\rangle) = |\downarrow\rangle\otimes \mid i-1\rangle.$$

Hence, we can say that S is a *conditional* shift that affects only the position state (it does not affect the coin space), however, its action depends on the coin state. In the entire space, the operator S can be formally defined by

$$S = |\uparrow\rangle\langle\uparrow| \otimes \sum_{i\in\mathbb{Z}} \mid i+1\rangle\langle i \mid + |\downarrow\rangle\langle\downarrow| \otimes \sum_{i\in\mathbb{Z}} \mid i-1\rangle\langle i \mid .$$

Finally, the unitary operator U, which is defined on the whole space $\mathscr{H}$ and describes the evolution of one step of the quantum walk, is given by $U = S \circ (C \otimes I)$, where I is the identity operator on $\mathscr{H}_p$ and $\circ$ denotes composition of operators.

It should be noted that, except Hadamard coin, one may utilize other unitary coin operators, which may be symmetrical or asymmetrical leading to unbiased or biased walks respectively. Furthermore, the above described framework corresponds to the Discrete Quantum walk on the line. One may also define quantum walks on graphs as well as continuous quantum walks, however, these disciplines are not addressed in this article. For more information on Quantum walks, we refer to Kempe (2003), Gram-Hansen (2014), Gută (2013), Barnett (2009) and Nielsen and Chuang (2010).

11.4 Implementation of Quantum Random Walks.

Preparing the quantum walk in the economic network, we take as initial state the vector $|\psi(0)\rangle = |\uparrow\rangle \otimes |0\rangle$ (which actually reflects the fact that no transaction of money has been made yet within the network). If we let the unitary operator U act on the initial state, we obtain:

$$|\psi(1)\rangle = U\,|\uparrow\rangle\otimes \mid 0\rangle) = \frac{1}{\sqrt{2}}\Big(\,|\uparrow\rangle\otimes \mid 1\rangle + |\downarrow\rangle\otimes \mid -1\rangle\Big).$$

Performing a measurement on $|\psi(1)\rangle$, we find the probability that the walker is at the position 1 is equal to $\frac{1}{2}$ and the probability the walker is at the position -1 is also $\frac{1}{2}$. Continuing this process of applying the operator U and then measuring and starting from the new state, we obtain the classical random walk. Therefore, because the Hadamard coin is balanced, if we measure the final state after each application of U, we obtain nothing more than the classical random walk on a line.

In order to obtain Quantum Random Walks (still with the Hadamard coin), we will not measure after each application of U. Instead, we will let the system perform several iterations. For instance, carrying out the second application of U, we get

$$|\psi(2)\rangle = U|\psi(1)\rangle = \frac{1}{2}\Big(\,|\uparrow\rangle \otimes |2\rangle + (|\uparrow\rangle + |\downarrow\rangle)|0\rangle - |\downarrow\rangle \otimes |-2\rangle\Big),$$

while for the third iteration we have

$$|\psi(3)\rangle = \frac{1}{\sqrt{2}}\Big(\ |\uparrow\rangle \otimes |3\rangle + 2\ |\uparrow\rangle \otimes |1\rangle + |\downarrow\rangle \otimes |1\rangle - |\uparrow\rangle \otimes |-1\rangle + |\downarrow\rangle \otimes |-3\rangle\Big).$$

Table 1 shows the probability of being at position i after 1, 2 and 3 steps of the quantum random walk. We observe a drift to the right. This is due to the fact that we started with the initial condition $|\uparrow\rangle \otimes |0\rangle$. If we started with the initial condition $|\downarrow\rangle \otimes |0\rangle$, then we would noticed a skew to the left.

Table 1: The probability of being at position i after 1, 2 and 3 steps with the initial condition $|\uparrow\rangle \otimes |0\rangle$.

t i	-3	-2	-1	0	1	2	3
0				1			
1			$\frac{1}{2}$		$\frac{1}{2}$		
2		$\frac{1}{4}$		$\frac{1}{2}$		$\frac{1}{4}$	
3	$\frac{1}{8}$		$\frac{1}{8}$		$\frac{5}{8}$		$\frac{1}{8}$

11.5 Mean vertex degree and money distribution

In this section, we utilize the model developed in order to study the effect of the connectivity of the economic network to the distribution of money. More specifically, we examine how the mean vertex degree of the graph affects the money distribution.

Firstly, let us recall from graph theory that the degree of a vertex is defined as the number of edges incident to it. In the context of economic networks, this magnitude indicates the number of other agents with whom a particular agent can conduct transactions. Hence, a higher mean vertex degree might suggest a more interconnected economy, while a lower mean vertex degree could suggest a more disjointed economy. The mean vertex degree is particularly useful when analyzing large networks, since it can provide insight into the overall connectivity of the network, and can sometimes be used to infer properties about the network's structure or behavior.

To apply the ideas of the previous section, we perform Monte-Carlo simulations. We consider a closed economic network consisting of $N = 100$ agents. Initially, we assume that the set of agents is uniformly distributed according to their wealth (i.e. paper money). Roughly speaking, we have the same (more or less) number of agents in the several economic categories (see the first diagram of Figure 35).

Then, we apply the Quantum Random Walk. As it is explained, if we perform measurement at each step, we simply obtain the classical random walk. To avoid this, we perform measurements after $t = 5$ steps of the random walk. At this point, one of the possible positions of the walker is selected at random. Immediately after the measurement, this selection shows the state of the system and the procedure of Quantum Random Walk is repeated. Furthermore, the position of the walker determines the transactions that will take place through the network. More precisely, for every agent, we randomly choose another agent adjacent the first one. Then, one of these agents is randomly picked to be the "winner", which implies that the other one becomes the "loser". The amount $i \cdot \Delta m$ is transferred from the loser to the winner, where Δm is determined and fixed from the beginning and i is given by the position of the walker.

The computer simulations show that in the previous model the economic network does nor reach any stationary distribution. This is a standard feature when Quantum Random Walks are implemented. Therefore, we fix a time horizon T and we perform the computer simulations for this predefined time horizon. The second diagram of Figure 35 shows one of all possible final (i.e. at the end of the time horizon) distribution of the economic agents.

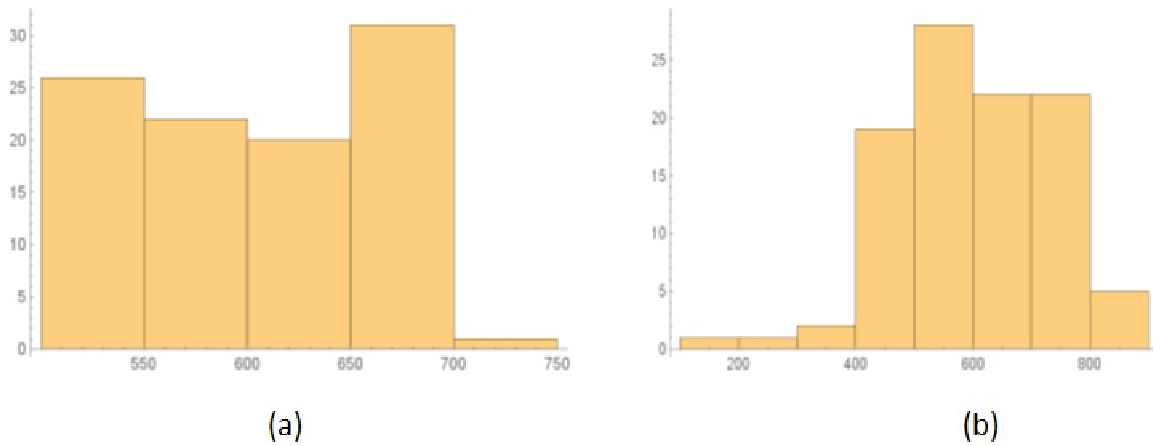


Figure 35: (a) The initial distribution. (b) One of the final distributions at the end of the time horizon.

Because of the lack of some stationary distribution, we utilize the Gini coefficient to measure the inequality of the wealth distribution among the agents in the network. As it is well-known, a value of Gini coefficient equal to 0 indicates perfect equality, while a coefficient equal to 1 implies maximum inequality.

In order to connect the mean vertex degree with the Gini coefficient, we start with an economic network and apply the Quantum Random Walk several times for the predefined horizon. Then, for each final distribution (at the end of the horizon) we calculate the corresponding Gini coefficient. Finally, we consider the average of all these calculations. In other words, for every network, we obtain the mean value of the Gini coefficients for several scenarios that follow from the implementation of the Quantum Random Walk. The result are shown in the diagram of Figure 36 and Table 2.

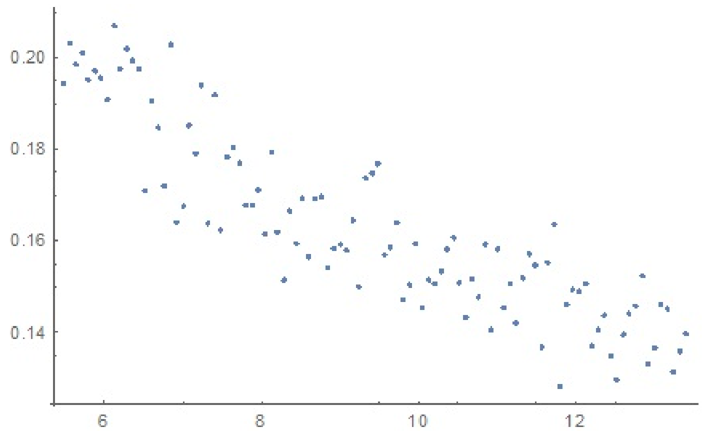


Figure 36: Terminal Gini coefficient vs mean vertex degree.

The elements of the correlation matrix in Table 36 are: Mean Vertex degree, standard deviation of vertex degree, global clustering coefficient and Gini coefficient. WE can observe that the Gini coefficient is negatively correlated to connectivity.

Table 2: Correlation matrix

1	0.841794	0.960274	-0.886377
0.841794	1	0.867775	-0.584538
0.960274	0.867775	1	-0.820892
-0.886377	-0.584538	-0.820892	1

The above results clearly show a trend that can be observed:the Gini coefficient is inversely proportional to the mean vertex degree. Therefore, as the average connectivity (mean vertex degree) in the economic network increases, the inequality (as represented by the Gini coefficient) decreases. This implies that as more connections are formed among economic agents (higher mean vertex degree), wealth is more evenly distributed. This could be because the increased connectivity allows for more transactions, possibly facilitating a more balanced wealth distribution. Conversely, if the mean vertex degree is low, indicating fewer connections among agents, wealth may be more concentrated, leading to a higher Gini coefficient. This could be because fewer transactions occur, potentially allowing wealth to accumulate with certain agents.

11.6 Conclusions

Modern economic systems are characterized by intricate complexities that often challenge the limits of traditional analytical models. As we grapple with these multifaceted dynamics, there's a growing consensus on the need for more sophisticated analytical instruments that can capture these intricacies. In this context, the integration of graph theory with quantum random walks emerges as a promising approach, offering a potentially richer and more detailed lens to study and model economic networks.

Quantum random walks are a quantum analogue to classical random walks, but they exhibit some unique characteristics. Notably, unlike their classical counterparts, quantum walks do not converge to a stationary distribution. Their movement pattern differs as well; a quantum walk, for instance, can cover a greater distance in a given time compared to a classical random walk. However, this also introduces challenges, especially when trying to discern the long-term behavior of a quantum walk.

In the scope of our research, we ventured to employ quantum walks in modeling monetary transactions within a closed economic system or network. The primary motivation was to probe whether this quantum approach could offer deeper, previously unexplored insights into the innate complexities and random nature of economic systems. Specifically, there's an anticipation that quantum-based models might unveil new emergent phenomena in economic systems, especially ones that traditional models might overlook. This could significantly enrich our comprehension of dynamics like wealth distribution.

One of the principal revelations from our study was the pivotal role the structure or topology of the economic network plays in influencing wealth distribution. When visualized as a graph, the connections and configurations within the network had a marked impact on how wealth was disseminated. It points to the exciting potential that further research in this domain holds. By delving deeper into the attributes of these graphs or by experimenting with diverse types of quantum random walks, we might unearth even more nuanced insights. Ultimately, this line of research could not only deepen our theoretical understanding but might also guide us in architecting more balanced and equitable economic frameworks.

12 Conclusion of each paper separately

The progression of modern economic study is often demarcated by its gradual shift from generalized models to more specialized and intricate ones. This shift reflects the increasing complexity and diversity of real-world economic systems. Across a range of studies conducted, some recurrent themes and methodologies emerge that illuminate our efforts to understand this complexity better.

12.1 An agent-based model of consumer choice: Some preliminary results

Our study focused on consumer choice, challenging the traditional view in classical demand theory through agent-based modeling (ABM). This approach offered a detailed analysis of consumer behavior, emphasizing the importance of individual differences and sequential decision-making in understanding consumption patterns. Heterogeneous Agent Behavior: ABM reveals the complexity of consumer behavior, highlighting individual differences and decision-making processes, which differ significantly from traditional models. Dynamic Expenditure Patterns: The model shows how consumer spending changes over time, influenced by price variations, providing a more realistic view of decision-making compared to the static approach of classical demand theory. Real-world Economic Scenario Simulation: ABM effectively replicates real economic situations, demonstrating how price changes influence consumer spending and decision timelines, offering insights into actual market dynamics. Comparison with Classical Models: ABM provides a dynamic and evolving perspective of consumer behavior, contrasting with the static nature of classical demand theory. In summary, our research demonstrates the effectiveness of ABMs in modeling consumer choice, highlighting the importance of agent heterogeneity and dynamic decision-making. ABM proves to be a valuable tool for understanding consumer behavior, offering insights for future research and economic policy-making.

12.2 An agent-based data-driven model of consumer demand

Our research aimed to model consumer demand dynamics using a novel data-driven, agent-based approach. We focused on creating demand functions for various agents and commodities, using advanced modeling techniques to handle complex market scenarios. Advanced Modeling Framework: We successfully modeled demand functions for diverse agents and commodities, showcasing the flexibility and effectiveness of our agent-based, data-driven approach. Overcoming Non-Trivial Constraints: Traditional optimization methods were unsuitable due to non-differentiability issues. We employed advanced techniques to navigate these complexities, demonstrating the capability of our approach in complex scenarios. Demand Dynamics Insights: Our methodology revealed detailed demand functions for different commodities, providing clear visualizations of how demand varies by agent type, product price, and income. Engel Curves Analysis: The Engel curves we constructed offered insights into how consumer spending varies with income, enhancing our understanding of market behavior. Random Forest Algorithm's Role: We highlighted the effectiveness of the random forest algorithm in handling nonlinear data, while also acknowledging its limitations. Broader Applicability: Our approach, though tested on self-constructed data, shows promise for real-world applications, suggesting its potential in forecasting demand based on various market factors. In summary, our study represents a significant advancement in applying agent-based, data-driven models to understand consumer demand. By overcoming complex challenges and utilizing sophisticated algorithms, we gained valuable insights into market dynamics, with implications for real-world economic analysis and forecasting.

12.3 A bounded rational agent-based model of consumer choice

This paper explores consumer behavior in a multi-product market, diverging from classical demand theory. We investigated how an agent-based model, enhanced with Gaussian Process Interpolation, can offer a detailed understanding of consumer demand amid varying preferences and incomes. Importance of Heterogeneity: The study underscores the need to consider consumer diversity in economic models, moving beyond the traditional "representative agent" approach. Emergent Market Behaviors: The agent-based model highlighted emergent, non-intuitive market patterns, challenging classical economic theories. Role of Computational Techniques: Gaussian Process Interpolation proved effective in capturing complex market dynamics, demonstrating the value of machine learning in economic analysis. Foundation for Future Research: The paper sets a groundwork for further studies, suggesting exploration into dynamic consumer preferences, a broader range of products, and real-world data validation. In summary, this research integrates computational methods with economic theories to provide new insights into consumer behavior, contributing significantly to the field of computational economics.

12.4 An agent-based study on the dynamic distribution and firms concentration in a closed economy

This research article focused on analyzing market concentration dynamics in a closed economy using a Monte Carlo simulation approach. The study examined how factors like capital productivity, expenditure coefficient, and price fluctuations affect the market structure, particularly among 100 non-interacting firms. Impact of Price and Cost Coefficients: Market concentration is heavily influenced by the relationship between price and cost coefficients, with firms having better profit margins tending to dominate. HHI as a Competition Indicator: HHI distribution is a key indicator of industry competition, with higher HHI indicating greater concentration and potential for reduced competition and efficiency. Policy Implications: The findings offer valuable insights for policymakers to develop strategies that encourage competition and market efficiency. Variable Market Concentration: Market concentration, as indicated by HHI, varies with changes in price and cost coefficients. In summary, the study provides a detailed examination of market concentration dynamics, combining empirical data with theoretical insights to enhance understanding of economic market structures and their implications.

12.5 Simulating Market Concentration and Capital Distribution: Insights into Firm Dynamics and Economic Inequality

This paper investigates how economic parameter shifts affect capital distribution and market concentration, focusing on the interactions between firms, banks, and market conditions. Key Findings: Capital Distribution Dynamics: The simulation showed a shift from equal capital distribution to a geometric-type distribution, where a few accumulate most wealth, mirroring real-world economic inequality. Impact on Economic Inequality: The study highlighted how market dynamics contribute to economic disparity, offering insights crucial for policymakers. Model Limitations: The research acknowledged the limitations of the simulation, dependent on specific assumptions and parameters. In summary, your paper effectively explores the complex dynamics of market concentration and capital distribution, providing valuable insights into economic inequality and setting a direction for future research in this field.

12.6 Understanding money distribution in closed economic systems: A graph approach with quantum random walks

This study aimed to understand money distribution in closed economic systems using a novel approach combining graph theory and quantum random walks. Conclusions: Advantages of Quantum Walks: Quantum random walks provide a sophisticated way to model economic transactions, capturing the complexities of economic systems more effectively than traditional models. Role of Network Structure: The structure and connectivity of economic networks are crucial in determining wealth distribution, with increased interconnectivity fostering more balance. Challenges and Future Directions: While quantum walks offer deep insights, their long-term behavior is complex to predict. Future research could explore other graph attributes, different quantum walk models, and real-world data comparisons. In summary, the study presents a groundbreaking approach to modeling economic systems, highlighting the importance of network structure in wealth distribution and opening avenues for further research in this field.

13 Main conclusion of thesis

The main conclusion, which encompass different studies on economic systems and consumer behavior, can be summarized as follows:

Integration of Advanced Computational Methods and Reinforcement of Economic Theories: The collective conclusion from the various studies is the pivotal role of advanced computational methods, particularly agent-based modeling, in deepening our understanding of economic systems, with respect to micro and marco computational economics. These methods, including Monte Carlo simulations and quantum random walks, offer a more intricate and realistic portrayal of economic dynamics than traditional models, thereby not only supporting but also expanding upon established economic theories.

Crucial Role of Agent-Based Modeling: Agent-based modeling emerges as a crucial tool in these studies, providing insights into individual behaviors and interactions within economic systems. This approach allows for a more granular analysis of consumer behavior, market concentration, and wealth distribution, highlighting the diversity and complexity of agent actions and decisions. It challenges the oversimplified ”representative agent” approach and underscores the importance of considering heterogeneity and dynamic interactions in economic analysis.

Enhanced Understanding of Consumer Behavior and Market Dynamics: The research consistently focuses on a deeper understanding of consumer choices and market dynamics. By simulating the behaviors of individual agents, these studies reveal the dynamic nature of market interactions and the significant impact of individual preferences and income levels on economic outcomes.

Network Structure's Influence on Economic Outcomes: A significant finding across the studies is the impact of network topology and connectivity on economic outcomes, such as wealth distribution and market concentration. This underscores the importance of considering the structural aspects of economic networks in policy-making and economic theory development.

Practical Implications and Theoretical Advancements: These studies highlight the need for economic policies and theories to incorporate the complexities and stochastic nature of real-world economic systems. The insights gained are not just academically enriching but also have practical implications for designing policies aimed at creating more equitable and competitive economic environments.

Future Research and Methodological Complexity: The research opens new avenues for exploration, emphasizing the importance of complex methodologies in furthering economic understanding. Future research could explore various computational models and validate findings against real-world data, thereby enriching and advancing economic theories.

In summary, the overarching conclusion of the thesis is the critical importance of employing advanced computational methods, especially agent-based modeling, in economic research. These methods provide a more nuanced understanding of economic phenomena, challenging and enriching traditional economic theories, and offering valuable insights for both policy-making and future academic inquiry.

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